Aharonov-Bohm Interaction of Cosmic Strings with Matter

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It is shown that generically cosmic strings will interact with matter via their pure gauge potential. The cross section is purely geometrical, and does not vanish in the limit of zero string size. The mechanism is the Aharonov-Bohm effect, which is already known to cause scattering of charged particles off an infinitesimally thin solenoid. A related mechanism for particle production is also discussed.

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1. Introduction.— In a famous paper,[1] Aharonov and Bohm emphasized that the vector potential, which in classical physics is merely a convenient parametrization of the field strength, takes on independent physical meaning in quantum mechanics. A particularly clear illustration of this is the example discussed in their paper, the case of a charged-particle scattering off an infinitesimally thin solenoid. The field strength is zero everywhere outside the solenoid, and (as is easily shown) the wave function of the particle vanishes at the solenoid, so the particle is never located in a region of nonvanishing field strength. Classically no force acts upon it. Yet the particle does scatter, with a cross section per unit length

$$\frac{d\sigma}{d\theta}=\frac{\sin^2(\pi\alpha)}{2\pi k\sin^2(\theta/2)}, \tag{1.1}$$

where $\alpha=\Phi/\Phi_0=(e/2\pi)\Phi$ is the ratio of actual flux Φ through the solenoid to the basic flux unit $\Phi_0=2\pi/e$ associated with the charge e of the particle, k is the momentum in the plane perpendicular to the string, and θ is the scattering angle. Note that this cross section is nonvanishing as long as α is not integral, that it has a simple universal form, and that it diverges both at low energy and for forward scattering.

In the interaction of cosmic strings with matter, essentially the same situation arises. Outside a tiny core region we have vanishing field strengths, but nonvanishing vector potentials. These arise in a unified gauge theory with gauge group G, coupling constant g, with a scalar field φ which acquires a vacuum expectation value v (breaking $G\rightarrow H$) and winds around the z axis once, i.e., as $r\rightarrow\infty$, $\langle\varphi(r,\phi)\rangle\rightarrow\exp(i\phi\kappa R_\varphi)v$. R is some broken generator of the gauge group, and the scalar φ couples to the corresponding gauge boson field A^μ with charge gR_φ. We choose κ so that $\exp(i2\pi\kappa R_\varphi)$ is in a disconnected component of H. The line integral $\oint\mathbf{A}\cdot d\mathbf{l}$ gives the enclosed flux $\Phi=2\pi/gR_\varphi$. For a fermion ψ that couples to A^μ with strength gR_ψ

$$\alpha=R_\psi/R_\varphi, \tag{1.2}$$

and so there may be a gauge interaction with the string just as with a solenoid. The vector potential is not necessarily associated with ordinary electromagnetism (in fact the gauge boson may have a large mass), and we will use relativistic kinematics, but the essence remains. In this Letter we shall demonstrate that the generalized Aharonov-Bohm effect gives in some sense the dominant interaction between matter and cosmic strings, and in particular leads to scattering cross sections and to particle production rates that do not go to zero as the geometrical size of the string goes to zero.

2. Example grand unified theory.— To give our consideration a definite context we will calculate the effective values of α for various particles in the presence of one particular cosmic string, the usual one contemplated in SO(10) grand unified theories.[2]

The group theory we require is contained in the decomposition of the representations of the symmetry-breaking Higgs fields and the fundamental fermion multiplet under an $SU(5)\otimes\tilde{U}(1)$ subgroup of Spin(10):

$$\begin{aligned}&\mathbf{126}=(1,10)\oplus\cdots, \quad \mathbf{45}=(24,0)\oplus\cdots,\\&\mathbf{10}=(5,2)\oplus(\bar{5},-2),\\&\mathbf{16}=(1,-5)\oplus(\bar{5},3)\oplus(10,-1),\end{aligned} \tag{2.1}$$

where in the case of **126** and **45** we have indicated only those pieces that acquire vacuum expectation values. The first stage of symmetry breaking is implemented by the **126**, and breaks $\mathrm{Spin}(10)\rightarrow SU(5)\otimes Z_2$. We can form a Z_2 string at this stage, by winding the nonzero component of the **126** through a phase 2π as we circle the string at infinity:

$$\Phi(\phi)=\exp(i\phi\tilde{Q}/10)\Phi(0). \tag{2.2}$$

This is most energetically efficient if the vector potential is pure $\tilde{U}(1)$ at infinity, so that the string contains only $\tilde{U}(1)$ flux. The various components of the fermion field will then wind through phase angles of $-\pi$, $6\pi/10$, $-2\pi/10$, etc., so they have acquired exotic angular momenta.

