Vacuum Tunneling Probe: A Nonreciprocal, Reduced-Back-Action Transducer

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(Received 7 March 1988)

The vacuum tunneling probe used in the scanning tunneling microscope represents a new class of nonreciprocal electromechanical transducers. Nonreciprocity implies reduced back action and consequently increased sensitivity over conventional, reciprocal transducers. A vacuum tunneling probe may reach the quantum limit for a measurement of the position of a macroscopic mechanical oscillator even with use of a non-quantum-limited amplifier. The quantum limit is enforced by the momentum shot noise associated with the tunneling current.

PACS numbers: 61.16.Di, 06.30.Bp, 06.70.Mx, 85.30.Mn

The application of vacuum tunneling in the scanning tunneling microscope[1] and the atomic force microscope[2] has made it possible to monitor the motion of extremely small masses, even down to single atoms. A recent proposal to use a vacuum tunneling probe as the transducer for a gravitational-radiation detector[3] has prompted our investigation of the sensitivity limits of transducers which are based upon vacuum tunneling, and we have found some surprising results. The vacuum tunneling probe is representative of a *new class of active, nonreciprocal electromechanical transducers* which have greatly reduced back action on the mechanical element being monitored. Consequently the precision in a position measurement of a harmonic oscillator can be virtually independent of the noise of the amplifier following the tunneling probe. This follows from the nonreciprocity of the tunneling probe and is in sharp contrast with conventional reciprocal transducers in which the back action enforces an "amplifier limit" for the sensitivity. Our analysis shows that the quantum limit for a position measurement of a harmonic oscillator is enforced by the momentum shot noise associated with the tunneling current.

Weber-bar gravitational-radiation detectors are intended primarily to search for impulsive gravitational-wave events from supernova explosions which are expected to be the strongest common astrophysical sources.[4] Thus, in our analysis, we restrict our attention to impulse detection, although in other applications measurements of sinusoidal or steady forces may be required. A useful figure of merit for the detection of an impulse by a harmonic oscillator is the impulse noise number, n_I, which is defined as the number of quanta which would be deposited in the unexcited oscillator by the minimum detectable impulse, i.e., an impulsive force which gives a signal-to-noise ratio of 1. Note that, in general, both the energy and the phase of the mechanical oscillator will change upon reception of an impulse depending upon its arrival time relative to the oscillator phase.

Conventional transducers which operate by the modulation of a capacitance, inductance, or a piezoelectric crystal are reciprocal[5] and consequently obey the following constraint[6]:

$$n_I \geq n_A \geq 1, \tag{1}$$

where n_A is the amplifier noise number defined by $n_A = k_B T_A / \hbar\omega_0$ in which k_B is Boltzmann's constant, ω_0 is the angular resonant frequency of the mechanical oscillator which is being monitored, and T_A is the noise temperature of the amplifier which is used to monitor the transducer output. Thus, the smallest impulse noise number that one can achieve is equal to the noise number of the amplifier. Phase-insensitive linear amplifiers must obey the quantum limit,[7] $n_A \geq 1$, so that one can view the amplifier as enforcing the quantum limit for the harmonic oscillator. We show below that a transducer which uses a nonreciprocal tunneling probe may have an impulse noise number, n_I, which is much less than n_A, but that the shot noise which accompanies the tunneling current enforces the quantum limit $n_I \geq 1$. The quantum limit must be obeyed by the tunneling-probe transducer because the position of the harmonic oscillator, which is not a quantum nondemolition observable,[8] is the measured quantity.

How may a tunneling probe be used in a practical transducer? The area which a tunneling probe monitors is extremely small, and so to construct a gravitational-wave antenna transducer some type of mechanical impedance-matching element, which transforms the motion from the large antenna to a much smaller mass, must be used. A number of such schemes have been proposed and studied[3,9,10] but it is difficult to achieve a large effective-mass ratio. We believe that the tunneling probe will probably be of most use in integrated microelectromechanical devices which measure forces. The back-action noise of the amplifier becomes a dominant factor for very-low-mass devices and so the reduced back action of the tunneling probe may offer a substantial improvement in sensitivity.

