Instabilities in Hot Nuclear Matter: A Mechanism for Nuclear Fragmentation

H. Heiselberg,[1] C. J. Pethick,[1,2] and D. G. Ravenhall[2]
[1]*NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*
[2]*Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801*
(Received 26 May 1988)

We analyze the process of fragmentation in heavy-ion interactions from the viewpoint of growth of instabilities in hot, expanding nuclear matter. The growth rates of modes in a medium unstable to small density perturbations of finite wavelengths are calculated from a kinetic equation. This enables us to treat the case where the nucleon mean free path is comparable to or greater than the wavelength of the fluctuation, and the temperature is arbitrary. We estimate for hot nuclear matter the initial conditions which, after expansion, will result in fragmentation, evaporation, or vaporization.

PACS numbers: 25.70.Np, 05.20.Dd, 21.65.+f

Theoretical studies of heavy-ion reactions at laboratory energies of order 100 MeV per nucleon have used a variety of different approaches.[1] This work has led to a better understanding of the initial conditions that give rise to the various possible final states: a nuclear residue containing a significant fraction of all the particles plus a small number of individual nucleons (evaporation), a large number of intermediate-size nuclei (fragmentation), or a cloud containing mainly single nucleons (vaporization).[2] In this Letter we show that much insight into these processes may be obtained by studying the growth of small density inhomogeneities in a blob of expanding hot nuclear matter.[3] The characteristic size of fragments, if any, observed in the final state will be directly related to the length scale of those density fluctuations that grow most rapidly to the stage where they are comparable in magnitude to the mean density. Our approach makes direct connection with the usual concepts for describing properties of bulk many-body systems, allows one to describe dynamical effects, and is computationally simple. Numerical simulations based on the Vlasov equation and its extensions and on molecular dynamics give detailed information about the final distribution of fragment sizes in a reaction, and it is not our intention to compete with them.[4] Our aim is rather to provide a framework for elucidating the basic physical processes that come into play in the fragmentation process. In this spirit we treat the results of simulations as "experimental" data, to be understood in physical terms.

The basic assumption is that fragmentation arises because of density instability in the central parts of the blob. There are three stages to the calculation: the determination of the time dependence of the density and temperature of the expanding matter, the evaluation of the instantaneous growth rate of small density fluctuations superimposed on the uniform expansion, and the calculation of the factor by which modes grow during the expansion process. We assume for simplicity that Coulomb and surface effects may be neglected in calculating the trajectory of matter, and that the entropy per particle of matter in the central part of the expanding blob is conserved. For the equation of state of hot dense matter, we use a fit of the Skyrme type to Friedman and Pandharipande's calculation.[5] In Fig. 1 we show for symmetric nuclear matter the coexistence curve, which is the boundary of the two-phase region in thermodynamic equilibrium, and the isothermal spinodal line, on which $(\partial P/\partial n)_T$ vanishes. Here P is the pressure, n the number density of nucleons, and T the temperature. Outside the spinodal line matter is thermodynamically stable to small long-wavelength density fluctuations, whereas inside it is unstable.

To explore the properties of finite-wavelength disturbances, we need the free energy of nonuniform nuclear

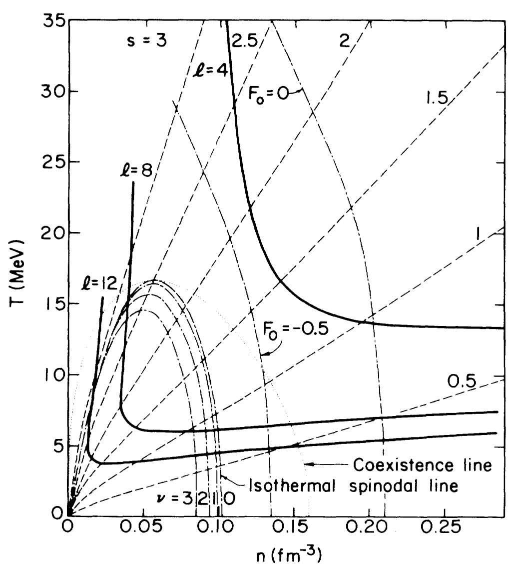

FIG. 1. The coexistence line (dotted), and adiabats (dashed) for different values of the bulk entropy per particle, s; contours of constant mean free path l, in units of fm, and of $F_0(T)$. The lines on which a mode with $q=\nu\pi/R$ becomes unstable are the dash-dotted lines within the coexistence line.