The next stage of symmetry breaking, $SU(5)\otimes Z_2\rightarrow SU(3)\otimes SU(2)\otimes U(1)_Y\otimes Z_2$, is implemented by the **45**, which transforms as a singlet under $\tilde{U}(1)$, so it does not have any phase change around the string.

Finally we want to reproduce the symmetry breaking of the standard model: $SU(3)\otimes SU(2)\otimes U(1)_Y\otimes Z_2\rightarrow SU(3)\otimes U(1)_Q\otimes Z_2$, which we do by giving a vacuum expectation value to the components of the **10**, that correspond to the ordinary Higgs doublet. From (2.1) we see that the **10**, has $\tilde{Q} = \pm 2$, which means that the **10** acquires a phase difference of $2\pi/5$ in circling the pure $\tilde{U}(1)$ string, which is incompatible with its condensing with a definite vacuum expectation value.

The solution to this problem is to add other forms of flux to the string in such a way that the component of the **10** that condenses acquires no phase difference around it. We know that the relevant part of the **10** is a color singlet and isospin doublet annihilated by the electromagnetic charge Q. To express Q in terms of the generators of Spin(10) and its SU(5) subgroup we must know how they act on the **16** of Spin (10). Then we can see which components of the **10** condense, and adjust the flux in the string accordingly.

The energetically favored answer is

$$R = \tilde{Q} + 4I_3 - \tfrac{3}{2}Q\,. \tag{2.3}$$

This gives $R=0$ for the condensed component of the **10**, $R=10$ for the **126** (i.e., $R_\varphi = 10$), and for the fermions we find, using (1.2),

	u	d	e	ν_L	N_R
R_ψ	0	$-\frac{5}{2}$	$+\frac{5}{2}$	$+5$	-5
α	0	$-\frac{1}{4}$	$+\frac{1}{4}$	$+\frac{1}{2}$	$-\frac{1}{2}$

Since these values of α are nonintegral, there can be significant Aharonov-Bohm scattering.

For the ordinary components of matter, protons, electrons, and neutrons, we find $\alpha = -\frac{1}{4}$, $\frac{1}{4}$, and $-\frac{1}{2}$, respectively. A neutral atom will therefore have α integral or half-odd integral, depending on whether the number of neutrons in its nucleus is even or odd. Since in these cases the scattering is zero or maximal, respectively, we come to realize that this particular cosmic string is an excellent isotope separator. It is, however, evident that the α values for fermions interacting with strings depend on the details of the grand unified theory under consideration.

3. Differential scattering cross section.—In this section we calculate the cross section for the scattering of a relativistic fermion off a vortex in 2+1 dimensions. This will be the same as a normally incident fermion scattering off a cosmic string along the z axis in 3+1 dimensions. The Dirac equation is $(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi = 0$, and the flux associated with a solution ψ is $J^\mu = \bar{\psi}\gamma^\mu\psi$. To calculate the differential cross section we must find the complete set of solutions to the Dirac equation, and construct a superposition of them that makes an incident wave $\psi^{(i)}$ an infinity. This will also contain an outgoing circular wave, the scattered wave:

$$\lim_{r\to\infty} \psi = \psi^{(i)} + \psi^{(s)}\,. \tag{3.1}$$

Following Aharonov and Bohm we will choose the incident wave to describe a flux of particles parallel to the x axis, coming from $x = +\infty$, so $J_x^{(i)}$ will be negative. The differential cross section will then be

$$\frac{d\sigma}{d\phi} = \lim_{r\to\infty} \frac{\mathbf{r}\cdot\mathbf{J}^{(s)}(r,\phi)}{-J_x^{(i)}}\,. \tag{3.2}$$

To exploit the symmetry of the string under z translations, we use the following representation of the γ matrices[3,4]:

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix},$$
$$\gamma^2 = \begin{pmatrix} -i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{3.3}$$

Since there is no ∂_3 or A_3 in the Dirac equation, it falls apart into two independent equations for two-component spinors. We write A_μ in Lorentz gauge,

$$A = \frac{1}{gr^2}\begin{pmatrix} 0 \\ -y \\ x \\ 0 \end{pmatrix}, \tag{3.4}$$

so that with the *Ansatz*

$$\psi = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} e^{-i\omega t}, \tag{3.5}$$

the equations of motion become

$$(\omega - m)u_1 = -ie^{-i\phi}\left(\frac{\alpha}{r} + \partial_r - \frac{i}{r}\partial_\phi\right)u_2\,,$$
$$(\omega + m)u_2 = -ie^{i\phi}\left(\frac{-\alpha}{r} + \partial_r + \frac{i}{r}\partial_\phi\right)u_1\,. \tag{3.6}$$

