Long-Range Cross-Field Ion-Beam Propagation in the Diamagnetic Regime

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It is shown that high-kinetic-β neutralized ion beams with transverse width less than an ion gyroradius and density larger than the background plasma density can propagate uninhibited through a magnetized plasma over many ion gyroradii. The model is confirmed by computer simulations using a two-dimensional hybrid code.

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A classic question in plasma and space physics concerns the conditions, if any, under which charge-neutral ion beams, often called plasmoids, can propagate across magnetic fields.[1,2] Such beams represent a phenomenon which occurs in nature (astrophysical jets, flares, precipitated ion beams, and magnetotail physics), and man-made situations (the high-altitude nuclear tests of the 1950's, active space release experiments, and beam injection from rockets). Laboratory analogs occur in laser-plasma interactions and cross-field injection experiments.

It was demonstrated[2,3] that a beam with transverse width $\Delta \gg R_b$, where R_b is the gyroradius of the beam ions, will in general compress the ambient magnetic field $\mathbf{B}_0$ but will not propagate significantly. Propagation can potentially occur when $\beta_0 \equiv 4\pi n_b M_b u_b^2/B_0^2 \gg 1$, where n_b, M_b, and u_b are the density, mass, and cross-field velocity of the beam. This propagation mode is similar to that of a solid conductor moving across the magnetic field $\mathbf{B}_0$ and is properly described by MHD theory. Another mode of propagation relies on the beam having a small width such that $R_e < \Delta < R_b$, where R_e is the electron gyroradius and occurs in the so-called electrostatic (or nondiamagnetic) $\beta_0 \ll 1$ regime. In this case the flow is too weak to alter the magnetic field, and a polarization electric field develops by the differential motion of the magnetized electrons ($R_3 \ll \Delta$) and the unmagnetized ions ($R_b > \Delta$), allowing cross-field motion by an $\mathbf{E}\times\mathbf{B}_0$ drift.[4,5] This mode of propagation has been demonstrated experimentally by Baker and Hammel.[6] As noted by Peter and Rostoker,[5] the presence of an ambient plasma does not affect the propagation if $V_{Ap}/V_{Ab} < 1$, where V_{Ap} and V_{Ab} are the plasma and beam Alfvén speeds. In this Letter we demonstrate that long-range cross-field ion-beam propagation is possible in the important diamagnetic regime ($\beta_0 \gg 1$) for a particular combination of beam and plasma conditions. These conditions are (i) finite ion-beam size Δ, with $R_e \ll \Delta < u_b/\Omega_0 \equiv R_0$, where Ω_0 is the cyclotron frequency of the ambient-plasma ions, and (ii) large beam-to-plasma density ratio ($n_b/n_0 \gg 1$), which puts us in the regime $\Delta \gg \delta_e$ where δ_e is the electromagnetic skin depth. A simple physical model of the propagation mode is presented, followed by results from two-dimensional (2D) are computer simulations.

Consider a cold, dense ($n_b \gg n_0$), super-Alfvénic ($u_b \gg V_{Ap}$) beam with $\beta_0 \gg 1$ interacting with an ambient magnetoplasma. Figure 1(a) illustrates this situation in the beam reference frame. In this frame the magnetofluid flows with a cross-field velocity $\mathbf{u} = -u_b \hat{\mathbf{e}}_z$ with the aid of a motional electric field $E_{0x} = u_e B_0/c \approx -u_b B_0/c$, where u_e is the fluid velocity of the plasma electrons and of the magnetic flux. The plasma ions, with charge e and mass M_0, move according to

$$du_x/dt = (e/M_0)(E_x - u_z B_y/c), \tag{1a}$$

$$du_z/dt = (e/M_0)(E_z + u_x B_y/c). \tag{1b}$$

The value of the motional electric field $E_x(z)$ is given by

$$E_x(z) = u_e(z) B_y(z)/c. \tag{2}$$

In the region $z > 0$ ahead of the beam-plasma interface [Fig. 1(a)], $u_e(z) = -u_b$ and $u_z = -u_b$ so that the right-hand side of (1a) is zero; namely, the ions, the electrons, and the flux follow straight ballistic orbits. At the plasma-beam interface ($z \approx 0$), the fluid velocity u_e reduces to

