Liang and Ding Reply: Our Letter[1] shows mainly a fact that the theory of anyons would result in fractional magnetic-flux quantization. It has been pointed out in our Letter that there indeed exists a controversy in the original description of anyons,[2] namely, the distinction between canonical and kinetic angular momentum and the singular gauge transformation by which the difference is removed.[1] The ambiguity, however, does not create serious difficulty and the singular gauge transformation is not necessary to derive anyons.[3,4] A topological phase, which gives rise naturally to the fractional angular momentum (anyon), can be obtained from the homotopy theory of path integrals in multiply connected spaces.[4]

The fractional magnetic-flux quantization predicts only a total shift of the quantized magnetic flux from the zero point. The shift is not observable by experiments in usual circumstances. In the anyon case, for lack of an applied, accessible magnetic field to superelectrons, there exists a zero-current solution which corresponds to the lowest-energy state of superelectrons. Therefore, the existence of a field-free region between the applied magnetic flux and the surrounding superconductor is a crucial condition for the test of the fractional quantum of magnetic flux and anyons. In the experiments of Tonomura *et al.*,[5] however, the Permalloy toroid was covered with the niobium layer.[6] Superelectrons can touch the magnetic field of the ferromagnet on the inner surface of the superconducting layer and the boundary condition is different from that in the anyon case. For the sake of simplicity, suppose the toroid has a round cross section. Solving Eqs. (2)–(5) in Ref. 1 up to the first-order perturbation, one obtains the density of diamagnetic, superconducting current and the induced magnetic field in a macroscopic, superconducting cylinder. The total magnetic flux is approximately quantized in the conventional way,

$$\Phi_T \sim \frac{m}{2}\Phi_0 - \frac{\lambda_L}{R}\Phi^{\text{ext}} \sim \frac{m}{2}\Phi_0 ,$$

where $\Phi^{\text{ext}} = \pi R^2 H^{\text{ext}}$ and R is the radius of the cylinder. Since the applied field is accessible to superelectrons, the fractional magnetic-flux quantization as well as anyons cannot be rejected by the result of the experiments of Tonomura *et al.* It is emphasized in the preceding Comment[7] that the top and bottom surfaces of the magnetic toroid were covered with SiO layers. In order to reduce the coercive force, the Permalloy film was, indeed, covered with SiO (500 Å thick) on the top and bottom surfaces. When the toroidal shape was cut out and sputtered with Nb, the side surfaces of the Permalloy toroid were directly exposed to the superconductor Nb.[5] It makes no difference, in principle, whether the Permalloy toroid is wholly or partly exposed to the superconductor. The condition for the test of anyons is not satisfied.

The beautiful experiments of Tonomura and co-workers[5,6] is, of course, a crucial one confirming the Aharonov-Bohm interference. Since the leakage flux from the magnet was shielded by Nb and Cu covering from the electron wave of interferometer, most physicists no longer believe that the experiments were flawed. The experiments in the present design, however, are, in principle, not able to test anyons. In our proposed experiment the field of Permalloy should be shielded, however, from the surrounding superconductor Nb. The leakage outside the Nb (2500 Å thick) and Cu ($\sim$1000 Å) covering is $\Phi_0/20$, and inside the covering should be much greater than this amount, if one realizes that the strength of the leakage field decreases rapidly from the surface of the tiny toroidal magnet. Even if one disregards the side surfaces exposed to Nb, the samples of Tonomura and co-workers are still useless for the test of anyons because of the possible great leakage field from the tiny Permalloy toroid inside the Nb and Cu shielding. In order to test anyons, we would suggest that Permalloy toroids with magnetic flux $\delta\Phi_0/2$ $(0<\delta<1)$ must be completely covered with an SiO layer, the thickness of which should be about 0.5 μm. Before the toroid is sputtered with Nb, one should measure that leakage flux is less than $\delta\Phi_0/20$.

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