A basis for the set of positive-energy solutions is

$$P_{kn}^{(\pm)}(r,\phi) = A_k \begin{pmatrix} J_{\pm(n+\alpha)}(kr)e^{in\phi} \\ \dfrac{\pm ik}{\omega+m} J_{\pm(n+\alpha+1)}(kr)e^{i(n+1)\phi} \end{pmatrix} e^{-i\omega t},$$

where J_ν are Bessel functions. We will impose the boundary condition that the top component of the spinor be regular at the origin. Gerbert and Jackiw[5] have pointed out that this is not the most general boundary condition; however, it is the correct one for a solenoid containing pure magnetic flux $B_z > 0$. A more detailed discussion will be published elsewhere.

A suitable form for the incident wave is

$$\psi^{(i)} = \begin{pmatrix} 1 \\ \dfrac{-k}{\omega+m} \end{pmatrix} e^{-i(kx+\alpha\phi)}\,. \tag{3.8}$$

This is a solution to the Dirac equation with

$$J_x^{(i)} = \frac{-2k}{\omega+m}, \quad J_y^{(i)} = 0\,, \tag{3.9}$$

so it corresponds to a flux of particles coming down the x axis, as required. To calculate the scattering cross section we now need to find a superposition of the P_{kn} that tends to the incident wave at infinity. Luckily this has already been done for us by Aharonov and Bohm. They showed that

$$u_1(r,\phi) = \sum_{n\geq 0} (-i)^{(n+\alpha)} J_{n+\alpha}(kr) e^{in\phi} + \sum_{n<0} (-i)^{-(n+\alpha)} J_{-(n+\alpha)}(kr) e^{in\phi}$$
$$\to e^{-i(kx+\alpha\phi)} - \frac{ie^{ikr}}{(2\pi ikr)^{1/2}} \frac{e^{i\phi/2}}{\cos(\phi/2)} \sin|\pi\alpha| , \qquad (3.10)$$

where $0<\alpha<1$. The final term is the top component of $\psi^{(s)}$. We can obtain the bottom component by using (3.6), and keeping only terms of order $r^{-1/2}$. This yields the full scattered wave:

$$\psi^{(s)} = -i \frac{e^{ikr}}{(2\pi ikr)^{1/2}} \frac{e^{i\phi/2}}{\cos(\phi/2)} \sin(\pi\alpha) \begin{pmatrix} 1 \\ \dfrac{k}{\omega+m} e^{i\phi} \end{pmatrix} . \qquad (3.11)$$

For α outside the range 0 to 1 extra phase factors appear, but do not affect the cross section. Using this and (3.9) in (3.2) we find the scattering cross section. Note that ϕ is not the scattering angle, since the particles were traveling in the negative x direction. The scattering angle is $\theta = \pi - \phi$, so

$$\frac{d\sigma}{d\theta} = \frac{\sin^2(\pi\alpha)}{2\pi k \sin^2(\theta/2)} . \qquad (3.12)$$

4. Particle production.— We expect there to be a particle production mechanism closely related to the Aharonov-Bohm scattering, essentially by crossing the initial particle over into the final state. The computation is substantially more difficult, however, so here we only outline our approach to the problem, which reduces it to quadratures; detailed results will be presented elsewhere.

We are interested in the effect of a moving string on the quantum fields around it. The problem naturally divides itself into two parts: calculation of the gauge potential associated with the moving string, and calculation of the effect of this potential on the matter fields.

For the first part we work with the field equation

$$\partial^\mu F_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} \partial^\mu S^{\alpha\beta} , \qquad (4.1)$$

where

$$S_{\alpha\beta}(r) = \frac{\partial \tilde{r}_{[\alpha}}{\partial t} \frac{\partial \tilde{r}_{\beta]}}{\partial z} \delta(r_x - \tilde{r}_x(z,t)) \delta(r_y - \tilde{r}_y(z,t)) \qquad (4.2)$$

is a source of vorticity along the string. Here $\tilde{r}(z,t)$ describes the world sheet of the string. This source is manifestly covariant [regarding $\delta(x)$ as $1/dx$], and, as is easily checked, gives the correct result in the static limit. We readily solve (4.1) in Lorentz gauge $\partial^\mu A_\mu = 0$, to find

$$A_\nu = \epsilon_{\mu\nu\alpha\beta} \partial^\mu \frac{1}{\partial^2} S^{\alpha\beta} . \qquad (4.3)$$

