PHYSICAL REVIEW LETTERS

VOLUME 62 | 9 JANUARY 1989 | NUMBER 2

Superconducting Cosmic Strings and Primordial Nucleosynthesis

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(Received 9 June 1988; revised manuscript received 17 October 1988)

We show that the presence of superconducting cosmic strings in the early Universe may have dramatic consequences for primordial nucleosynthesis. Due to the enormous currents that they potentially can carry, very large magnetic fields can be produced in the vicinity of such strings. As they then move through the primordial plasma, charged particles are deflected away by the magnetic pressure surrounding the strings. We show that the predicted primordial abundances can differ radically from standard big-bang predictions, and may even be consistent with an $\Omega_b = 1$ universe.

PACS numbers: 98.80.Ft, 95.30.Qd, 98.80.Cq

If they exist, superconducting cosmic strings[1] (SCS's) can play a dramatic role in cosmology.[1-6] It has been shown how they could be crucial to the formation of large-scale structure in the Universe[2] and how they can be copious producers of radio waves, γ rays, and ultrahigh-energy cosmic rays.[2-6] Indeed, it has been proposed that a SCS could readily account for a long "threadlike" radio source observed at the galactic center.[3] In this paper, we find another important consequence of SCS's for cosmology. We show that during the nucleosynthesis era of the early Universe, the strong magnetic fields surrounding SCS's as they move through the primordial plasma can perturb the relative distributions of charged and uncharged particles. Studies of a similar effect, arising from a first-order QCD phase transition,[7,8] indicate that such a perturbation can have a dramatic effect on primordial nucleosynthesis, and in fact may even reconcile observations of the light primordial isotopes with an $\Omega_b = 1$ universe.[8-10] We show that similar conclusions can be drawn from the consideration of SCS's in the early Universe based on a simple model of their properties and their interactions with the ambient plasma.

Let us first review some of the relevant parameters related to SCS's. The mass per unit length of a cosmic string is $\mu = c\eta^2/\hbar$, where η is the mass scale at which the string was produced as a result of symmetry breaking. Very long strings intersect with each other forming loops of string, which can then intersect with themselves to form even smaller loops. The loops can oscillate and lose energy in the form of gravitational or electromagnetic radiation, which in turn leads to their contraction and eventual disappearance. The ratio of the energy radiated by electromagnetic waves to that of gravitational waves is given by[2]

$$f_r \sim (I/I_c)^2 \alpha (\gamma_{\rm em}/\gamma_g)(G\mu/c^2)^{-1}, \qquad (1)$$

where the ratio $\gamma_{\rm em}/\gamma_g$ depends on the shape and trajectory of the loop,[2] α is the fine-structure constant, I is the current carried by the string, and I_c is the critical current of the string (the current at which growth is terminated by particle production). The magnitude of this current is given by $I_c = e\eta c^2/\hbar$ for bosonic strings[1] (where e is the electron charge) and a similar form, related to the mass of the charge carriers, can be found for fermionic strings.[1]

For simplicity we will assume that the cosmic strings at the onset of nucleosynthesis can possess currents of $I = I_c$. There are several scenarios which could support such an assumption. For example, one simple mechanism that can generate current in the loop is one usually discussed for strings in galactic plasmas.[3,5,11] Neglecting any dissipative effects, the motion of a loop of string with radius R is periodic with a period $T \sim R/2c$. The rms velocity of a string segment in the center-of-mass frame of the oscillating loop is $v_0 \sim c/\sqrt{2}$. References 5 and 11 discuss the generation of currents via the coupling of local segments of the string to an electric field. If a string segment has a velocity v relative to a plasma possessing a magnetic field of strength B, then it sees an electric field $E \sim vB$ (ignoring any inhomogeneities in B). Since the center-of-mass velocity of a string is small, then $v \sim v_0$. Reference 11 has shown that the coupling to such a field is resonant, and generates an alternating current which grows linearly with time to give

