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Transition between Incompatible Properties: A Dynamical Model for Quantum Measurement

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We use an electronic four-level model, optically prepared in a coherent superposition of energy eigenstates, to discuss the dynamics of a measurement process leading to the preparation of pure energy eigenstates. The coupling strength to the environment (test laser) is found to control the speed at which information can be gained. An explicit detection strategy is given which can be evaluated as regards its average error probability. The model helps to clarify the conditions under which axiomatic measurement theory (i.e., the collapse of the wave function) applies; it also gives some clues on the recently revived discussion on "quantum jumps."

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There has recently been a growing interest in the properties and observation of *individual* quantum systems. This type of investigation, motivated by a number of ambitious experiments,[1-4] promises to give new insight into old and still controversial questions: It provides a testing ground not only for the predictions of quantum theory, but even for certain interpretations thereof. A striking example is the revived discussion on "quantum jumps,"[5-10] which stresses the stochastic nature of quantum processes despite the continuous dynamical evolution of the respective density matrix. We thus felt encouraged to reexamine a fundamental problem of quantum mechanics: The sequential measurement of *incompatible* observables A, B, the operators of which do not commute.

Quantum measurement defines a class of interaction processes between the subsystem to be measured and its environment.[11,12] So-called measurements of the first kind[13] can be identified with a preparation process which, according to axiomatic measurement theory,[14] is reduced to a mere definition of the operators $\hat{A}$ ($\hat{B}$), in the eigenstates of which the system is prepared (measured). A sequence of two such processes models the way in which the state of quantum objects can be manipulated.

This generally accepted concept, which is at the heart of the interpretation of quantum mechanics, has a number of serious deficiencies: (1) It is not constructive, i.e., it does not give any hint on how the system-environment interaction should be designed in order to have $\hat{B}$ measured. This question is usually related to the destruction of coherence in the measurement basis.[15-18] (2) It is not dynamical, i.e., there are no statements as to how a correlation between the object state and the detector evolves in time, and (3), nothing can be said about possible limits or systematic sources of errors in analyzing this correlation as regards its information content about the object state.

It is the purpose of this Letter to discuss a quantum optical model in which these deficiencies can be overcome: We do not define the observable to be prepared or measured, but rather the interaction from which the physical properties *and* the information about these properties will follow. This is in the spirit of Zurek[15]: It is the coupling to the environment which eventually determines the properties of quantum systems. We then demonstrate how the measurement process modifies the physical properties of the object, and how our information about the object, which is supposed to be complete at the beginning, changes with time.

The energy spectrum of our electronic quantum system, corresponding to the fixed basis $|v\rangle$, is sketched in the inset of Fig. 1. A realization could be based on nanostructured semiconductor materials,[19,20] but more

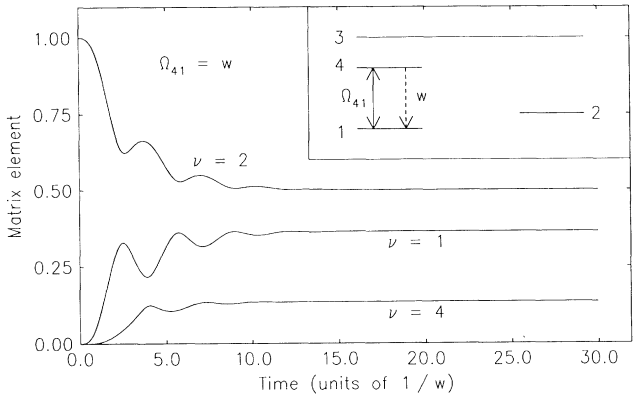

FIG. 1. Decay of an initially prepared periodic attractor state (cf. text) after the test laser Ω_{41} has been switched on. The matrix elements $\tilde{\rho}_{\nu\nu}$ in the diagonal representation $|\tilde{\nu}\rangle_t$ are shown. Inset: The level scheme and the relevant couplings for this measurement step. The transition matrix elements are taken to be proportional to the overlap of the respective wave functions, the localization of which are indicated by the extension of the horizontal lines. The delocalized level 3 is needed for the preparation of the initial state (cf. text).

likely on a molecular or atomic system.[21] For our present purpose it is essential that, because of selection rules (based on symmetry or different localization behavior), the optical transitions are basically of two types, either allowed, with spontaneous decay rate $w=w_{31}=w_{32}=w_{34}=w_{41}$, or "forbidden," with rate $w_d=w_{21}=w_{42}\ll w$ (time scale spreading). For what follows we will assume resonant interactions; the results can easily be generalized to include nonzero detuning. To describe the dynamics of the electronic system, we use a master equation approach,[20] in which the background radiation field has been traced out.[22]

