Bright Squeezing

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We show how the squeezed vacuum fields generated in collective fluorescence within a high-finesse cavity can be transformed into squeezed coherent states with high brightness but with the same degree of noise suppression. The transformation is achieved by an injected signal which creates a mean polarization and by squeezing in the high, rather than the low branch of the optical bistable collective radiation.

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Squeezed light, characterized by a level of noise smaller than the vacuum fluctuations, has attracted a great interest in recent years[1] and has recently been detected in several laboratories.[2-6] The squeezed radiation generated to date has been either squeezed vacuum states (with zero mean field) or squeezed light with an extremely small mean field. It is therefore of interest to see how squeezed light with a *large coherent mean field* may be produced. This type of squeezed coherent-state light generation we will call "bright squeezing." Amplification of previously squeezed light adds spontaneous-emission noise from the amplifier; several authors[7] have shown that this limits the amplification to a factor of 2 if squeezing is to be preserved, although "rigged reservoir" techniques may circumvent this limitation. Here we study one method of producing intense squeezed light with nonzero mean amplitude within a single nonlinear device through the action of an injected signal which induces a nonzero steady-state polarization in addition to the cavity-mode field interaction. We show how this injected signal displaces the center of the generated squeezed vacuum Wigner contour to realize for the first time the squeezed coherent state proposed in the early days of squeezed-light studies.[1] One particular process used to generate squeezed light is the bistable interaction between N two-level atoms and a single mode of a high-Q optical cavity.[5,8,9] In this Letter, we propose a modified version of the experiment performed by Orozco and co-workers[5] that, while conserving the fluctuation characteristics in the output light, would *increase its brightness by a factor of more than 1 order of magnitude.*

Our model is derived basically from the model used by Refs. 10 and 11. We consider N two-level atoms interacting with a cavity mode (see Fig. 1). The cavity is driven by an external field $E_d(t)$ and damped at a rate κ' by the transmission through the mirrors. The inversion decay rate is γ and the polarization decay rate is γ_A. The modification proposed by us is the introduction of an external classical field $E_p(t)$ to pump the atoms directly.[12] The master equation for the reduced density operator $\hat{\rho}$ can be obtained by our projecting out the bath variables.[13] In the rotating-wave, dipole, and Born-Markoff approximations, and as before ignoring transverse spatial field variations and phase-matching considerations, we obtain

$$i\hbar\, \partial\hat{\rho}/\partial t = [\hat{H}_0 + \hat{H}_d + \hat{H}_p, \hat{\rho}] + \hat{L}_f[\hat{\rho}] + \hat{L}_a[\hat{\rho}]. \tag{1}$$

The Hamiltonian $\hat{H}_0$ for the atoms, the cavity field, and the interaction with each other is

$$\hat{H}_0 = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_a \hat{J}_z + i\hbar g(\hat{J}_- \hat{a}^\dagger - \mathrm{H.c.}), \tag{2}$$

where $\hat{J}_z$, $\hat{J}_-$, and $\hat{J}_+$ are collective atomic operators, ω_a is the resonance frequency of the N two-level atoms, $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators for the mode of frequency ω_c and $g = (\omega_c \mu_0^2 / 2\hbar\epsilon_0 V)^{1/2}$ is the coupling constant for the atom-cavity mode in the dipole approximation. The Liouville operators, representing the interactions of field ($\hat{L}_f$) and atoms ($\hat{L}_a$) with heat baths, are given by

$$\hat{L}_f[\hat{\rho}] = \kappa'\{[\hat{a}\hat{\rho}, \hat{a}^\dagger] + [\hat{a}, \hat{\rho}\hat{a}^\dagger] + 2n_{\mathrm{th}}[[\hat{a}, \hat{\rho}], \hat{a}^\dagger]\}, \tag{3}$$

$$\hat{L}_a[\hat{\rho}] = \sum_\mu \{\tfrac{1}{2}\gamma([\hat{\sigma}_-^\mu \hat{\rho}, \hat{\sigma}_+^\mu] + [\hat{\sigma}_-^\mu, \hat{\rho}\hat{\sigma}_+^\mu]) + \gamma_p([\hat{\sigma}_z^\mu \hat{\rho}, \hat{\sigma}_z^\mu] + [\hat{\sigma}_z^\mu, \hat{\rho}\hat{\sigma}_z^\mu])\}, \tag{4}$$

where γ_p describes dephasing collision broadening, n_{th} is the thermal average number of bosons for the field bath, and $\hat{\sigma}_z^\mu$, $\hat{\sigma}_-^\mu$, and $\hat{\sigma}_+^\mu$ are Pauli operators. The driving of the cavity and the pumping of the atoms by classical fields of frequency ω_l is described by

$$\hat{H}_d = i\hbar(\hat{a}^\dagger E_d e^{-i\omega_l t} - \hat{a} E_d^* e^{i\omega_l t}), \tag{5}$$

$$\hat{H}_p = i\hbar(\hat{J}_+ E_p e^{-i\omega_l t} - \hat{J}_- E_p^* e^{i\omega_l t}), \tag{6}$$