The tunneling probe itself may be modeled as a resistor in which the resistance is exponentially dependent upon the separation of the probe tip from the surface of the object to be monitored. If the separation of the tip from the object is $d-x(t)$, in which d is the nominal gap and $x(t)$ is the small time-dependent part of the gap, we may express the probe resistance as

$$R=R_0\exp[-2\kappa x(t)], \tag{2}$$

where R_0 is typically $10^7\ \Omega$ for a nominal separation d of several angstroms and $\kappa=(2m_e\phi)^{1/2}/\hbar$, with ϕ the probe work function and m_e the electron mass. In our analysis we assume that the tunneling probe is dc voltage biased by the source V_{dc} so that a current $I_{\mathrm{dc}}=V_{\mathrm{dc}}/R$ flows through the probe. We also assume that the resistance of the tunneling probe is much greater than the internal resistance of the bias source or amplifier. The electrical schematic of the transducer is shown in Fig. 1. We represent the noise intrinsic to the tunneling probe by a voltage source in series with R. Since we assume that the probe is voltage biased and grounded through the current amplifier the fluctuations in the tunneling probe may be adequately represented by the voltage noise source, $S_{V_T}=2eI_{\mathrm{dc}}R^2$. In the limit of vanishing dc current we must have $S_{V_T}=4k_BTR$, which is the open-circuit Johnson noise of the tunnel resistor, but we will assume throughout that this limit is avoided. The signal is sensed by a current amplifier with noise that is represented by an input voltage noise generator, with spectral density S_{V_I}, and a purely additive current noise source with a spectral density S_{I_I}. We also include a capacitor, C, which shunts the tunnel junction to represent the stray capacitance of the probe.

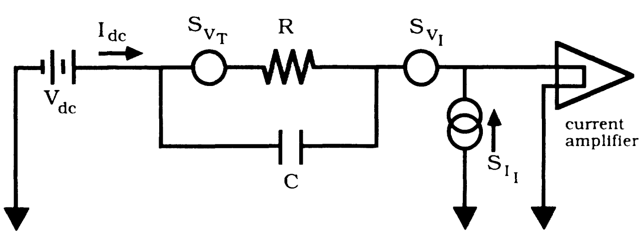

FIG. 1. The electrical schematic of the vacuum-tunneling-probe transducer and the amplifier. The tunneling probe represented by R is dc voltage biased by V_{dc} and the shot noise associated with the dc current is represented by a voltage source with spectral density S_{V_T}. The capacitance of the probe is represented by C. The amplifier is shown as an ideal current amplifier with an input voltage noise generator with spectral density S_{V_I} and a purely additive current noise source with spectral density S_{I_I}. All noise sources have a frequency-independent spectral density.

The analysis proceeds as follows. If we assume that $2\kappa x(t)$ is small, so that the exponential may be expanded, we find that the current developed through the tunneling probe due to a displacement $x(t)$ is given by

$$i=2\kappa I_{\mathrm{dc}}x(t). \tag{3}$$

Referring to the equivalent circuit in Fig. 1 one may calculate the fluctuating current at the output of the amplifier, and with Eq. (3) we find the spectrum of the apparent displacement fluctuations:

$$S_{x_{\mathrm{app}}}=\frac{1}{4\kappa^2I_{\mathrm{dc}}^2}\left[S_{I_I}+\frac{S_{V_I}}{R^2}(1+\omega_0^2R^2C^2)+\frac{S_{V_T}}{R^2}\right]. \tag{4}$$

The apparent displacement, x_{app}, is obtained from Eq. (4) by

$$x_{\mathrm{app}}=S_{x_{\mathrm{app}}}^{1/2}(\tau_{\mathrm{meas}}/2)^{-1/2}, \tag{5}$$

in which we multiply by the square root of the measurement bandwidth to find the apparent displacement.

In addition to the apparent motion of the mechanical oscillator represented by Eq. (4) there is an actual fluctuating displacement of the mechanical oscillator caused by the back-action force of the transducer and its Brownian motion.

