Study of Bound Nucleons by Quasiexclusive Scattering with Large Momentum Transfer

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We describe a method for determining the four-momentum distribution of nucleons in nuclei which is particularly sensitive to large momenta. It may also prove useful in probing the differences between bound and free nucleons.

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In this paper we propose to study the nuclear wave function, especially the large-momentum component and properties of bound nucleons, by means of quasiexclusive scattering processes where the nuclear transparency phenomenon is expected to occur.[1,2] Nuclear quasiexclusive scattering is defined to be a *large-momentum-transfer* lepton-nucleon or hadron-nucleon exclusive scattering in which the target nucleon is in a nucleus and the rest of the nucleus acts as a spectator. The residual $A-1$ nucleus is required to have small excitation energy ($\ll m_\pi$) and no soft particles may be produced in the process. An example of such a process is $pA \to p'N(A-1)$, with the momentum transfer t, and also u, greater than a few GeV.[2] Since the elementary exclusive cross section at fixed t is an extremely rapidly falling function of s, most of the scattering events occur in those kinematic configurations of the target nucleon four-momentum which significantly reduce the s of the elementary reaction below its value for an on-shell target at rest, $\sim 2m_pE_{\text{beam}}$. In typical experimental conditions, as we shall see below, target nucleons have $p_{\text{lab}} \sim 1$ GeV/c in the direction of the beam. Thus the target momentum distribution could be probed with high sensitivity to large momenta if it were possible to ignore interactions of the incoming and outgoing particles in the nucleus and if binding effects could be ignored.

This simple scenario is disturbed in several ways. First, the target nucleon is not on the mass shell. We expect that the elementary exclusive cross section for given s and t in the large s, t, and u regime of interest is only weakly dependent on the hadron masses, since the successful dimensional scaling rules[3] rely on s, t, and u being the only relevant scales with dimension of mass squared. (Otherwise one would not find the dimensional scaling behavior.) However, the exclusive scattering amplitude does depend on the quark wave function of the target, which could in principle depend weakly on the nucleon mass, e.g., via m_N/Λ_{QCD}. Effects such as this are properly categorized as "binding effects" and are just the sort of phenomena one would like to probe by this technique, as we discuss in conclusion below.

Secondly, determination of s and t is, in general, not possible due to interactions of the final hadrons. In this regard, the nuclear quasiexclusive scattering we are proposing here has an advantage over other schemes such as deep-inelastic or inclusive scattering, or quasiexclusive scattering at low t or u.[4] According to perturbative QCD, exclusive scattering at large transverse momentum occurs via components of the wave functions of the participating hadrons of transverse size $O(1/t^{1/2})$. Mueller[1] has pointed out that as a consequence of this, and the dilation of the time scale in which the hadrons regain their normal size, there should be negligible nuclear absorption for sufficiently large s, t, and u. This effect has been called "color transparency" and "nuclear transparency"; we shall adopt the latter term. It is a minor generalization to note that the small-sized hadrons are not only absorbed less, but have reduced interactions of all kinds. The precise extent of their interaction and absorption depends in detail on the mechanism by which the small-transverse-size states of the hadrons return to more typical configurations. We have examined this question quantitatively,[2] using several different models of the expansion process. We find that at present energies there should be a significant reduction in the absorption and interactions due to this effect. Also, a recent experiment[5] has reported an indication that there is less absorption than would be expected without the transparency phenomenon.

Nonetheless, some correction for absorption and final-state interactions is necessary in order to infer the Fermi-motion distribution from nuclear quasiexclusive scattering experiments in the kinematic range feasible at present. This can be done by using models to describe the evolution of the small-sized hadrons, as in Ref. 2. The correctness of the models can be verified without needing to know the target momentum distribution by selecting events in which the target nucleon is at rest and near the mass shell. Since the degree of absorption depends on s and t of the elementary reaction, and on the laboratory momenta of the incoming and outgoing hadrons, an extensive set of measurements is desirable. In this connection quasiexclusive electron scattering, $eA \to e'p(A-1)$, will be invaluable, since there is only one final hadron whose development and interaction needs to be considered.