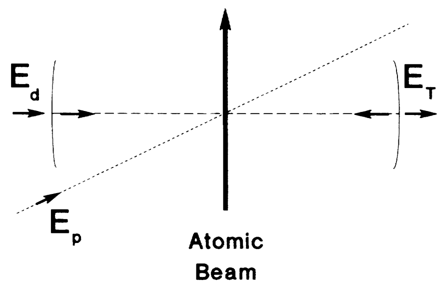

FIG. 1. Schematic outline of driven optical bistability, where an atomic beam interacts in a Doppler-free manner with a driven cavity mode (excited by a field E_d) and a pump field, E_p. The transmitted squeezed field is E_T.

where E_d and E_p are scaled amplitudes. We show that when considering only the pump E_p, without directly driving the cavity, the fluctuations and therefore the squeezing are the same as in the driven-cavity model (DCM), but a displacement of the cavity field amplitude occurs. We make a unitary transformation to a rotating frame where there is no explicit time dependence in the Hamiltonian. We define the further transformation

$$\hat{b}=\hat{a}+E_p/g, \tag{7a}$$

$$E'=E_d+\kappa E_p/g. \tag{7b}$$

Here, $\kappa=\kappa'(1+i\phi)$ and the normalized cavity detuning is $\phi=(\omega_c-\omega_l)/\kappa'$. The displacement given by Eq. (7a) is obtained by the application of the Glauber displacement operator $\hat{D}(\lambda)=\exp(\lambda\hat{a}^\dagger-\lambda^*\hat{a})$, with $\lambda=E_p/g$. By transformation (7), the problem becomes mathematically equivalent to the one considered in Refs. 5 and 11, with an effective driving field E' and bosons $\hat{b}$ and $\hat{b}^\dagger$. Therefore, the expectation value and variance for $\hat{b},\hat{b}^\dagger$ will be the same as in the DCM, and the conditions for squeezing are the same. But the amplitude of the cavity field is displaced, and can be much greater here than in the DCM, as we show.

With use of standard methods,[10,11,13] a Fokker-Planck equation for a generalized P-distribution function for the system is found, from which Îto stochastic differential equations are obtained. These are solved by a linearization of the fluctuations.[11,13] The mean-field solution gives the state equation relating input and cavity fields:

$$y=x[1+i\phi+2C(1-i\delta)/(1+\delta^2+|x|^2)], \tag{8}$$

where

$$\begin{aligned} &y=n_s^{-1/2}E'/\kappa'=n_s^{-1/2}(E_d+\kappa E_p/g)/\kappa', \\ &x=n_s^{-1/2}\bar{\beta}=n_s^{-1/2}(\bar{\alpha}+E_p/g), \\ &\delta=(\omega_a-\omega_l)/\gamma_A, \\ &C=Ng^2/2\kappa'\gamma_A, \\ &n_s=\gamma_A\gamma/4g^2. \end{aligned} \tag{9}$$

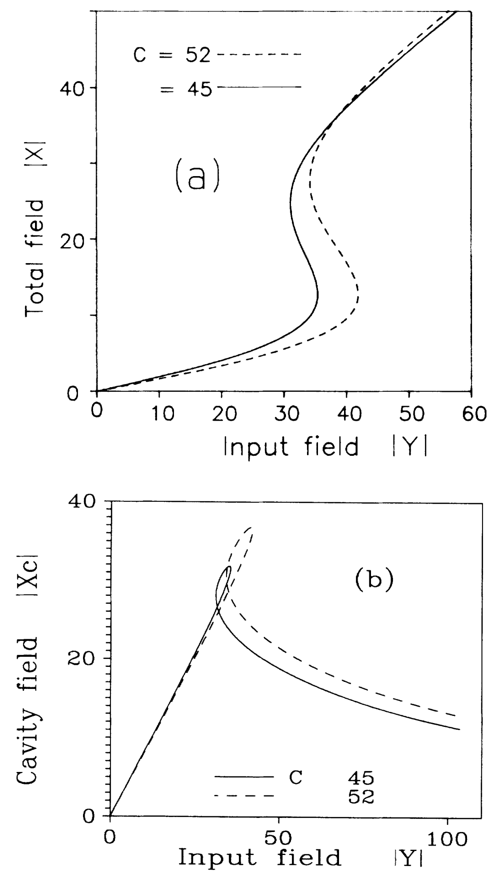

FIG. 2. (a) Variation of normalized total field $|x|$ given by Eq. (8) with normalized input field $|y|$ for two values of the cooperativity C. We set the normalized atomic detuning as $\delta=-14.6$, and the normalized cavity detuning as $\phi=-1$. (b) Variation of the normalized cavity field $|x_c|$ with normalized input field $|y|$; the parameters are chosen as in (a).