matter. As calculated by a Skyrme interaction, it contains terms of the type $(\nabla n)^2$. To supplement the uniform-density calculations of Ref. 5 we have added such terms under the assumption that they are related to the kinetic-energy density functional in the usual Skyrme manner, with no additional spin dependence, an assumption that produces a reasonable surface energy. These terms have the effect of adding to the free energy of a density fluctuation a term proportional to q^2. As a consequence the lines denoting the onset of instability at finite wavelength are displaced from the isothermal spinodal, for infinite wavelength. The results shown in Fig. 1 are for wave numbers given by $q=\nu\pi/R$, where R is the radius of a sphere containing 200 particles, a typical nucleon number in a heavy-ion collision. The longest-wavelength modes correspond to $\nu\sim 1$: If such a mode develops sufficiently to disrupt a hot blob of nuclear matter it leads to the production of a few large fragments (as in fission). A $\nu\sim 3$ mode leads to fragments comparable in size to α particles. The lines given in Fig. 1 show the reduction of the unstable region for finite-wavelength modes.

To estimate growth rates we employ a natural generalization to finite temperatures and finite wave numbers of Landau's kinetic equation,[6] which when linearized is given by

$$(\omega-\mathbf{v}_\mathbf{p}\cdot\mathbf{q})\delta n_\mathbf{p}+\mathbf{v}_\mathbf{p}\cdot\mathbf{q}\frac{\partial n_\mathbf{p}}{\partial\epsilon_\mathbf{p}}\delta\epsilon_\mathbf{p}=-\tau^{-1}\delta n_\mathbf{p}^{\mathrm{LE}}.$$

Here ω is the frequency, $\epsilon_\mathbf{p}$ is the quasiparticle energy in equilibrium, $\mathbf{v}_\mathbf{p}$ is the quasiparticle velocity, and $\delta n_\mathbf{p}$ is the deviation of the quasiparticle distribution from its equilibrium value $n_\mathbf{p}$. The right-hand side is the collision term, in the relaxation-time approximation. The relaxation time is τ, and $\delta n_\mathbf{p}^{\mathrm{LE}}$ is the deviation from the *local* equilibrium distribution function. This is a Fermi function evaluated with the actual quasiparticle energy $\epsilon_\mathbf{p}+\delta\epsilon_\mathbf{p}$, and shifted chemical potential, mean velocity, and temperature, chosen so that the total particle number, momentum density, and energy density corresponding to the local equilibrium distribution function are the same as for the actual distribution function. The shift in the quasiparticle energy is given by $\delta\epsilon_\mathbf{p}=4\sum_{\mathbf{p}'}f_{\mathbf{pp}'}\delta n_{\mathbf{p}'}$, where the factor 4 is the spin-isospin degeneracy factor and $f_{\mathbf{pp}'}$ is the quasiparticle interaction. The spin-isospin averaged interaction has the form