At any instant of time, the density matrix of our object in its N-dimensional Hilbert space can either be specified by N^2-1 independent real numbers, defining $\rho_{\nu\nu}$, with respect to any *fixed* complete basis $|\nu\rangle$, $\nu=1,2,\ldots,N$ (e.g., the energy eigenbasis), or by $N-1$ matrix elements $\tilde{\rho}_{\nu\nu}$ in the (instantaneous) diagonal representation $|\tilde{\nu}\rangle$, supplemented by the unitary transformation $\hat{U}(t)$ connecting this specific basis to the fixed one, $|\nu\rangle$. Both dynamical descriptions are equivalent. However, if we suppose that at any instant of time there exists a complete (zero entropy) description for a *single* quantum system (i.e., the density matrix has only one nonvanishing element after diagonalization), and if we further assume that alternative pure states are mutually orthogonal (so that we can decide which state is occupied by the individual quantum object), it is the instantaneous diagonal representation which allows for a simple interpretation in terms of probabilities: $\tilde{\rho}_{\nu\nu}(t)$ gives the probability for the single quantum object to be in state $|\tilde{\nu}\rangle_t$. Measurement thus reduces to hypothesis testing[23]; as in classical physics, the analysis must exploit the dynamical behavior derived from the measurement object and its interaction with the environment. Here four different classes can be distinguished: (i) $|\tilde{\nu}\rangle$ and $\tilde{\rho}_{\nu\nu}$ are both independent of time; (ii) $|\tilde{\nu}\rangle$ changes and $\tilde{\rho}_{\nu\nu}$ is independent of time (coherent time evolution); (iii) $|\tilde{\nu}\rangle$ is stationary, while $\tilde{\rho}_{\nu\nu}$ changes with time (coupled set of rate equations); (iv) $|\tilde{\nu}\rangle$ and $\tilde{\rho}_{\nu\nu}$ change with time.

The recent resonance fluorescence experiments[1-4] performed with single atoms can be ascribed to class (iii) with $\hat{A}=\hat{H}_0$, the atomic Hamiltonian (the interaction with the light field will make $|\tilde{\nu}\rangle$ different from the atomic basis, but this is not essential). These experiments indicate that the individual decay of the "shelved" state is appropriately interpreted as a stochastic point process with a waiting-time distribution equivalent to the constant decay rate. This process assumes that the atom is always in one of the states $|\tilde{\nu}\rangle$, while the transition responsible for the change of $\tilde{\rho}_{\nu\nu}(t)$ occurs in the form of discontinuous quantum jumps. This interpretation is confirmed by an analysis of the two-time photon correlation function[8,9] for a stationary scenario in which the "shelving" (or switching) laser is applied simultaneously with the test laser inducing the luminescence: In this case, a stochastic two-state process can be formulated for the photon field, see, e.g., Ref. 6.

Our present scenario belongs to the most general case (iv). Starting from the ground state $|1\rangle$, we can prepare a "periodic attractor state" (i.e., a coherent superposition with respect to basis $|\nu\rangle$, but a single state with respect to $|\tilde{\nu}\rangle$) by means of two correlated laser beams connecting state $|1\rangle$ with $|3\rangle$, and $|2\rangle$ with $|3\rangle$, respectively[20]:

$$\rho_{\nu\nu'}=0.5(\delta_{\nu1}\delta_{\nu'1}+\delta_{\nu2}\delta_{\nu'2}-\delta_{\nu1}\delta_{\nu'2}-\delta_{\nu2}\delta_{\nu'1}).$$

The time period for $\rho_{12}(t)$ is given by $\hbar(E_2-E_1)^{-1}$, where E_2-E_1 is the energy difference between level 2 and level 1. It decays on a time scale $1/w_d$ which is supposed to be long compared with the time scale of the measurement step.

This step consists of applying now the test laser Ω_{41} (Rabi frequency for the transition $|1\rangle\leftrightarrow|4\rangle$) to this initial state. Though the test laser cannot switch between $|1\rangle$ and $|2\rangle$ on the time scale considered, it will certainly change the prepared state: The basis $|\tilde{\nu}\rangle_t$ starts to move irreversibly, such that for large times $|\tilde{2}\rangle$ becomes $|2\rangle$, while $|\tilde{1}\rangle\rightarrow|1\rangle$ and $|\tilde{4}\rangle\rightarrow|4\rangle$ only after, in addition, the test laser has been switched off. It is in this sense that we can say to move $|\tilde{\nu}\rangle$ asymptotically into the basis $|\nu\rangle$. With respect to $|\tilde{\nu}\rangle$, the initial pure state decays into a mixed state (cf Fig. 1). This latter behavior, reminescent of that found for the class (iii) systems as discussed above, should again be simulated by a stochastic process, implying quantum jumps.

