Computer Simulation of Chiral-Symmetry Breaking in (2+1)-Dimensional QED with N Flavors

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Noncompact quantum electrodynamics in three Euclidean dimensions with N species of four-component Dirac fermions is simulated by lattice-gauge-theory techniques. On an 8^3 lattice we find chiral-symmetry breaking in the continuum limit for $N \le N_c$ but no symmetry breaking for $N \ge N_c$, with $N_c = 3.5 \pm 0.5$. A physical picture of our results is presented.

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Quantum electrodynamics in 2+1 dimensions (QED_3) is a surprisingly interesting and relevant field theory. It is often mentioned in high-energy-theory applications as an example of a model with chiral-symmetry breaking. In condensed-matter physics, it has been shown that there is an interesting relation between QED_3 [also $SU(2)_3$] and models of strongly interacting fermions that are good candidates to describe the new high-T_c superconductors.[1] Although the theory is superrenormalizable (the coupling e^2 has dimensions of mass), it has many similarities with four-dimensional field theories with dynamical-symmetry breaking.

The Lagrangian of QED_3 in the continuum is given by

$$L = -\tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} + \sum_{j=1}^{N} \bar{\chi}^j \gamma^\mu (i\partial_\mu - eA_\mu)\chi^j , \qquad (1)$$

where we have included N species of massless fermions. QED_3 has two main formulations: (i) If two-component spinors are used then the Clifford algebra is satisfied by the 2×2 Pauli matrices. The theory has no chiral symmetry and the mass term breaks parity explicitly. Interesting phenomena exist in this formulation because a mass for the photon is dynamically generated through a Chern-Simons term.[2] (ii) We can also consider spinors with four components. In this case there is a chiral symmetry in the model, as in four dimensions, since we have a γ^5 matrix.[3] For this reason we concentrate on the four-component model. The mass term does not break parity but only chiral symmetry as usual.

We report here a numerical study of the four-component theory which uses the lattice techniques recently applied to the more difficult problem of the existence of multiflavor QED in 3+1 dimensions.[4] Our simulations on relatively small lattices (6^3, 8^3, and limited data from 10^3 systems) indicate that QED_3 with N Dirac fermions breaks chiral symmetry only for a sufficiently small number of dynamical fermions with a critical number of fermions $N_c = 3.5 \pm 0.5$. For $N \le N_c$, measurements of the chiral condensate $\langle\bar{\chi}\chi\rangle$ indicate symmetry breaking in the continuum limit of the lattice theory. However, for $N \ge N_c$ no symmetry breaking is found.

There are some analytic studies of this model in the framework of the Schwinger-Dyson equations combined with the $1/N$ expansion. For some time it was believed that QED_3 had chiral-symmetry breaking for a large number of flavors although with a dynamical mass exponentially small with N.[5] We found no evidence of such behavior.

In fact it is interesting that our new numerical results are quantitatively similar to a recently revised[6] study of the continuum model in a more carefully analyzed $1/N$ expansion, where no chiral-symmetry breaking was found for large N. This study also illustrates the new fermion algorithms developed for lattice QCD.[7]

The lattice (Euclidean) version of this model using staggered fermions is given by the action

$$S = -\tfrac{1}{2}\beta \sum_p \theta_p^2 + \sum_{j=1}^{N} \sum_{x,y} \bar{\psi}_x^j M_{x,y} \psi_y^j , \qquad (2)$$

where M is the Dirac operator on the lattice defined as

$$M_{x,y} = \tfrac{1}{2} \sum_\mu \eta_{x,\mu} [e^{i\theta_{x,\mu}} \delta_{y,x+\mu} - e^{-i\theta_{y,\mu}} \delta_{y,x-\mu}] + m\delta_{x,y} , \qquad (3)$$

where x, μ, and p denote sites, directions, and plaquettes, respectively, of a three-dimensional cubic lattice. $\theta_{x,\mu}$ are dimensionless fields on the links of the lattice that are proportional to the gauge fields through the relation $\theta_{x,\mu} = eaA_\mu(x)$, where a is the lattice spacing which acts as an ultraviolet cutoff. $\bar{\psi}_x$ and ψ_x are (dimensionless) one-component Grassmann variables on sites (i.e., to reduce species doubling in the continuum we use staggered fermions). The lattice fermionic fields are related with their continuum counterparts through $\psi_x \to a\chi_x$. The rest of the notation is standard. A careful analysis of the naive continuum limit has been done in Ref. 8 showing that the continuum fermions coming from Eq. (2) correspond to the four-component theory.[9]

The dimensionless coupling constant β of the lattice action is related to the charge by $\beta = 1/e^2 a$. Then, the continuum limit of the model is recovered at $\beta \to \infty$ if we assume that e^2, which sets the scale of the theory, is finite.