$$f_{\mathbf{pp}'}(q)=f_0(n,T)+\beta q^2+\phi_1\mathbf{p}\cdot\mathbf{p}'.$$

The coefficient f_0 is chosen so that the compressibility calculated from the kinetic equation agrees with that from the equation of state, while $\beta(n,T)$ and $\phi_1(n,T)$ are directly related to the momentum-dependent terms in the Skyrme interaction. At zero temperature ϕ_1 is identical to f_1^s/p_F^2 in Landau theory, and βq^2 is the finite-wave-number correction, which is positive for nuclear matter. (The Coulomb contribution to the interaction, $\pi e^2/q^2$, is relatively unimportant for the q's that we consider.[3]) Our approach is close in spirit to that of the polarization potential, with both scalar and vector mean fields.[7] Since for nuclear matter the finite-wavelength effects coming from the potential energy are much more important than those from the kinetic energy, the quantity $(n_{\mathbf{p}+\mathbf{q}/2}-n_{\mathbf{p}-\mathbf{q}/2})/v_\mathbf{p}\cdot\mathbf{q}$ in the quantum kinetic equation has been approximated by $\partial n_\mathbf{p}/\partial\epsilon_\mathbf{p}$.

Details of the solution of the kinetic equation will be reported elsewhere,[8] and here we give selected results. The condition for instability to density fluctuations is $1+f_0(q)N(T)<0$, where $f_0(q)=f_0+\beta q^2$ and $N(T)=4\sum_\mathbf{p}(-\partial n_\mathbf{p}/\partial\epsilon_\mathbf{p})$ is the finite-temperature generalization of the density of states at the Fermi surface. This result is a natural generalization of the long-wavelength, zero-temperature condition $F_0^s=-1$ given by Landau-Fermi-liquid theory.[6] In Fig. 1 we show contours of $F_0(T)=f_0(0)N(T)$. The effective mass m^* is a function only of density, and has the values $\frac{5}{6}m$ and $\frac{2}{3}m$ at $n=0.071$ and $0.187\ \mathrm{fm}^{-3}$, respectively. (m is the nucleon mass.) For weakly unstable matter the growth rate $\Gamma=-i\omega$, when collisions are unimportant, is given by[9]

$$\Gamma=-(2q/\pi)[1+f_0(q)N(T)]/\langle v_\mathbf{p}^{-1}\rangle,$$

where $\langle O_\mathbf{p}\rangle$ denotes $\sum_\mathbf{p}O_\mathbf{p}(-\partial n_\mathbf{p}/\partial\epsilon_\mathbf{p})N(T)$. This is a generalization to quantum fluids with nonzero mean free paths and finite temperatures of hydrodynamic estimates for classical fluids,[10] and of zero-temperature calculations for Fermi liquids.[11]

The relative size of the nucleon mean free path l and the wavelength of a disturbance $2\pi/q$ is an important parameter, since the transition between collisionless and hydrodynamic behavior occurs at $ql\sim 1$. Danielewicz's impulse-approximation calculation[12] of the shear viscosity η provides a relaxation time τ, by means of the expression $\tau=15\eta/N(T)\langle p^2v^2\rangle$. (The thermal conductivity reported in Ref. 12 leads to a relaxation time not materially different from this one.) We have then estimated a mean free path from the expression $l=\langle v^2\rangle^{1/2}\tau$, contours of which are shown in Fig. 1. Presumably the omission of effective-mass effects in the impulse approximation yields a lower limit to the mean free path. We thus conclude that for nuclear matter lying within the coexistence curve, even for the modes $q=\pi/R$ with the longest wavelengths, the mean free path exceeds q^{-1}. Consequently, conditions are close to the collisionless limit, and far from the hydrodynamic one. It would therefore appear that the adiabatic spinodal line, which is relevant for discussion of instabilities in the hydrodynamic limit, plays no role.