$$I \sim acB(v_0 \Delta t) \sim ac^2 Bt , \tag{2}$$

where we have used the approximation $\Delta t \sim t$ (strings are formed at very early times). If at the onset of nucleosynthesis ($t \sim 100$ s) we assume that a primordial magnetic field $B \sim 10^9$ G exists (a simple scaling law coupled with the present day intergalactic magnetic field,[12] or consideration of small-scale rapidly fluctuating motions in the early Universe[13] are consistent with this value of B), then Eq. (2) results in a value of $I \sim 10^{20}$ A. This is approximately $\frac{1}{10}$ of the critical current of a SCS formed at $\eta c^2 = 10^{16}$ GeV.

At a distance $r \ll R$ from the string the strength of the magnetic field generated by the current I carried by the string is

$$B_s(r) \sim 2I/cr . \tag{3}$$

The moving string and its magnetic field move the cosmic plasma and, rather like a blunt body through a fluid,[14] deflect charged particles away (a similar phenomenon is the deflection of the solar wind around the Earth). In the rest frame of the string (neglecting small effects due to nucleon dipole moments) a region completely free of charged particles arises around it[3,4] (see Fig. 1 of Ref. 3). The distance r_f from the string at which the charged-particle-free region ends can be estimated by considering the balance, at time t, between the magnetic pressure and the radiation pressure of the Universe, viz.,

$$\frac{B_s^2(r_f)}{8\pi} = K_s P_\gamma \left\{ \approx \left[\frac{10}{t^{1/2}} \right]^4 c^2 \right\}. \tag{4}$$

For weak shocks or subsonic flow, K_s is of order 1 and we will neglect it from here on (our main interest is in the order of magnitude of the effects discussed here). We can evaluate this expression to find

$$r_f \sim 10^{-3} \frac{It}{c^2} \text{ cm} . \tag{5}$$

In order to determine the significance of this effect for nucleosynthesis, we must look at the number density of strings at that time, and the relevant length scales. The maximum radius of a loop at time t is given by $R_{\max} \sim ct$, and the smallest surviving loop at this time has $R_{\min} \sim \gamma_{\rm em} \alpha ct/2\pi$ if electromagnetic losses dominate (since we assume here $I = I_c$ then electromagnetic radiation will be the most important energy-loss mechanism). The number density of loops with radius between R and $R + dR$ at time t is given by

$$\rho(R) dR \sim \frac{\lambda}{(ctR)^{3/2}} \frac{dR}{R} , \tag{6}$$

where λ is a numerical factor estimated from numerical simulations of string evolution.[15]

Since their mean free path will be small relative to r_f, neutrons will also be transported to a distance r_f from the string. However, we will show later that the neutrons rapidly diffuse back into the charge-free region left behind by the string, whereas the corresponding proton diffusion occurs on a much longer time scale. As the string moves through the plasma its passage leaves a relatively long-lived neutron-rich wake of length L_w behind it. L_w will most likely be a complicated function of R. However, we consider here that L_w is independent of R and simply assume that $L_w \sim ct$ for all strings. Although this is a good approximation for the larger loops, it will be a bad overestimate of L_w for loops with $R \ll ct$ even after accounting for the many nonplanar relativistic oscillations undergone by such small loops (note that L_w for small loops roughly equals the total summed distance traveled by a string segment). However, since at the time of nucleosynthesis $R_{\min} \sim 0.1 R_{\max}$, loops with $R \ll ct$ no longer exist and the approximation remains valid. (This is verified by another calculation in which we assume $L_w = R$ for all strings, which yields similar results.)