Note that in Eq. (2) we use the noncompact formulation of lattice QED_3. This formulation proved to be very successful in the study of QED_4,[4] while the compact formulation had first-order transitions that prevent a nontrivial continuum limit. So it is natural to use the noncompact action also in this case although it would be interesting to know which of our conclusions apply to the compact case.[10]

Our numerical method is the hybrid technique with noisy fermions. Details of the method applied to QED have been presented in Ref. 11 for the compact case but its modification to study the noncompact action is trivial. Here we just mention some details about the counting of fermionic species in the continuum. After the integration of the fermions in the path integral we need to simulate an effective action given by

$$S = -\tfrac{1}{2}\beta \sum_p \theta_p^2 + \tfrac{1}{2} N \ln[\det(MM^\dagger)_{\text{even}}] , \tag{4}$$

where a factor $\frac{1}{2}N$ (instead of N) has been introduced in front of the determinant because the operator M represents two species of four-component fermions in the continuum limit.[8,9] Besides, the operator M in the determinant has been replaced by $MM^\dagger$ to have a positive-definite operator. The restriction to work over only half of the lattice (even sites) prevents additional fermion doubling. (Note that in the introduction of the factor N in front of the determinant we are assuming an analytic behavior in that variable. However, for N multiple of 2 our lattice Lagrangian is local and the algorithm requires no new assumptions. So our final estimate of N_c may depend on the continuity assumption but our qualitative results do not.)

Using the hybrid algorithm we measured local observables such as the plaquette and the chiral condensate on 6^3, 8^3, and 10^3 lattices. Our most accurate data were taken on the 8^3 lattices and will be discussed here. A larger paper which analyzes finite-size effects on the 6^3, 8^3, and 10^3 lattices is in preparation.[12] We simulated the theory with $N=0$ (quenched), 1, 2, 3, 4, and 5 Dirac flavors with bare fermion masses of $m=0.050$ and 0.025 at β values ranging from 0 to 1. The chiral limit of the condensate $\langle\bar\psi\psi\rangle$ was taken at each β by extrapolating the finite-mass results to $m=0$. The linear dependence of $\langle\bar\psi\psi\rangle$ in m was checked at several points by accumulating data at $m=0.0375$ in addition to $m=0.050$ and 0.025. To achieve the statistical accuracy needed to obtain quantitative results the hybrid algorithm was run with a time step $dt=0.025$ for 10^5 to 5.0×10^5 sweeps at each β and m value. Runs of 10^6 sweeps were made at several β and m values to check our statistical analyses.

In Fig. 1 we show the $\langle\bar\psi\psi\rangle$ (extrapolated to massless quarks) vs β curves for $N=0$, 1, 2, 3, 4, and 5. Note the "tails" in the $N=0$, 1, and 2 curves which extend to