Here, $\bar{\alpha}$ is the average cavity field, $\bar{\beta}$ is the total field at the position of the atoms, and $\gamma_A=\gamma_p+\gamma/2$. Equation (8) generalizes the state equation studied previously,[5,8-11,14] the differences being the presence of the normalized total field on the atom x, instead of the normalized cavity field $x_c=n_s^{-1/2}\bar{\alpha}$, and the normalized effective total field y instead of the normalized driving field $y_d=n_s^{-1/2}E_d/\kappa'$. In Fig. 2(a), we plot $|x|$ as a function of the total input field $|y|$. The curves present the same aspects as in the usual DCM, with an unstable region and two turning points for each curve. The parameters that we use are in the range of those used in Ref. 5. In Fig. 2(b), we plot the cavity field $|x_c|$ as a function of $|y|$, the total input field amplitude. There is an inversion of the behavior with respect to Fig. 2(a), with the high-transmission branch being the first one. This is the most important feature of this model, because it is in this branch that squeezing has been detected, and

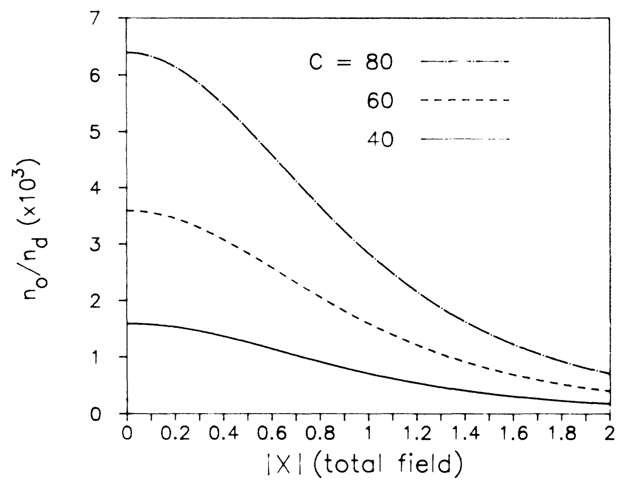

FIG. 3. Ratio of the square of the cavity field amplitude in the injected-signal model with $E_d=0$, $|\alpha_p|^2 \equiv n_0$, to that with the driving field E_d present only, $|\alpha_d|^2 \equiv n_d$, and zero pump field E_p, for three values of the cooperativity C. We choose parameters such that the normalized atomic detuning is $\delta=1$ and the normalized cavity detuning is $\phi=-1$.

this branch gives a brighter output light.

In order to compare the results in the DCM and in our model, we note that the points corresponding to the same value of the scaled input amplitude or to the same normalized total field at the position of the atoms will correspond to the same level of noise in both models. The ratio $|\alpha_p|/|\alpha_d|$ between the coherent squeezed cavity amplitudes in our model and in the driven model is

$$\frac{|\alpha_p|}{|\alpha_d|}=\frac{2C}{(1+\phi^2)^{1/2}}\frac{(1+\delta^2)^{1/2}}{1+\delta^2+|x|^2}, \tag{10}$$

where we have used the DCM model with $E_p=0$ and our model with $E_d=0$. The ratio is optimal when $|\delta|=|x|$ and $\phi=0$. Note that the ratio $|\alpha_p|/|\alpha_d|$ is proportional to the cooperativity C, and the squeezing increases as C is increased.[5] In Fig. 3, we plot $|\alpha_p|^2/|\alpha_d|^2$ as a function of $|x|$. We see that this ratio can be as big as 6.4×10^3. The signal detected in a balanced homodyne detection scheme, which is essentially proportional to $|\alpha_p|$, would be increased by roughly 3 orders of magnitude, using our injected-signal geometry.

Another feature favorable to our model is the intensity needed in order to achieve this effect. To compare the models, we put $E_p=0$ for the DCM and $E_d=0$ for our model. We define the normalized effective pump field as $y_p=(1+i\phi)E_p n_s^{-1/2} g^{-1}$. To have $|y_p|=|y_d|$, we need a ratio of the pump intensity in our model to the driving intensity in the DCM given by

$$\frac{I_p}{I_d}=\frac{g^2}{\kappa'^2(1+\phi^2)}=\frac{C}{\mu N(1+\phi^2)}, \tag{11}$$

where $\mu=\kappa'/\gamma$. This ratio is independent of the number of atoms, because C is proportional to N. Using the values of the parameters as given in Ref. 5 and choosing $\phi=0$, we obtain $I_p/I_d \sim 6/N$, so that proportionally less intensity is required in our model.

