

### Comment on "Fractional Angular Momentum and Magnetic-Flux Quantization"

In a recent Letter,<sup>1</sup> Liang and Ding predicted the possible fractional quantization of magnetic flux trapped within a superconductor resulting from the fractional angular momentum<sup>2</sup> of charged particles orbiting around a magnetic-flux tube. To test such a possibility, they proposed the following experiment. An infinitely long, thin solenoid is located inside the hole of a superconducting hollow cylinder to produce a magnetic flux inaccessible to the cylinder above the superconducting critical temperature  $T_c$ . Liang and Ding predicted that below  $T_c$ , the total magnetic flux  $\Phi$  trapped within the cylinder would remain unchanged even when  $\Phi$  was not an integral multiple of  $h/2e$ . They claimed that previous flux-quantization experiments<sup>3-6</sup> did not satisfy the conditions for this test, since "an axial, uniform magnetic field" applied was not localized inside the holes of the cylinders.

However, the argument to derive the fractional magnetic-flux quantization seems to involve difficult points. For example, they used a "singular gauge transformation" [Eq. (13) in Ref. 1], which was already criticized by Kobe<sup>7</sup> as invalid. This transformation is not between dynamically equivalent sets of  $(\mathbf{A}, \psi)$  but means just introducing the return flux of the infinitesimal diameter along the axis  $r=0$ . Therefore, the zero supercurrent density and zero additional magnetic flux are natural results from the cancellation of the effective flux but do not correspond to practical experimental arrangements.

In addition, the most important point is that the proposed experiment to test for the prediction has already been carried out in our experiments<sup>6,8,9</sup> confirming the Aharonov-Bohm effect. We used a tiny toroidal ferromagnet (Permalloy) rather than a straight solenoid to eliminate the return flux. Electron holography<sup>10</sup> confirmed that leakage flux from the magnet was less than 1/100 of the total flux of  $5h/e$ . The top and bottom surfaces of the magnet were then covered with evaporated SiO layers. The magnet was finally covered with a superconducting material (Nb) (see Fig. 1 in Ref. 6). In this way, the magnetic field was practically confined within the magnet, and did not leak into the surrounding superconducting layer. When the sample was cooled below  $T_c$  ( $\approx 9.2$  K), electron interferometry showed that the total magnetic flux always jumped to an integral multiple of  $h/2e$ . To take a concrete example (Fig. 12 in

Ref. 8), the total flux just above  $T_c$  (15 K) was given by  $n+0.4(h/e)$  ( $n$  denotes an integer). When the temperature decreased to below  $T_c$ , the flux suddenly changed to the nearest quantized value of  $n + \frac{1}{2}(h/e)$  at  $T=T_c$ . Thus, the absolute value of the trapped magnetic flux was confirmed in this experiment to be quantized as integral multiple of  $h/2e$  units by the supercurrent induced inside the cylindrical superconductor, even under the conditions where the magnetic flux was confirmed in the hole of superconductor.

*Note added.*—In their Reply to our Comment, Liang and Ding required a new experimental condition that there should be a space thicker than the penetration depth  $\lambda$  between the magnetic flux and the surrounding superconductor. However, this is wrong. First, the concept of the penetration depth  $\lambda$  is characteristic of superconductors and cannot be defined in vacuum space. Second, to our knowledge, it is already well established that superconductivity leads to integral-flux quantization so long as the superconductive shield is enough thicker than  $\lambda$ , whether or not the inner surface is exposed to the magnetic flux. In fact, we observed the flux quantization independently of the leakage flux value above  $T_c$ , which perhaps corresponded to the flux intensity on the surface below  $T_c$ .

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