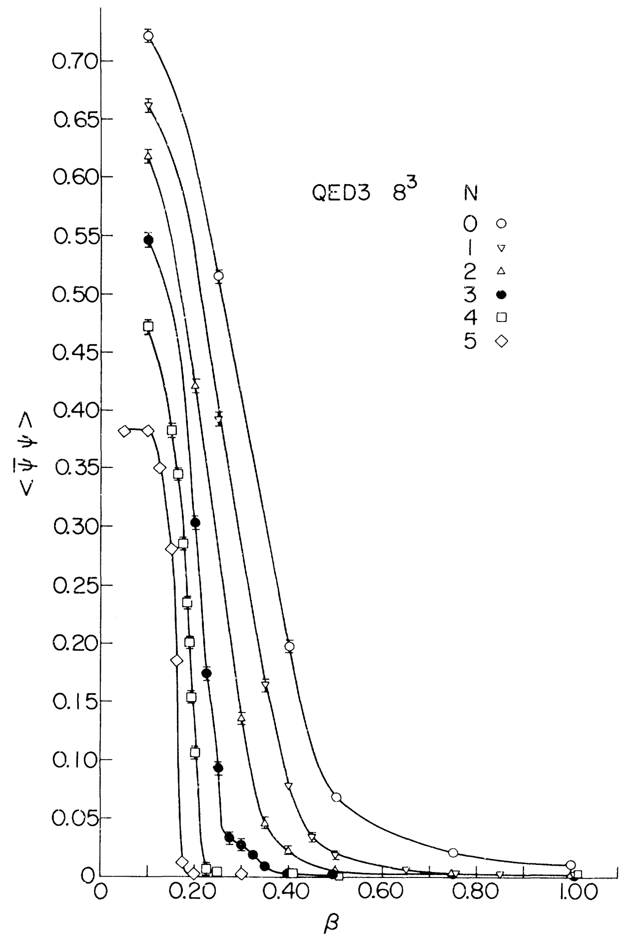

FIG. 1. $\langle\bar\psi\psi\rangle$ vs $\beta=1/e^2a$ for QED_3 on an 8^3 lattice for $N=0$, 1, 2, 3, 4, and 5.

weak coupling. These curves are smooth from strong coupling where chiral symmetry is broken (analytic methods such as the strong coupling expansions or mean-field theory on the lattice prove this point at least for the compact formulation) to the weakest coupling we can probe. So, in these cases there appear to be no nonanalyticities in $\langle\bar\psi\psi\rangle$ as we pass from strong to weak coupling. Therefore, the theory is predicted to reside in only one phase where chiral symmetry is broken.

From dimensional analysis it follows that the chiral condensate should behave like $\langle\bar\psi\psi\rangle = A\beta^{-2}$ near the continuum limit. If the constant A is nonzero we have chiral-symmetry breaking. In Fig. 2 we check this scaling law that allows us to extract the chiral condensate of the continuum theory. For the $N=0$ (quenched) theory the scaling window is easily reached and is quite broad. Although we cannot take β literally to infinity to extract the continuum limit, the plateau in the $\beta^2\langle\bar\psi\psi\rangle$ vs β curve, which extends from $\beta > 0.75$ to 2.00, is quite clear. For β larger than 2.0, $\langle\bar\psi\psi\rangle$ becomes very small and difficult to measure accurately by a statistical algo-

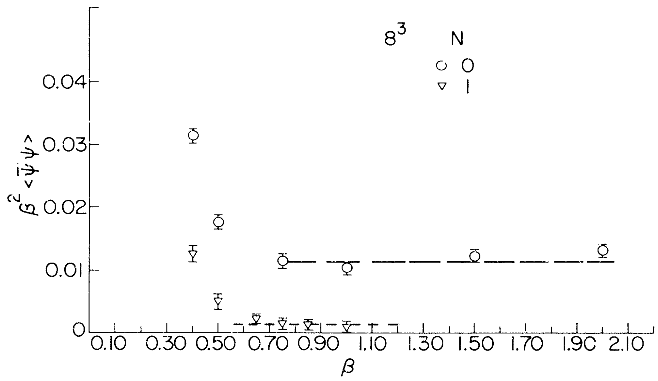

FIG. 2. $\beta^2\langle\bar\psi\psi\rangle$ vs β for $N=0$ and 1. The dashed lines are visual estimates of the chiral condensates of the continuum theory.

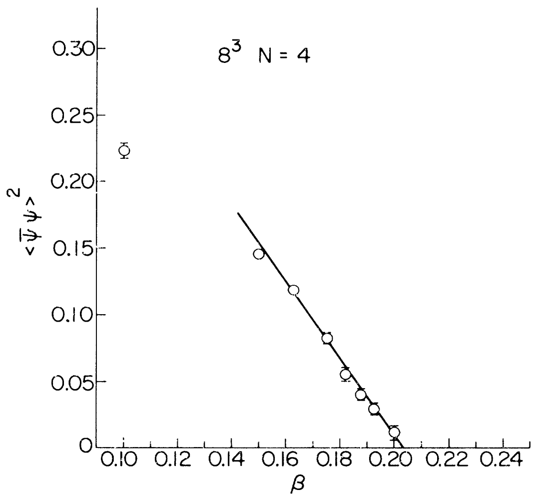

FIG. 3. $\langle\bar\psi\psi\rangle^2$ vs β data from Fig. 1 for the $N=4$ theory.