The $1/\partial^2$ is to be interpreted as the retarded Green's function. The reader might well wonder about our treatment of the massive gauge field using the ordinary Maxwell equations. In particular it seems there might be radiation from the string. Actually our solution (4.3) satisfies the stronger field equation

$$F_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} S^{\alpha\beta} \qquad (4.4)$$

that would follow from (4.1) by canceling the derivatives. This stronger field equation shows that $F_{\mu\nu}$ is localized on the string world sheet after all. The potential outside has no field strength, but is globally nontrivial as it should be. These are strong indications that (4.1) is the proper equation for the potential outside the string in a full Higgs theory as the mass scale of the heavy fields becomes large, and the string correspondingly small.

Now for the Dirac field we generate approximate solutions using

$$\psi(x) = \psi_0(x) - \int K(x,y) \delta \mathcal{A}(y) \psi(y) dy$$
$$\approx \psi_0(x) - \int K(x,y) \delta \mathcal{A}(y) \psi_0(y) dy , \qquad (4.5)$$

where $K(x,y)$ is the Green's function, ψ_0 is a solution of the Dirac equation in the presence of potential A_0 due to a static string, and $\delta A = A - A_0$ is the perturbation of A caused by a small localized motion of the string. It is necessary to treat the static field exactly, since it is in no way a small perturbation. The generalization of (3.7) to 3+1 dimensions provides us with the complete set of ψ_0 and with K in the usable form of an infinite series.

Having computed $\psi(x)$—or, operationally, the first term on the right-hand side of (4.5)—we take its inner product with the negative-frequency solutions to derive particle production amplitudes in the standard way.

While this procedure is explicit and straightforward in principle, the integrations are arduous and the results complex. We will present them elsewhere.

5. Comments.— (1) Until now the standard reference on the scattering of matter from cosmic strings has been Everett.[6] We believe that his analysis neglects the dominant effect, which is the Aharonov-Bohm interaction described above. He argued that the vector potential outside the string is negligible because it can be gauged away everywhere except for a singular plane (measure zero) whose location is arbitrary. As we have seen by explicit calculation, this plane nevertheless affects the cross section, for nonintegral α.

Everett went on to estimate a nonzero total cross section for a string of radius R, $\sigma \propto 1/[k \ln^2(kR)]$, from which Vilenkin[7] obtained the friction on a moving string. Our result dominates Vilenkin's by a logarithmic factor [see comment (5)].

(2) In Ref. 4 and subsequent papers based on it, matter scattering from cosmic strings was analyzed by modeling the string in an effective Abelian gauge theory. However, the effective fermion changes were taken to be commensurate with the flux (α=integer, in our notation), and so the Aharonov-Bohm effects calculated above were missed.

(3) The analysis above has concerned strings for which the left- and right-handed components of the fermion fields have the same effective charge α. If this is not the case, the mass becomes effectively angle dependent and there will be zero modes on the string.[8] Calculations of scattering and particle production will have to take account of the possible excitation of these modes. We leave these interesting problems for future investigation. A closely related problem can occur even for our Spin(10) string where a Majorana mass for the neutrino is angle dependent. This is a simpler problem because the neutrino zero modes do not come with the extra baggage of ordinary electromagnetic interactions.

(4) It is also interesting to consider scattering under the influence of vector potentials as discussed here, taking into account the conical distortion of space by the gravitational influence of the string. As Gerbert and Jackiw[5] have noted, the relevant equations are implicit in their calculation of the gravitational effects of a spinning cosmic string, since the spin contributes an Aharonov-Bohm-type scattering term to the cross section.

(5) After this work was completed, some previous published work by Rohm[9] was brought to our attention, in which he derived the result (3.12) for Aharonov-Bohm scattering of fermions off a cosmic string. He also calculated the frictional effect, namely, the rate of energy loss per unit length for a string moving through particles of number density n at speed v (Lorentz factor γ). By boosting (3.12) to the rest frame of the particles he found

$$\frac{dE}{dl\,dt}=2\hbar n\gamma v^2 . \qquad (5.1)$$

This is to be compared with Vilenkin's result,[7]

$$\frac{dE}{dl\,dt}\approx nv^2\hbar/\ln^2(kR) , \qquad (5.2)$$

where k is the momentum of the particles in the string rest frame. This has a similar form to the friction due to Aharonov-Bohm scattering, but suppressed by a logarithmic factor of about 1000 for typical grand unification scales, $k\sim 1$ GeV, $1/R\sim 10^{15}$ GeV.

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