$$u_e(z) = n_0 u_b/[n_0 + n_b(z)] \tag{3}$$

to maintain charge neutrality. This is accompanied by a diamagnetic current at the front and a field compression [Fig. 1(a)] such that $B(z)/B_0 \approx 1 + n_b(z)/n_0$. From Eq. (1a) the reduction in $u_e(z)$ produces a net force $(eB_y/M_0c)[u_b - u_e(z)]$ in the positive x direction, which diverts the plasma ions upwards [Fig. 1(a)]. In a high-β flow, the electrons and the flux follow the path of the ions. For $\Delta \ll R_0$, a change of the plasma frame speed $u_e(z)$ for the order of $\Delta u_e/u_b \approx \Delta/R_0$ should be sufficient to establish a quasistationary state in which the background magnetoplasma is diverted to one side of the beam. In the laboratory frame, the above interaction corresponds to the beam front's diverting the ambient plasma and magnetic field sideways and propagating freely. The beam front suffers an erosion due to the energy required to push the plasma sideways. The resulting loss is, however, minimal for $\beta_0 \gg 1$. An important ingredient of the proposed propagation mode is the fact that the steady state is highly *asymmetric,* contrary to the one expected on the basis of fluid or MHD models.

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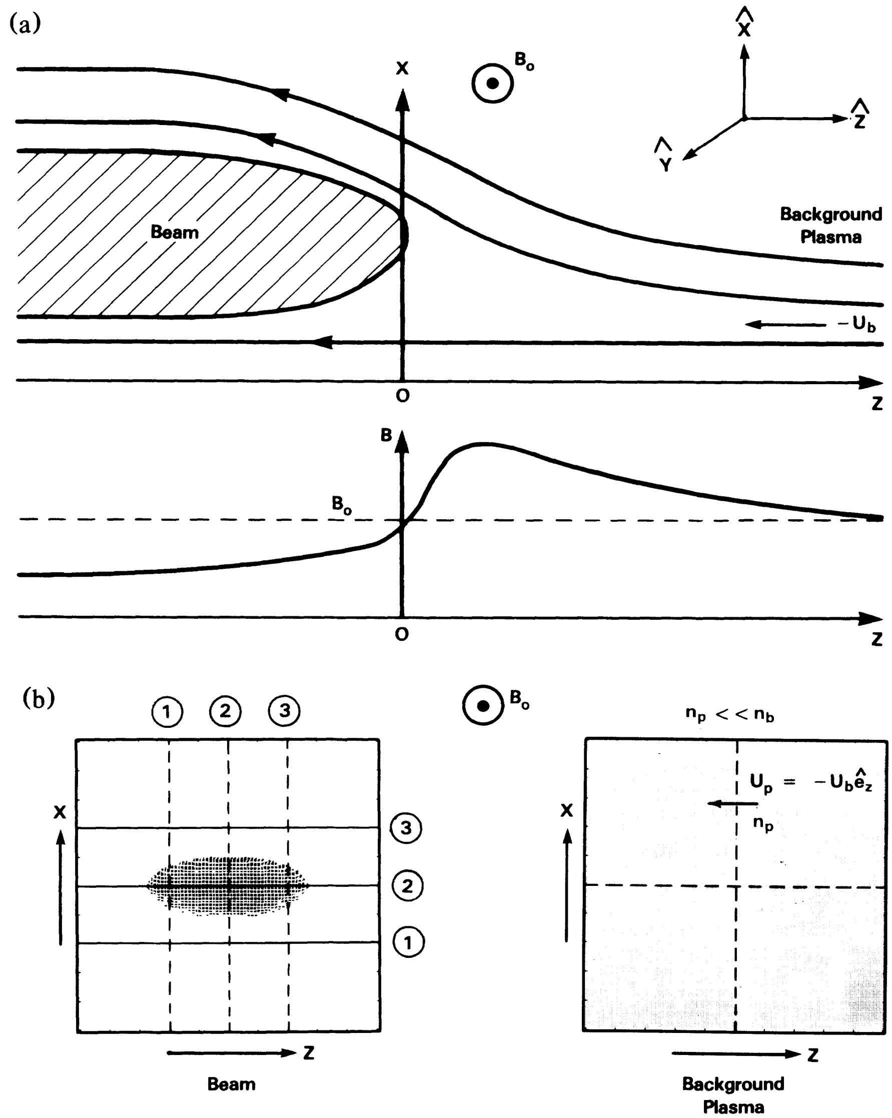

FIG. 1. (a) Schematic of beam-background plasma interaction physics. (b) Initial conditions and diagnostic cuts for simulations.