rithm. A similar plateau in the $N=1$ theory is also shown in Fig. 2. Because of screening, the chiral condensate is suppressed but $\beta^2\langle\bar\psi\psi\rangle$ appears to have a nonzero plateau although a study with a bigger lattice is necessary to clarify this point (in preparation). The character of the data did not change qualitatively for $N=2$. By contrast, the $N=4$ and 5 data show clear nonanalytic behavior. Inspecting Fig. 1 for $N=4$ we see that $\langle\bar\psi\psi\rangle$ is essentially zero for $\beta>0.25$ but is nonzero for stronger coupling. The nonanalytic character of the curve is emphasized in Fig. 3 which shows $\langle\bar\psi\psi\rangle^2$ vs β. Clearly the data are well fitted by a mean-field behavior,

$$\langle\bar\psi\psi\rangle\sim(\beta-\beta_c)^{1/2}, \tag{5}$$

with $\beta_c=0.204\pm0.001$ (remember that the order-parameter critical exponent is predicted by mean-field theory to be half independent of the dimension of the system). This is a theoretically satisfying result. In fact, we expected either a first-order or a mean-field chiral transition in those cases where these models have a transition at finite β. Such phase transitions would *not* lead to an interacting relativistic field theory in the strongly cut-off lattice model (although we cannot exclude the presence of logarithmic corrections that can make the continuum theory nontrivial). Finding an interacting continuum model at such a point would be very puzzling indeed. The $N=5$ data are qualitatively similar to the $N=4$ case. In this case $\langle\bar\psi\psi\rangle=0$ within statistical errors for each β value of 0.175 and larger. The $N=3$ curve may also be nonanalytic but this case is not as clear. Perhaps N_c is quite close to 3.

Can we develop a physical picture and a quantitative calculational technique to assimilate these numerical results? Models of chiral-symmetry breaking in two- and four-dimensional quantum chromodynamics show how the long-distance attraction due to flux-tube formation leads to a negative self-mass for constituent quarks. This effect leads to a chiral condensate and the existence of a multiplet of Goldstone pions.[13] We found, by performing the same calculations as in Ref. 13, that such a physical picture can be made for QED_3 in the quenched, $N=0$ limit, and that the inclusion of N species of dynamical fermion can be seen to generate a finite N_c.

Consider the energetics of pair condensation. Following Amer *et al.*,[13] it is easy to show that the logarithmically confining the Coulomb potential in two-space dimensions leads to an infrared singular self-energy for the electrons. For low momenta ($p\ll e^2$) the energy of an e^+e^- composite

$$E\approx 2p-(e^2/2\pi)\ln(e^2/p)-(e^2/2\pi)\ln(1/e^2r)$$

can be made negative, although both kinetic and potential energies are positive. Here, $-(e^2/2\pi)\ln(e^2/p)$ is the electron self-energy calculated in the ladder approximation, and $-(e^2/2\pi)\ln(1/e^2r)$ is the bare potential. As a consequence, the massless vacuum is destabilized and the condensation occurs at low momenta ($p\ll e^2$).

For $N>0$ the presence of virtual fermions makes the physics of chiral-symmetry breaking different in two ways. The general form of the full photon propagator is $D^{-1}(k)=k^2+e^2Nf(k)$. Using gauge invariance and dimensional analysis, we conclude that $f(k)$ vanishes linearly with k. At large distances ($r\gg 1/Ne^2$), the long-range static potential between the charges is partially screened from $(e^2/2\pi)\ln(e^2r)$ to $-\text{const}/Nr$. The precise value of the constant can be determined from the exact form of the vacuum polarization tensor. When the propagator corrections are well approximated by the one-loop contribution, the value of the constant is $4/\pi$.[5] The physical reason for the softer infrared behavior is easily understood. If we fix two charges at large separation, they will interact strongly since the bare interaction is confining. Virtual pairs from the vacuum will tend to neutralize the test charges, thus diminishing the effective strength of the interaction. The two immediate consequences are that the self-energy is softened in the infrared and the interaction is negative in that region. Therefore, as N increases from zero, the effect of the negative self-energy becomes less important and the condensation is driven by the $1/Nr$ attraction. Now, the energetics resemble four-dimensional QED with supercritical coupling.[14] In that case the energy of an e^+e^- pair is estimated from the uncertainty principle: $E=2p-\alpha/r \approx (1-\alpha)/r$, where α is the four-dimensional coupling constant. As long as the coupling is above some critical value, the condensation is energetically favored. The reason for the existence of the critical N is then clear from this analogy: The four-dimensional coupling is replaced with const/N, and chiral symmetry is broken only if this coupling is supercritical, i.e., $N < N_c$.

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