The $|\tilde{\nu}\rangle_t$ could be said to define the instantaneous measurement basis if we were able to get information about it. The only source of information (without further interactions) is the flow of incoherently emitted pho-

tons, which is expected to be correlated with the electronic state. Let hypothesis $\tilde{H}^{(0)}(t)$ denote the situation in which the system at time t is found in state $|\tilde{2}\rangle_t$ [which occurs with probability $\tilde{\rho}_{22}(t)$, see Fig. 1] and let $\tilde{H}^{(1)}(t)$ denote the system not being in $|\tilde{2}\rangle_t$ [expected with probability $1-\tilde{\rho}_{22}(t)$]. The corresponding normalized density matrices are

$$\tilde{\rho}^{(0)}_{vv}(t)=\delta_{v2}\,,$$

$$\tilde{\rho}^{(1)}_{vv}(t)=\frac{1}{1-\tilde{\rho}_{22}(t)}[\tilde{\rho}_{11}(t)\delta_{v1}+\tilde{\rho}_{44}(t)\delta_{v4}]\,.$$

We now want to decide from the photon counts, integrated over the period of interaction up to the time τ_m, whether at time $t=\tau_m$ hypothesis $\tilde{H}^{(0)}(\tau_m)$ or $\tilde{H}^{(1)}(\tau_m)$ is true. Such an integration is necessary, as photon counting cannot give instantaneous information. For $\tau_m\rightarrow\infty$ this amounts to deciding essentially between the system being in $|2\rangle$ ("dark") or in subspace $|1\rangle$, $|4\rangle$ ("bright").

The binary decision is now based on a reduced stochastic point process defined by the following transition rules: State $\tilde{\rho}^{(0)}(t)$ with conditional probability (time-dependent rate)

$$\alpha_0(t)=-\frac{1}{\tilde{\rho}_{22}(t)}\frac{d}{dt}\tilde{\rho}_{22}(t)$$

or from state $\tilde{\rho}^{(1)}(t)$ to $\tilde{\rho}^{(0)}(t)$ with

$$\alpha_1(t)=\frac{1}{1-\tilde{\rho}_{22}(t)}\frac{d}{dt}\tilde{\rho}_{22}(t)\,,$$

whichever of the two is positive.[24] As is obvious from Fig. 1, both transitions die out for large t. Any sequence of transitions within $0\le t\le\tau_m$ defines a path $i(t)=0,1$ with $i(0)=0$ and, by that, a realization of an instantaneous photon emission rate $\lambda(t)$ with

$$\lambda(t)=w_{41}\,\mathrm{Tr}[\tilde{\rho}^{i(t)}(t)\hat{P}_4]\,,$$

where $\hat{P}_4=|4\rangle\langle 4|$ is a fixed projection operator.

The resulting photon emission process is then simulated as a doubly stocahstic Poisson point process,[25]

$$P_i(N,t+\Delta t)=P_i(N,t)[1-\lambda(t)\Delta t]+P_i(N-1,t)\lambda(t)\Delta t\,,$$

with $P_i(0,0)=1$. $P_i(N,t)$ is the probability to observe N photons between 0 and $t\le\tau_m$ on a path ending in $\tilde{\rho}^{(i)}(\tau_m)$, $i=0,1$. In general, more than one transition may occur; however, as they come on a time scale close to $1/w$, they cannot be resolved by photon counting.

The resulting average photon number $\bar{N}$ for sampling over 500 realizations $\lambda(t)$ is shown in Fig. 2 as a function of τ_m. We see that the photon counting will lead either to an $\bar{N}$ increasing with τ_m {which is expected to occur in 500 $[1-\tilde{\rho}_{22}(\tau_m)]$ cases}, or to a *finite* $\bar{N}>0$ (in the remaining cases). This should be contrasted with the measurement of a mixed state for which $\bar{N}$ is either zero or increasing with τ_m. This statistical bifurcation, related to a quantum jump in the electronic system, is the basis of the following detection strategy for observing our single quantum object:

Choose hypothesis $\tilde{H}^{(0)}(\tau_m)$ if $N(\tau_m)\le N_c$,

choose hypothesis $\tilde{H}^{(1)}(\tau_m)$ if $N(\tau_m)>N_c$,

where $N(\tau_m)$ is the number of photons counted within the measurement interval τ_m. This strategy leads to the following error probability[23]

$$P_e(\tau_m,N_c)=\tilde{\rho}_{22}(\tau_m)P_0(N>N_c,\tau_m)+[1-\tilde{\rho}_{22}(\tau_m)]P_1(N\le N_c,\tau_m)\,.$$