We now present the formalism relevant to this problem. Several ways of formulating this problem have been developed in terms of nonrelativistic momentum distributions and spectral functions; see, e.g., Refs. 4 and 6 for hadrons and Refs. 7 and 8 for electrons. In the large-momentum transfer kinematics considered here, however, the process develops near the light cone, so that it is natural to use nuclear wave functions quantized on the light cone as in Refs. 9 and 10. Define the light-cone spectral function, $\rho_A^N(\alpha,p_t,M_{\rm res}^2)$, to be the probability of finding a nucleon in the nucleus with light-cone momentum α,p_t (where α is the fraction of the longitudinal momentum of the nucleus carried by the nucleon and is between 0 and A), with the residual system having the invariant mass $M_{\rm res}$. It is normalized by the requirement that baryon number is conserved,

$$\int \rho_A^N(\alpha,p_t,M_{\rm res}^2)\frac{d\alpha}{\alpha}\,d^2p_t\,dM_{\rm res}^2=A\,. \tag{1}$$

Assuming perfect nuclear transparency to simplify the discussion, we write the cross section at fixed t as

$$\frac{d\sigma^{h+A\to h'+N'+(A-1)}}{dt}=\int\frac{d\alpha}{\alpha}\,d^2p_t\,dM_{\rm res}^2\,\frac{d\sigma^{h+N\to h'+N'}}{dt}(s',t)\,\theta(s'+t-2m_N^2-2m_h^2)\rho_A^N(\alpha,p_t,M_{\rm res}^2)\,, \tag{2}$$

where s' is the invariant energy squared of the $h'N'$ system:

$$s'=2E_h m_A\frac{\alpha}{A}+m_h^2+\frac{\alpha}{A}\left[m_A^2-\frac{M_{\rm res}^2+p_t^2}{1-\alpha/A}\right]. \tag{3}$$

Note that the p_t component of the momentum is not conserved in the intermediate state; i.e., $(d\sigma/dt)(s',t)$ is off the p_t shell. Although this nonconservation is small, one cannot neglect it because of the rapid variation of $d\sigma/dt$ with s. In the relevant kinematical region, the third term in the expression for s', which we denote $(\alpha/A)\overline{m}^2$, depends only weakly on the form of the nuclear recoil. In fact, two extreme models, coherent recoil and one-nucleon recoil (valid in the two-nucleon correlation approximation), give similar results. Therefore we can replaced $\overline{m}^2$ by its mean value in one of these models:

$$\overline{m}_{\rm coh}^2=(2-\alpha)\alpha m_N^2+2\alpha(m_{A-1}+m_N-m_A)m_N\,,$$

where m_{A-1} is the mass of the nuclear system $A-1$, or

$$\overline{m}_{\text{two-nucl}}^2=\frac{3-2\alpha}{1-\alpha/2}\frac{\alpha}{2}m_N^2+2\alpha(m_{A-2}+2m_N-m_A)m_N\,.$$

It is then possible to integrate Eq. (3) over $M_{\rm res}^2$ to get

$$\frac{d\sigma^{h+A\to h'+N'+(A-1)}}{dt}=\int\frac{d\alpha}{\alpha}\,dp_t^2\,\rho_A^N(\alpha,p_t)\frac{d\sigma^{h+N\to h'+N'}}{dt}(s',t)\,\theta(s'+t-2m_N^2-2m_h^2)\,.$$

For fixed t, $d\sigma/dt$ is in general a rapidly decreasing function of α. Therefore the region where $\alpha\sim\alpha_{\rm min}$ ($\alpha_{\rm min}$ is determined from the θ function in the above integration) will give a large contribution to the cross section in spite of the smallness of $\rho_A^N(\alpha,p_t)$. As a result, this reaction is sensitive to the small-α component of the nuclear wave function. It is noteworthy that in practically all other processes, including $x\gtrsim 1$ deep-inelastic scattering, cross sections are determined largely by the $\alpha>1$ component of the nuclear wave function. Thus nuclear quasiexclusive scattering is complementary to other processes such as inclusive or deep-inelastic scattering off nuclear targets in terms of the information it yields.