The volume fraction of the Universe left proton-free due to the passage of strings of radius R can be written as

$$f_v^R dR \sim 2 r_f L_w (2\pi R) \rho(R) \, dR . \tag{7}$$

After the summation over all the existing loops, the total fractional volume of the universe which is proton-free is given by

$$f_v = \int_{R_{\min}}^{R_{\max}} f_v^R dR . \tag{8}$$

Equations (5), (6), (7), and (8) with $I = I_c$ then give

$$f_v \sim 0.08 \frac{\lambda e \eta}{\hbar c (\gamma_{\rm em} \alpha)^{1/2}} . \tag{9}$$

By putting constraints on the parameters of Eq. (9) we can estimate from our simple model whether SCS's in the early Universe play a role in primordial nucleosynthesis calculations. Adopting the constraints $\gamma_{\rm em} > 2$, $\lambda < 0.05$, and $\eta c^2 < 10^{17}$ GeV (see Refs. 2, 15, and 16, respectively), we find that values of $f_v \sim 0.1$ are plausible. Since values of $f_v \gtrsim 5 \times 10^{-2}$ will have an effect on primordial nucleosynthesis, we can see that it is possible that the SCS's could indeed have a significant nucleosynthesis role (for smaller values of I and λ, f_v will be reduced accordingly and the effects on primordial nucleosynthesis will be less dramatic). It is also worthwhile to note that, at the onset of nucleosynthesis, the magnetic energy around the strings pushes against the charged particles, and does not displace the background radiation energy density. As such, the energy per unit volume required to remove the charged particles from the volume fraction f_v will be on the order of $\rho_b v_0^2 f_v$, which is much less than the kinetic energy of the string network.

Admittedly our model is extremely crude and neglects several important effects. For example, the use of a sharp boundary between the neutral- and charged-particle regions is a gross simplification to what actually occurs. Also, the interactions of SCS's with each other may be affected by their electromagnetic properties, consequently altering their further evolution. In addition, electromagnetic radiation might have the effect of increasing radiation pressure around the strings and will likely affect the temperature. Reference 2 discusses this phenomenon for strings at red shifts $z \sim 10^4$, and finds dramatic effects. If the frequency of the radiation is significantly less than the plasma frequency, the radiation cannot propagate, but instead creates large voids on the scales of superclusters. If a similar effect occurs at the time of nucleosynthesis, it will create large-scale inhomogeneities in the baryon density similar to those found in discussions of the QCD phase transition[8] (we do note, however, that the high ambient radiation pressure at the time of nucleosynthesis will seriously inhibit the formation of large voids). It is clear that a more refined calculation and a detailed determination of the properties and evolution of SCS's will be required before the importance of SCS's on perturbing charged and uncharged particles in the early Universe is more accurately known.

Before discussing the effects of such a perturbation on nucleosynthesis we show that the diffusion of protons back into the wakes will be unimportant, while the diffusion of neutrons will be very important. The time scale for diffusion can be estimated with use of the methods of Ref. 8. The characteristic time scale for proton diffusion in a plane (a reasonable approximation for the long wake structure) is $t_d = r_f^2/D_p$, where D_p is the diffusion constant for protons in the cosmic plasma. With use of the fact that the most important scattering mechanism for protons is Coulomb collisions with electrons and positrons (e^+e^- annihilation will not seriously affect D_p until after nucleosynthesis),[17] Ref. 8 gives

$$D_p = \frac{3\pi\hbar x e^{1/x}}{8\alpha^2 m_e g(x) \ln(2/\theta_0)}, \tag{10}$$

where m_e is the electron mass, $x = T/m_e c^2$, $g(x) = 1 + 2x + 2x^2$, and θ_0 is the cutoff angle due to charge screening. (We have assumed that electrons and positrons, being much lighter and faster than the protons, diffuse into the wake more quickly.) At the onset of nucleosynthesis ($t \sim 100$ s) $T \sim 10^9$ K and $\ln(2/\theta_0) \sim 5$, resulting in $D_p \sim 10^5$ cm^2 s^{-1}. Using Eqs. (5), (10), and $t_d = r_f^2/D_p$ we find t_d(protons) $\sim 10^6$ s. Since nucleosynthesis is close to completion at about 10^3 s, the effects of proton diffusion will not play any role in the process. The neutron diffusion coefficient (based on neutron-proton scattering, and ignoring the somewhat smaller electromagnetic scattering with electrons and positrons) is given by