We next investigate how the growth of density fluctuations depends on the initial conditions for the expansion. The contour corresponding to an energy per particle, ϵ, equal to zero (measured relative to rest masses), in the absence of loss mechanisms, divides initial conditions with $\epsilon < 0$, for which bulk matter oscillates about the saturation density at that entropy, and those with $\epsilon > 0$, for which bulk matter can expand to infinity. For initial densities sufficiently close to saturation density, matter never oscillates into the unstable region, while for initial densities close to the $\epsilon = 0$ line, matter spends a long time in the unstable region. To estimate the factor by which a mode can grow we assume that the instantaneous growth rate in the expanding medium may be approximated by our results for a static medium. The growth factor is then simply e^G, when $G = \int \Gamma \, dt$, the integral being taken over the part of the trajectory inside the unstable region for the mode considered. In Fig. 2 we show boundaries, in the (n,T) plane for the initial state, of regions where G for some mode exceeds the values 0, 1, and 3 for a single traversal of, or excursion into, the unstable region. On the lines where $G = 0$, matter just reaches the boundary of the region where the lowest mode is unstable. As the initial density increases, at constant entropy per particle, more modes become unstable, and the first modes to reach growth corresponding to $G = 1$ and $G = 3$ have $\nu \simeq 2$ and $\nu \simeq 3$. The growth factor for a given mode has a maximum near the $\epsilon = 0$ line, as we explained, and above this line G decreases again. The last mode to drop below both $G = 3$ and $G = 1$ has $\nu = 3$.

To determine which modes can grow sufficiently to cause fragmentation of the drop, we need to estimate the level of density fluctuations. For a sound mode in thermal equilibrium, the fluctuation of the particle density is given classically by $\langle \delta n_q^2 \rangle / n^2 = T / A m c_s^2$, where A is

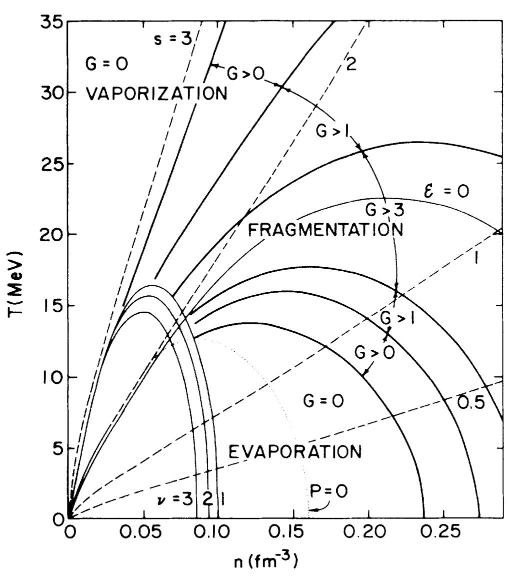

FIG. 2. Contours of initial conditions for which the growth exponent G, for the mode for which it is largest, falls within the specified range. The zero-pressure line, which gives the saturation density as a function of temperature, is shown together with adiabats, selected instability lines, and the zero-energy contour.

the number of nucleons and c_s the sound velocity. A plausible lower bound on the level of fluctuation as a mode becomes unstable is given by this expression evaluated when the mode is stable. If we take c_s to be $\simeq 0.16c$, the sound speed at saturation density, we find $\langle \delta n_q^2 \rangle^{1/2} / n \sim 0.04$ for all modes with $\nu > 1$. Such a fluctuation would require a growth exponent $G \sim -\ln(0.04) \simeq 3$ for it to disrupt the nucleus. The initial "seed" fluctuations present when the matter enters the unstable region may be larger if (1) the matter stays in thermal equilibrium down to lower densities, since c_s will then be smaller, or (2) before expansion the nuclear matter is not a stationary spherical blob. For example, a central collision between two heavy ions would enhance fluctuations for the longer-wavelength, e.g., quadrupolar, modes. Breakup could then occur for $G \sim 1$.

The initial conditions in the expansion process may be classified according to the expected final products, as was done by Vicentini, Jacucci, and Pandharipande[2] in their simulations for classical argon atoms interacting via a Leonard-Jones potential. For conditions corresponding to points on the low-temperature side of the low-temperature branch of the $G = 0$ contour, matter never reaches the unstable region, but in a finite system some particles may be lost by evaporation from the surface. There is the range of initial conditions, corresponding to points above this contour, for which matter enters the unstable region but density fluctuations do not grow enough to disrupt the matter. This corresponds to violent evaporation. The high-temperature boundary of this region depends on the level of fluctuations present when matter enters the unstable region, and our estimates above suggest that it should lie between the $G = 1$ and $G = 3$ contours. For initial conditions in which G exceeds the critical value for disruption, the outcome of the expansion is fragmentation.