We shall now interpret these results. The total field acting on the atom is the same as in the DCM, and thus one could expect a similar atomic response in both models, with fluctuations mathematically identical in both models. The cavity field, in our non-driven-cavity model, is generated only by the radiation emitted by the atoms and is triggered initially by the spontaneous emission, and in steady state is essentially stimulated dipole emission. We do not mean by this that gain is present, but that dipole phase coherence of the stimulated scattering is responsible for the spatial directionality. The dipole field and the pump field at the position of the atoms partially interfere; therefore at the position of the atoms there is a destructive interference. When the total field at the atoms, x, is small, in the initial branch of the bistability cycle the atoms develop a larger polarization than when they are saturated and located in the second branch. This larger polarization is favorable to our model, because it leads to increased dipole radiation in the cavity mode when in this squeezing branch. In the DCM, the strong absorption leads to less transmission. On the other hand, in the less interesting nonsqueezing second branch, the bleached atoms let the radiation through, leading to a greater transmission in the DCM, while in our model this leads to a smaller interaction between atoms and input pump field, and therefore less efficient generation of photons in the cavity mode. A similar inversion of behavior has been studied in Ref. 14, where the hysteresis cycle for the phase conjugation reflectivity is a scaled version of our Fig. 2(b).

In conclusion, we have shown that a rearrangement of the experiment of Ref. 5 should increase, by more than 1 order of magnitude, the coherent power output in the squeezed branch of the bistable transmission of a cavity interacting with N two-level atoms, while maintaining the same squeezing in the output light. Formally, this is achieved by a simple application of the Glauber displacement operator to the cavity mode. Experimentally, this could be done with essentially the same values for the cooperativity, the intensity of the input laser field, and other relevant parameters, changing only from a configuration with driven cavity to the one with directly pumped atoms. Injection of a coherent input into other squeezed-light systems (e.g., parametric oscillators) may also generate bright squeezing.

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Norte, Brazil.

[1]*Squeezed States of the Electromagnetic Field,* J. Opt. Soc. Am. B **4** (1987) (special issue); *Squeezed Light,* J. Mod. Opt. **34** (1987) (special issue). Squeezing is reviewed by R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987).

[2]R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. Lett. **55**, 2409 (1985).

[3]R. M. Shelby, M. D. Levenson, S. H. Perlmutter, R. G. DeVoe, and D. F. Walls, Phys. Rev. Lett. **57**, 691 (1986).

[4]L. Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986).

[5]M. G. Raizen, L. A. Orozco, M. Xiao, T. L. Boyd, and H. J. Kimble, Phys. Rev. Lett. **59**, 198 (1987); L. A. Orozco, M. G. Raizen, Min Xiao, R. J. Brecha, and H. J. Kimble, J. Opt. Soc. Am. B **4**, 1490 (1987).

[6]A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, C. Fabre, and G. Camy, Phys. Rev. Lett. **59**, 2555 (1987).

[7]See, e.g., J. Vaccaro and D. T. Pegg, J. Mod. Opt. **34**, 855 (1987); C. R. Gilson, S. M. Barnett, and S. Stenholm, J. Mod. Opt. **34**, 949 (1987). The discussion of the limit for amplification is given by S. Friberg and L. Mandel, Opt. Commun. **46**, 141 (1983) and R. Loudon and T. J. Shepherd, Opt. Acta **31**, 1243 (1984). J. Vaccaro and D. T. Pegg [Opt. Acta **33**, 1141 (1986)] have suggested that bright squeezing may be produced by prepared atoms in a linear amplifier or attenuator.

[8]See, e.g., L. A. Lugiato, in *Progress in Optics,* edited by E. Wolf (North-Holland, Amsterdam, 1984), Vol. 21, p. 69.

[9]H. M. Nussenzveig, in *Nonlinear Phenomena in Physics,* edited by F. Claro (Springer-Verlag, Berlin, 1985).

[10]P. D. Drummond and D. F. Walls, Phys. Rev. A **23**, 2563 (1981); M. Reid and D. F. Walls, Phys. Rev. A **28**, 332 (1983).

[11]H. J. Carmichael, Phys. Rev. A **33**, 3262 (1985). F. Castelli, L. Lugiato, and M. Vadacchino [Nuovo Cimento (to be published)] have examined the possibility of squeezing in the upper and lower branches in optical bistability, especially in the bad-cavity limit, without adiabatic elimination.

[12]For the importance of the initial preparation of the atoms, see, e.g., P. L. Knight, Phys. Scr. **T12**, 51 (1986), and also J. Krause, M. O. Scully, and H. Walther, Phys. Rev. A **34**, 2032 (1986).

[13]W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).

[14]G. P. Agrawal and C. Flytzanis, IEEE J. Quantum Electron. **17**, 374 (1981).