To illustrate the sensitivity of this technique, and demonstrate the correctness of the assertion above that the $\alpha<1$ region dominates, we take three model Fermi-motion distributions and calculate the ratio between the differential cross section on the nuclear target and the elementary cross section as a function of s' at fixed t, assuming the target nucleon is on mass shell. G_{M^p} and the elementary cross section $d\sigma^{pp}/dt$ are taken from experimental measurements[11,12]; we assumed $G_{E^p}=G_{M^p}/\mu_p$. The three model Fermi-motion distributions which we have used are the following:

(A) Simple Fermi-gas model with $n(p)=$const for $p<k_F$ and zero otherwise.

(B) Few-nucleon correlation model developed by two of the present authors.[9] If we only consider two-body correlations, the nuclear wave function is given by

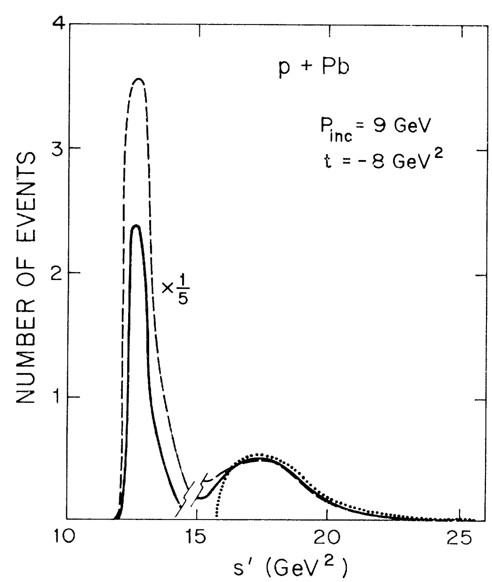

FIG. 1. Number of events as a function of s' calculated using three different models: Fermi-gas model (dotted line), Moniz's model (solid line), and the few-nucleon correlation model (dashed line).

$n(p)$ =const for $p < k_F$ and $\lambda_A \Psi_D^2(p)$ for $p > k_F$, where $\lambda_A \sim A^{0.15}$ for $A \geq 12$ and $\Psi_D^2(p) = Ce^{-\gamma p}$ with $C=0.85$ GeV^{-3} and $\gamma = 7$ GeV^{-1}.

(C) Moniz's parametrization based on calculations of nucleon-nucleon correlations in nuclear matter as given by Bodek and Ritchie.[13] Here, $n(p)=(1/c)[1-6(k_F a/\pi)^2]$ for $0<|p|<k_F$ and $n(p)=(1/c)[2R(k_F a/\pi)^2(k_F/p)^4]$ for $k_F<p<4$ GeV/c and 0 otherwise, with $a=2$ GeV/c^{-1}, $c=\frac{4}{3}\pi k_F^3$, and $R=1/[1-k_F/(4\text{ GeV})]$. α and p are related as $\alpha = 1+p_3/(m^2+p^2)^{1/2}$ and the relation between $\rho_A^N(\alpha,p_t)$ and $n(p)$ is given in Ref. 9.

The results of our calculations are presented in the figures. Figure 1 shows the distribution of the actual invariant energy of the elementary reaction s' for the three models used. The two important features to be appreciated are (i) the sensitivity of the predictions to the target momentum distribution and (ii) the fact that most events are at low s', coming from the tail of the Fermi-motion distribution, except in the Fermi-gas model which has no high momenta. The structure of the s' distribution of two peaks separated by a dip is an artifact of the sharp transition in the form of the simple models at k_F, and should probably not be taken seriously. We use the Moniz model for illustration in subsequent figures since its predictions fall between those of the other models. Figures 2 and 3 show the sensitivity of pA and eA to beam energy and to A. As one would expect, proton beams give greatest sensitivity to the large-momentum components of the nuclear wave function, because the pp elementary cross section falls more rapidly with s than do the πp or ep cross sections. The analogous figures for pion-induced reactions $\pi A \rightarrow \pi + N + (A-1)$ are not shown for reasons of space. They are less sensitive to large Fermi momenta, but nevertheless very useful to unraveling the myriad effects of transparency, binding, etc. We also see that the larger beam energy gives greater sensitivity to large target momenta.