With increasing initial temperature and/or decreasing initial density, G decreases until matter is not disrupted, but expands to low density without appreciable growth of instabilities. This corresponds to complete vaporization.

Thus the qualitatively different kinds of behavior expected in heavy-ion reactions as the initial conditions are varied find a natural explanation in terms of the growth of instabilities. It is surprising that the behavior obtained for hot nuclear matter is remarkably similar to that obtained for droplets of argon atoms,[2] for which the interaction and statistics are completely different, but the equation of state is similar.

We have couched our discussion in terms of the initial density and temperature of the blob of nuclear matter. These quantities depend on the equation of state at densities well above nuclear matter density, where it is rather uncertain. It is much better known at subnuclear densities, where the instabilities occur. In our calculations the high-density region contributes by determining the velocity with which the matter enters the unstable region. Our results may be expressed as functions of specific entropy and energy density, and one can then easily map out the initial conditions leading to different final states, and the curves corresponding to those in Fig. 2, for other high-density equations of state. By experimentally identifying the initial densities and temperatures (or entropies) that lead to the different outcomes,[13] one therefore has the possibility of determining the high-density and high-temperature equation of state.

Simple estimates of Coulomb and surface energies suggest that, although their inclusion changes the energy per particle significantly, the modification of the trajectory of the expanding matter, down to the region of the instability thresholds, is quite small, and the basic scenario that we have described is unaltered. The dynamics of the initial stage of a heavy-ion reaction and the creation of the hot blob require separate consideration, as does the nonlinear development of the instabilities.

We thank G. Baym, J. P. Bondorf, A. D. Jackson, J. A. Lopez, E. G. Lübeck, V. R. Pandharipande, D. Pines, K. Sneppen, J. Wambach, and A. Wirzba for helpful discussions. This research was supported in part by National Science Foundation Grants No. PHY86-00377 and No. PHY84-15064.

[1]For a review, see Nucl. Phys. **A471**, Nos. 1,2 (1987).

[2]A. Vicentini, G. Jacucci, and V. R. Pandharipande, Phys. Rev. C **31**, 1783 (1985).

[3]Preliminary estimates of these effects were reported by C. J. Pethick and D. G. Ravenhall, Nucl. Phys. **A471**, 19c (1987), and in *Hadrons in Collision,* edited by P. Carruthers and D. Strottman (World Scientific, Singapore, 1986), p. 277.

[4]The literature on this topic is so extensive that we have no space to quote it in detail, but content ourselves by citing the recent review by G. Bertsch and S. Das Gupta, Phys. Rep. **160**, 189 (1988).

[5]B. Friedman and V. R. Pandharipande, Nucl. Phys. **A361**, 502 (1981).

[6]For a review of the Landau kinetic equation see, e.g., G. Baym and C. J. Pethick, in *The Physics of Liquid and Solid Helium,* edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1978), Pt. 2, p. 1.

[7]See, e.g., D. Pines, Can. J. Phys. **65**, 1357 (1987).

[8]H. Heiselberg, C. J. Pethick, and D. G. Ravenhall, to be published.

[9]It is possible to include the momentum dependence of $f_{\mathbf{pp}'}$ given by the Skyrme interaction, but the calculations are simpler if one adopts the temperature-dependent form. Care is then needed in calculating certain thermodynamic quantities like the thermal expansion coefficient, but use of this interaction rather than the full one has little effect on the dispersion relation for collective modes.

[10]F. F. Abraham, Phys. Rep. **53**, 93 (1979).

[11]C. J. Pethick and D. G. Ravenhall, Ann. Phys. (N.Y.) **183**, 131 (1988).

[12]P. Danielewicz, Phys. Lett. **146B**, 168 (1984).

[13]M. Mahi *et al.*, Phys. Rev. Lett. **60**, 1936 (1988).