It is dominated by *kinetic* ion effects. Conversely, in the electrostatic (or nondiamagnetic) case where $\Delta B \cong 0$ and $\Delta u_e = 0$, flow diversion cannot occur and the plasmas will interpenetrate.

Progress past this intuitive physical picture requires computer simulations. A 2D hybrid code, described in Mankofsky *et al.*,[7] was used. In the hybrid code, the ions are treated as discrete particles. The electrons are treated as a massless fluid, described by the momentum and energy equations. These equations, along with the ion equation of motion, are solved self-consistently on a uniform 2D grid for the ion velocity vectors, the electromagnetic fields, and the electron pressure, in the nonradiative limit (i.e., Darwin Hamiltonian). The system has periodic boundary conditions in the direction transverse to the flow (i.e., x axis). The boundaries were placed sufficiently far from the interaction region so that they did not affect the results. In the flow direction (z axis) plasma is injected from the right boundary at a rate $n_0 u_b$ and permitted to leave on the left at the local flux rate. The magnetic field is allowed to float at these boundaries. The motional electric field is imposed on the simulation region according to the prescription in Eq. (2). We describe below simulation results designed to illustrate quantitatively aspects of the high-β propagation mode.

The geometry and initial conditions for the simulations are shown in Fig. 1(b). The calculations are performed in the beam frame. The beam, composed of protons, is initially given a Gaussian density distribution in the x and z directions. The run parameters represent a beam with $V_b = 10^8$ m/sec injected into the ionosphere at approximately 300 km. The ambient plasma, initially homogeneous in the x-z plane, is composed of oxygen ions ($M_0 = 16M$) and flows with velocity $\mathbf{u} = -\hat{\mathbf{e}}_z u_b = -1.95 v_{Ap} \hat{\mathbf{e}}_z$. The peak value of the beam density corresponds to $n_b/n_0 = 50$. To facilitate the understanding of the physics, diagnostic cuts labeled 1, 2, and 3 were taken along the x and z axes [Fig. 1(a)].

A global view of the interaction can be abstracted from Fig. 2, which shows scatter plots of the beam particles [Fig. 2(a)] and of the plasma particles [Fig. 2(b)] at times $\Omega_p t = 0.7$, 5.8, and 7.2, where Ω_p is the proton cyclotron frequency. The interaction of the beam front with the incoming plasma results in diversion of the plasma and the magnetic field asymmetrically around the