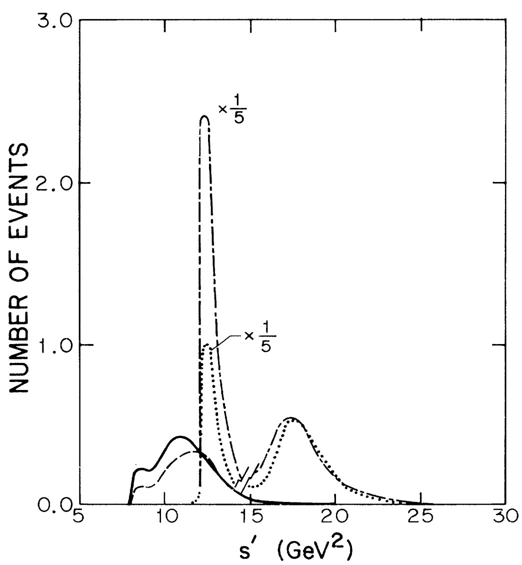

FIG. 2. Number of events as a function of s' using Moniz's model of Fermi motion for different beam energies of proton and target masses: $p_{\text{inc}}=6$ GeV, $t=-5$ GeV^2, and $A=200$ (solid line); $p_{\text{inc}}=9$ GeV, $t=-8$ GeV^2, and $A=200$ (dot-dashed line); $p_{\text{inc}}=6$ GeV, $t=-5$ GeV^2, and $A=12$ (dashed line); $p_{\text{inc}}=9$ GeV, $t=-8$ GeV^2, and $A=12$ (dotted line).

For the pA reactions at $p_{\text{inc}}=9$ GeV/c, the most probable value of the momentum of the struck nucleon is about 1 GeV/c. We an also estimate the characteristic excitation energy of the final nuclear system, as follows. In the pair-correlation approximation, the nucleon which balances the momentum of the target nucleon will be ejected with roughly the same light-cone fraction it had in the target nucleus. The excitation energy $M_{\text{res}} - M_{A-1}$ in this case is determined from the equation for the laboratory four-momentum p_R of the recoil nucleon $2-\alpha=(m^2+p^2)^{1/2}/m_N$. Since the average α for the low-s peak is about 0.5–0.7, we have

$$M_{\text{res}} - M_{A-1} \sim (m^2+p_R^2)^{1/2} - m \sim 100 \text{ MeV}$$

in this peak.

Including the effects of incomplete transparency would modify the predicted shape of the s' distribution in several ways. It would favor scattering by nucleons near the nuclear surface, where the nuclear density is smaller. Since both λ_A and k_F increase with average nuclear density, this effect narrows the s' distribution.[2] Furthermore, events which have low values of s', such that u is less than some $u_{\min}$ ($u_{\min} \sim 2$–4 GeV^2), are not expected to take place in small-transverse-size configurations, so for them the standard Glauber description should hold. This has the effect of cutting off the s' distribution presented above, which was obtained assuming complete transparency, at some value of $s_{\min}$ corresponding to $u_{\min}$. There exist still other events in which p_{lab} of the recoiling nucleon is small, due to the difference between

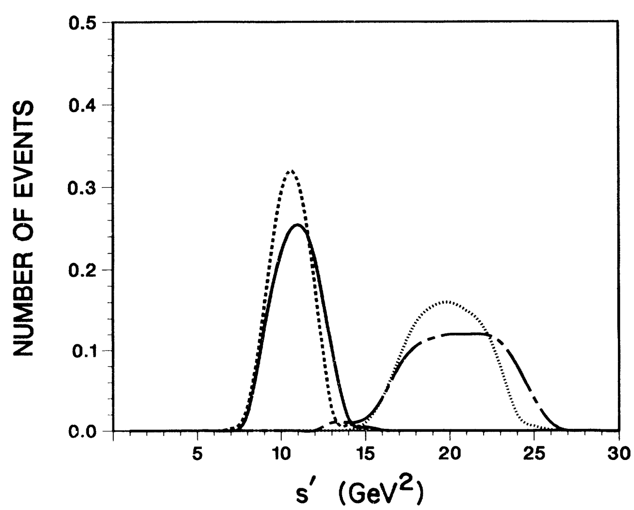

FIG. 3. Same as for Fig. 2, but for electron beam.

the nucleon frame and the laboratory frame, so that its lifetime in a small-size configuration is small even though s' and u are big enough that the actual event is legitimately described as having small transverse size. However, events of this type make a negligible contribution since they necessarily have s' larger than the nominal value so that it is not necessary to correct for their greater absorption.