$$D_n = \frac{653 T_e^{1/2}}{(1 - X_n)\phi\sigma_{np} T_\nu^3} \text{ cm}^2 \text{ s}^{-1}, \tag{11}$$

where X_n is the neutron mass fraction, ϕ is the present baryon-to-photon ratio, σ_{np} is the neutron-proton cross section (in fm^2), and T_e and T_ν are the electron and neutrino temperatures (in MeV), respectively. By a similar argument to that given for protons, we find t_d(neutrons) ~ 10 s. This justifies our earlier statement that the neutrons will diffuse into the charge-free region very quickly relative to the nucleosynthesis time scale.

We now briefly discuss what effect the perturbation caused by the SCS's can have on primordial nucleosynthesis calculations. The neutron mass fraction and mass density within the wake will be 1 and $\Omega_b/6$, respectively (we assume that, to first order, no protons whatsoever can penetrate into the wake), where Ω_b is the ratio of the baryon density to the critical density. In the region exterior to the wake the neutron mass density will be the same, but its mass fraction will be given by $(1 - f_v)/(6 - f_v)$. Now consider a universe in which 10% of its volume is in the form of neutral-particle wakes (i.e., $f_v = 0.1$) just before the onset of nucleosynthesis. Such conditions are somewhat similar to the possible configuration of the Universe following the cosmological QCD phase transition.[7-9] Indeed, when the nucleosynthesis, within and exterior to the neutral-particle wake, is carried out using the initial conditions given above, we recover the salient features of the nucleosynthesis[8-10] associated with the QCD phase transition. That is, for an $\Omega_b = 1$ universe the observed primordial abundances of D, ^{3}He, and ^{4}He can be reproduced, ^{7}Li, however, being overproduced by at least a factor of 5. This is of course a dramatic departure from standard big-bang nucleosynthesis[18] in which the rapid destruction of D at high densities rules out the possibility of $\Omega_b = 1$.

Compared to the nucleosynthesis which follows the QCD phase transition, however, there are two important differences in the SCS scenario which must be considered. First, in the latter scenario the diffusion of neutrons *during* the nucleosynthesis is likely to be much more important because of the different geometry and density of the proton-rich perturbation. It has been previously shown how such a late supply of neutrons can alter the D and ^{3}He abundances[19] as well as the ^{7}Li abundance.[10] Second, on account of the continuous movement of the SCS's the nucleosynthesis becomes a very dynamic process. The time t_f that a point in space-time remains a distance r_f or greater from a SCS is given by

$$\frac{1}{t_f} \sim \int_{R_{\min}}^{R_{\max}} v_0 2 r_f (2\pi R) \rho(R)\, dR . \qquad (12)$$

Using Eqs. (5) and (6), and $v_0 \sim c/\sqrt{2}$ we find that for $\eta c^2 < 10^{17}$ GeV all points in space-time are likely to experience, at least once during the nucleosynthesis, the expulsion of all charged particles from the nearby region. That is, for example, all charged particles produced initially in the neutral-particle wake will at some point be rapidly transported to a proton-rich region. Such an effect will have important consequences on the resulting nucleosynthesis.

Although we have adopted a simple model, we believe it is sufficient to show that the presence of high-current-carrying SCS's in the early Universe may well play an important role in primordial nucleosynthesis. Clearly, a detailed calculation of the nucleosynthesis will be very complicated and will require a much more quantitative study than that presented here. However, it is already evident that if such strings exist, the production of the primordial isotopes can be radically different from standard big-bang calculations.

The authors are very grateful to Roger D. Blandford, Milan B. Mijić, John P. Preskill, and Martin J. Savage for helpful discussions and comments. This work has been supported by NSF Grants No. PHY85-05682 and No. PHY86-04197 (R.A.M.), and the Natural Sciences and Engineering Research Council, Canada (M.N.B.).

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