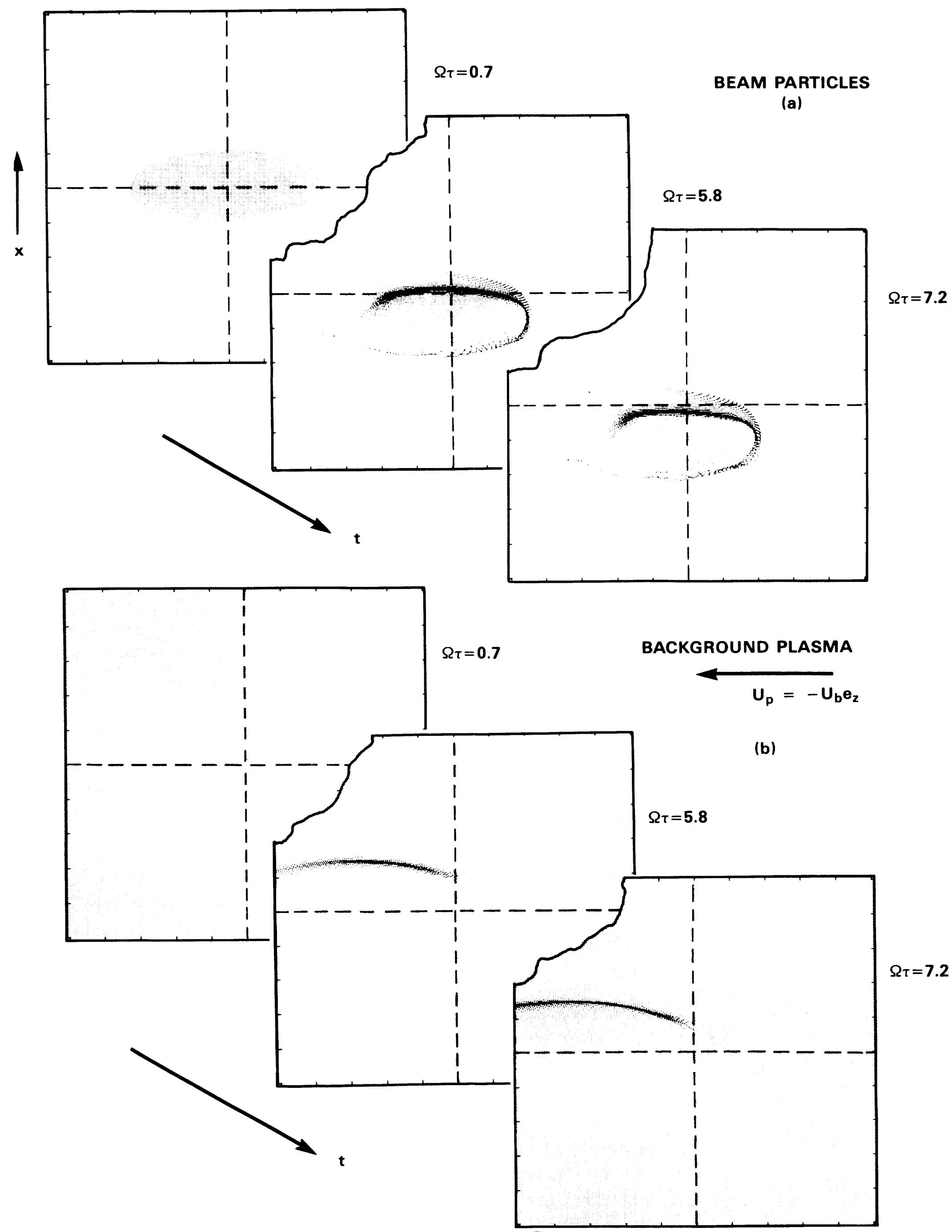

FIG. 2. Scatter plots of the time evolution of (a) ion-beam particle projections onto the x-z plane and (b) ambient-plasma particle projections onto the x-z plane.

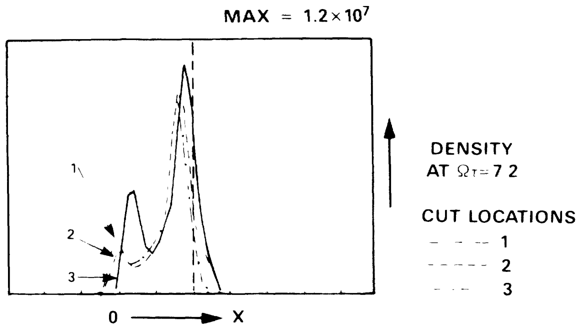

FIG. 3. Evolved transverse ion-beam profiles for the cuts shown in Fig. 1(b) at $\Omega_p t = 0.7$ and 7.2.

beam, thereby allowing the beam to propagate with minimal magnetic displacement in the x direction over times of $\Omega_p t \approx 7.2$. This corresponds to the axial distances of $7.2 u_b/\Omega_b$. The erosion of the front is clearly visible at late times in Fig. 2(a). Figure 2(b) shows the flow behavior of the ambient plasma. The one-sided non-MHD diversion of the ambient plasma starts immediately and stationarity is established before $\Omega_p t \approx 2$. Figure 3 shows profiles of the beam density along x in diagnostic cuts taken at the points marked 1, 2, and 3. It can be seen that the beam particles have remained, to a major extent, inside their original envelope. A detector measuring the beam flux would find over 96% of the beam particles within the original beam radius.

The electric and magnetic field structure which develops in the simulations, after a short transient, is stationary to within 10% over an entire run. The field profiles show field exclusion over the bulk of the beam and a maximum compression $\Delta B/B_0 \sim 0.15$ at the front. The beam excludes the motional electric field Ex_0, with the shielding factor scaling as N_b/N_0 for the range $N_b/N_0 = 2$–50 explored in the simulations, thus allowing denser beams to propagate over longer distances.