In the discussion above we have ignored the differences between the properties of bound and free nucleons. However, models accounting for the European Muon Collaboration (EMC) effect generally have important consequences for the s' dependence of nuclear quasiexclusive scattering. We intend to discuss this in greater detail in another publication, but note the conclusions briefly here.

(1) Models in which bound nucleons swell (Q^2 rescaling models, etc.): Here the quasiexclusive cross section is expected to be suppressed, due to the reduction of the normalization of the short-distance part of the target nucleon wave function since $r_N^*/r_N - 1 \sim 0.1$–0.2. This effect imitates absorption for light nuclei, but has a considerably weaker A dependence than nuclear transparency, since in these models $r_N^*/r_N - 1 \propto \langle \rho_A \rangle$, the average nuclear density.

(2) Pion model (x rescaling)[14]: In this model the EMC effect is interpreted as being due to the depletion of nucleon light-cone fractions, which is assumed to be approximately homogeneous: $\alpha \to \alpha(1-\eta)$ where $\eta_{\rm Fe} \sim 0.05$. The main effect here would be to shift the maximum in the lower s' peak to larger s'.

(3) "Minidelocalization" model of two of the present authors[10]: Here the probability of pointlike configurations in a bound nucleon with momentum p is suppressed by the factor $\delta(p) = (1+x/2)^{-2}$, where $x = 4(\epsilon_A + p^2/2m_N)/\Delta E_A$ and ϵ_A is the binding energy per nucleon (~ 8 MeV for $A \gg 1$). Fits to the EMC effect lead to $\Delta E_A \sim 0.6$–0.8 GeV, the characteristic energy excitation for pointlike configurations. This implies that the height of the low-s' peak should decrease by at least a factor of 2 while the peak at $s \sim s_0$ is suppressed very little. To distinguish this model from a Fermi-motion model with a more steeply decreasing wave function, one should study $h + d \to h + p + n$ for nucleon spectator momenta $0.2 < p < 0.5$ GeV/c: For these momenta, the deuteron wave function is known reasonably well ($\sim 10\%$ for $p \sim 0.2$ GeV/c, $\sim 30\%$ for $p \sim 0.5$ GeV/c) and $\delta(p)$ changes the cross section by a factor of ~ 3 for $p \sim 0.5$ GeV/c. Note that for $p \gtrsim 0.3$ GeV/c, the average internucleon distances in the deuteron are $\lesssim 1.2$ fm, so the nuclear transparency condition is easy to satisfy. Thus according to the minidelocalization model, this process is an alternative probe of the dominance of small-transverse-size hadrons in large-momentum-transfer exclusive scattering.

(4) Multiquark ($6q$, $9q$...) models[15]: In these models it is assumed that at small internucleon distances, i.e., large Fermi momenta, nucleons form six-quark bags with probability $P \sim 20\%$ (30%) for $A \sim 60$ (200). Consequently, these models lead to the suppression of the small-s' peak by a factor $1-P$, since the cross section of the reaction $h + (6q) \to h + N + N$ is suppressed due to the small overlap of the $6q$ and two-nucleon configurations. There is no effect on the $s' \sim s_0$ peak.

In conclusion, we have shown that nuclear quasiexclusive scattering is a very sensitive probe of the large-momentum component of the nuclear wave function, and in some models to the effects of binding on the properties of nucleons. Thanks to the nuclear transparency effect expected for large-momentum-transfer quasiexclusive scattering, initial- and final-state interactions should be less troublesome than in most other methods of obtaining such information. Furthermore, the $\alpha < 1$ part of the nuclear wave function is accessible, making this approach complementary to deep-inelastic and other techniques. Interpreting the results will be nontrivial, requiring an extensive set of data taken at low Fermi momenta in order to unfold the effects of absorption. For these purposes electron- as well as hadron-induced reactions are crucial. In general, it is difficult to separate effects of binding from effects of Fermi momentum. While experiments such as these will permit study of an important new domain of nuclear physics, a great deal of theoretical as well as experimental work will be necessary to fully understand the results.

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