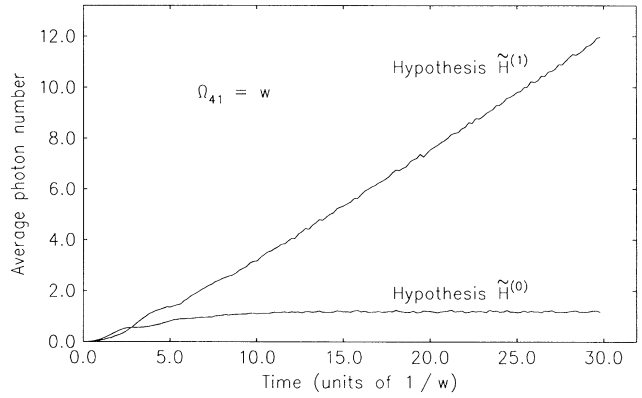

FIG. 2. Average photon number $\bar{N}$ [sampled over 500 realizations $\lambda(t)$] for an initially prepared periodic attractor state (cf. text). Hypothesis $\tilde{H}^{(1)}$ leads to an $\bar{N}$ increasing with τ_m, while hypothesis $\tilde{H}^{(0)}$ leads to a finite $\bar{N}>0$.

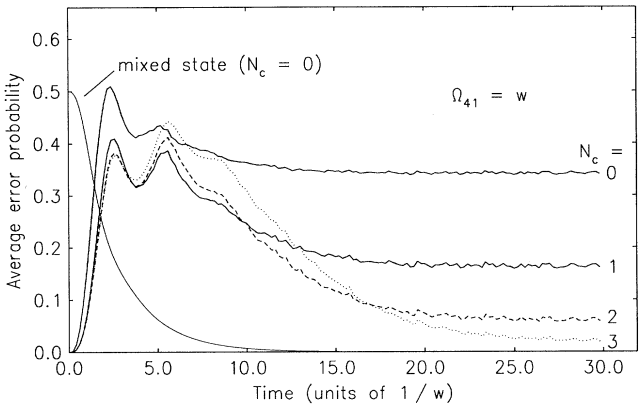

FIG. 3. Average error probability P_e [samples over 500 realizations $\lambda(t)$] for an initially prepared periodic attractor state (cf. text) and for various thresholds N_c. Also shown is P_e for a mixed initial state $[\tilde{\rho}_{vv}(0)=0.5(\delta_{v1}+\delta_{v2})]$.

The first term is the false alarm probability; the second term is the nondetection probability. P_e is shown in Fig. 3 for various thresholds N_c. We see how the strategy is, in principle, limited to finite-error probabilities, in particular for short-time intervals τ_m. This is due to the fact that for small τ_m both hypotheses are almost indistinguishable with respect to $\bar{N}$ (cf. Fig. 2). But even for large τ_m, P_e does not approach zero (unless $N_c \to \infty$): The reason is that there is always a finite probability to observe $N > N_c$ photons under hypothesis $\tilde{H}^{(0)}(\tau_m)$. This is in contrast to the same detection strategy applied to a mixed state, prepared in the $|v\rangle$ basis: As is also shown in Fig. 3, for this case P_e asymptotically approaches zero for any N_c (including $N_c = 0$).

We should remark that other detection strategies could easily be designed and evaluated. The present strategy was chosen in order to explicitly show the early-time behavior. If we were interested only in the asymptotic behavior, we might have preferred to count photons with $N_c = 0$, but starting at a later time when the probability to register "wrong" photons is already small. For such an analysis, however, the difference between the detection of a mixed and a coherent state would have almost disappeared.

In conclusion, we have shown how the stochastic formulation of quantum dynamics in terms of the instantaneous diagonal representation of the density matrix can be used for a detailed discussion of the measurement process of *single* quantum objects, prepared in a coherent, mixed, or pure state. The early-time behavior is significantly different for the measurement of a coherent state as opposed to a mixed state: The former may be said to involve quantum jumps which direct the long-time behavior to either the "dark-" or the "light-emitting" state. The experimental proof would consist in finding a finite photon count $N > 0$ for $\tau_m w \gg 1$. This last behavior is at variance with the often made assumption that the electron is in its ground state (i.e., in state $|1\rangle$ in our case) immediately after the emission of *any* spontaneous photon.[7]

Finally, the time-resolved analysis of the error probability, P_e, shows that the time scale on which the original information is destroyed and new information is gained increases as the coupling strength to the environment (test laser intensity Ω_{41}) decreases. The minimum time scale is set by the spontaneous decay time, $1/w$. Only for large interaction times τ_m, $\tau_m w \gg 1$ (but $\tau_m w_d \ll 1$), could P_e be made arbitrarily small; axiomatic measurement theory describes this asymptotic behavior, i.e., the collapse of the wave function.

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[25]Such a class of processes is known to describe, e.g., photoelectrons; see B. Saleh, *Photoelectron Statistics* (Springer-Verlag, Berlin, 1978).