The sideways displacement Δx of the beam can be estimated by the requirement of momentum conservation in the x direction, i.e.,

$$L_z n_b M_b \frac{d^2 \Delta x}{dt^2} = -\Delta z n_0 M_0 \frac{\Delta u_x}{\Delta T} = n_0 M_0 u_b \Delta u_x, \quad (4)$$

where Δz is the penetration length of the plasma into the beam, and L_x and L_z are its x and z dimensions, respectively. In a simple approximation ΔT is the time required to displace a background ion by L_x, permitting it to gain a velocity Δu_x, so that $\Delta Z = u_b \Delta T$ and $\Delta u_x = L_x/\Delta T$. It follows from Eq. (2) that

$$\Delta u_x = u_b (|e| B_0 / M_0 c)(1 - |u_e/u_b|) \Delta T. \quad (5)$$

Integrating Eq. (4), and relying on the inequality $|1 - U_{ez}/U_b| < 1$, we find that the key parametric dependence of sideways beam displacement is given by

$$\Delta x / R_b < (n_0/n_b)[(L_x R_0)^{1/2}/L_z] \tfrac{1}{2} (\Omega_b t)^2. \quad (6)$$

From inequality (6) we see that the ion beam will propagate over many ion gyrotimes with small lateral displacement for large values of n_b/n_0 and large aspect ratios L_z/L_x. The results of various runs with different n_0/n_b and L_x/L_z are consistent with the above scaling.

In summary, we have presented a simple model that indicates that ion beams can propagate over large distances through magnetoplasmas in the high-kinetic-β regime for $n_b/n_0 \gg 1$, $\Delta < R_0$, and $L_z \gg L_x$. The model was confirmed by hybrid simulations. The detailed scaling, as well as applications to laboratory, magnetospheric, and astrophysical plasmas, will be published elsewhere.

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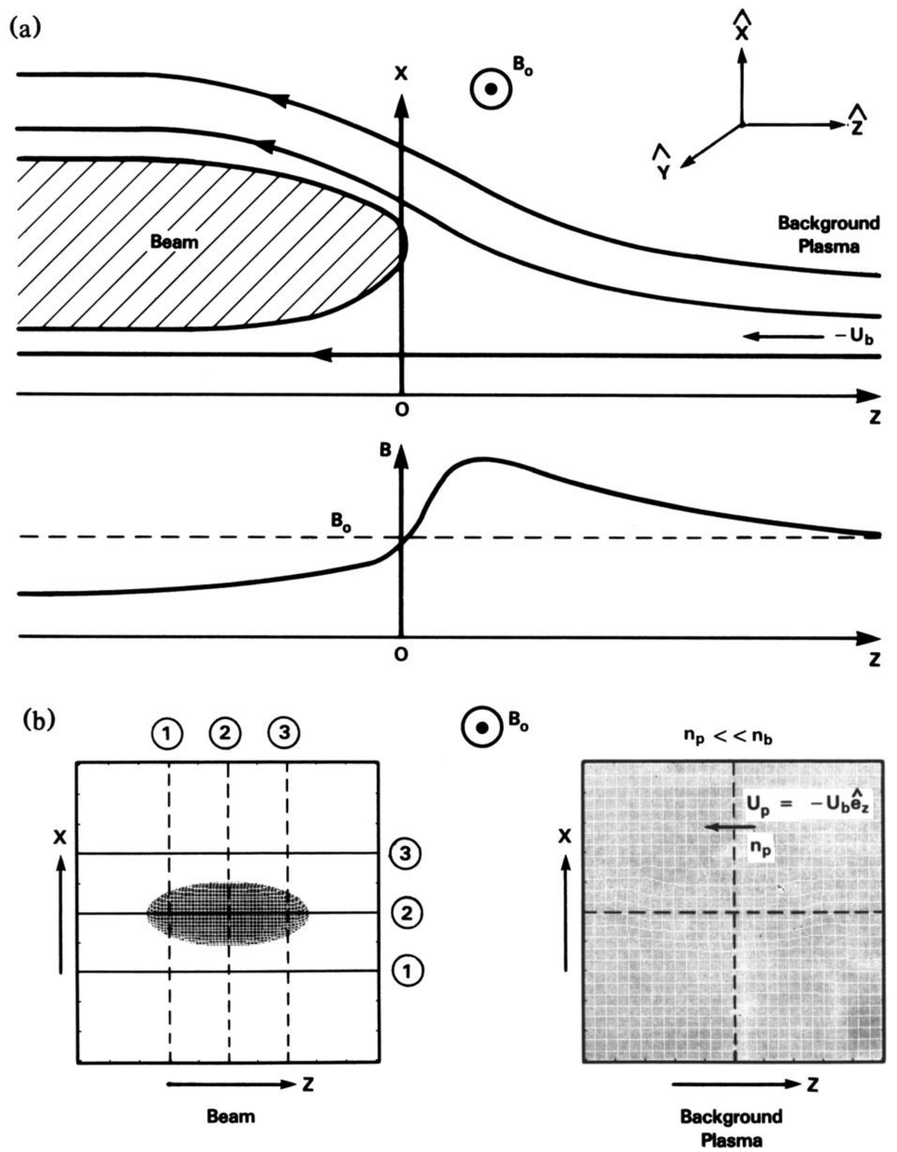

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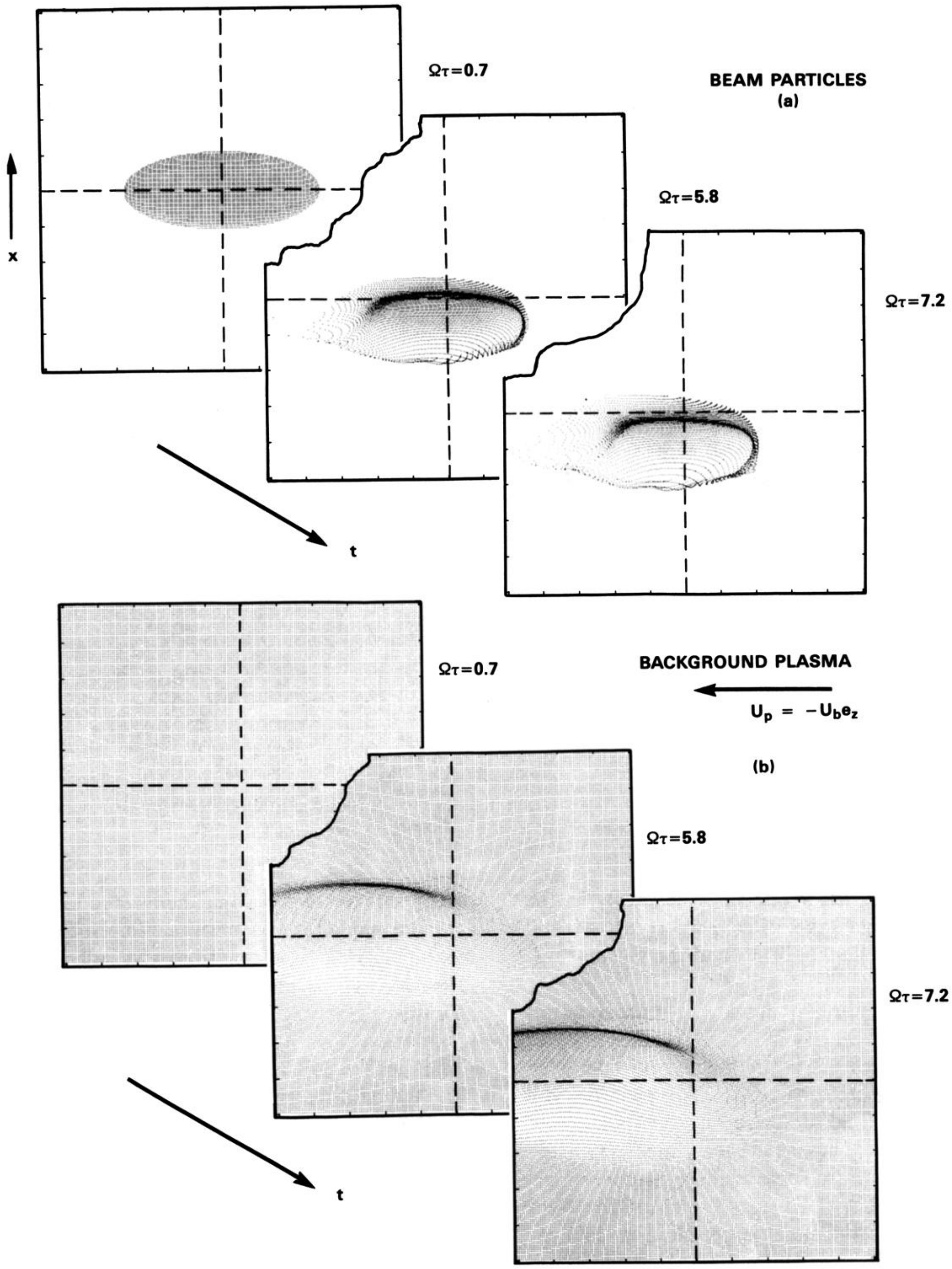

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