There are two sources of the back-action force. Each electron which tunnels from the probe to the mechanical oscillator carries momentum. The fluctuating current in the tunneling probe therefore gives a fluctuating momentum transfer. The other source of back action is the capacitance of the probe. There is an electric field in the capacitor so that the fluctuating amount of charge on the tunneling probe produces a fluctuating force. The spectral density of the fluctuating force is therefore

$$S_f=\frac{p^2}{e^2}\left[\frac{S_{V_T}}{R^2}+\frac{S_{V_I}}{R^2}\right]+C^2E^2S_{V_I}, \tag{6}$$

in which p is the momentum transferred to the mechanical oscillator by one electron, e is the electron charge, and E is the electric field in the capacitor C. The first terms in Eq. (6) represent the back-action force from the tunneling probe and the last term, the back action from the capacitor. The displacement of the oscillator in a time interval τ_{meas} which is much less than the oscillator relaxation time is given by

$$x_{\mathrm{ba}}=\frac{S_f^{1/2}}{m\omega_0}\left[\frac{\tau_{\mathrm{meas}}}{2}\right]^{1/2}, \tag{7}$$

where m is the mass of the mechanical oscillator and ω_0 is its resonant angular frequency.

The displacement of the oscillator, in a time interval

$\tau_{\rm meas}$, due to the Brownian motion is given by[11]

$$x_{\rm BM} = [kT\tau_{\rm meas}/m\omega_0 Q]^{1/2}, \tag{8}$$

where Q and T are respectively the quality factor and the temperature of the mechanical oscillator. This result is valid for a measurement time which is much less than the relaxation time of the oscillator, $\tau_{\rm meas} \ll Q/\omega_0$.

We combine these results and have the actual displacement of the harmonic oscillator in a time interval $\tau_{\rm meas}$,

$$x_{\rm act} = \left[\frac{2k_{\rm B}T}{m\omega_0 Q} + \frac{p^2(S_{V_T}+S_{V_I})}{e^2 m^2 \omega_0^2 R^2} + \frac{C^2E^2S_{V_I}}{m^2\omega_0^2} \right]^{1/2} \left[\frac{\tau_{\rm meas}}{2} \right]^{1/2}. \tag{9}$$

Thus we have an expression for the total displacement noise of the form

$$x_{\rm noise} = A\tau_{\rm meas}^{1/2} + B\tau_{\rm meas}^{-1/2}, \tag{10}$$

where A is given by Eq. (9) and B is obtained from Eqs. (4) and (5). The optimum $\tau_{\rm meas}$ is given by $\tau_{\rm meas} = B/A$ and therefore the minimum value of $x_{\rm noise}$ is $x_{\rm min} = 2(AB)^{1/2}$. To form an expression for n_I, the impulse noise number, we note that the minimum detectable displacement, $x_{\rm min}$, is related to the minimum detectable impulse, $p_{\rm min}$, by $x_{\rm min} = p_{\rm min}/m\omega_0$ and that $n_I = (p_{\rm min}^2/2m)/\hbar\omega_0$. After some manipulation we find the following expression for n_I:

$$n_I = \frac{2p}{\hbar\kappa} \left[1 + \frac{S_{I_I}R^2}{S_{V_T}} + \frac{S_{V_I}}{S_{V_T}}(1+\omega_0^2R^2C^2) \right]^{1/2} \left[1 + \frac{S_{V_I}}{S_{V_T}} + \frac{S_{V_I}R^2C^2E^2e^2}{S_{V_T}p^2} + \frac{2k_{\rm B}Tm\omega_0 e^2R^2}{S_{V_T}Qp^2} \right]^{1/2}. \tag{11}$$

We have not yet discussed p, the momentum transfer per electron tunneling event. Measurements of the tunneling transit time are consistent with the tunneling electrons having the Fermi velocity,[12] $p = \hbar k_{\rm F}$. If we use $p = \hbar\kappa$ we find that the leading factor is 2.[13] On the other hand each tunneling electron introduces a momentum uncertainty to the bar. The electrons which tunnel from the tip are localized to near the surface of the bar. This constitutes a measurement of the electron position which has a position uncertainty of $\Delta x = (2\kappa)^{-1}$ and hence, with use of the uncertainty relation $\Delta x\,\Delta p \geq \hbar/2$, an uncertainty of the momentum transferred by each electron of $\Delta p = \hbar\kappa$. Even in the limit where the Fermi momentum approaches zero the minimum value of p is therefore $\hbar\kappa$. Both contributions to p in Eq. (11) give a result close to the quantum limit, $n_I = 1$. We do not attempt to settle the issue of the exact value of the quantum limit in this Letter.

The above expression accounts for the Brownian motion intrinsic to the mechanical oscillator and the back action of the fringing capacitance of the tunneling probe. We now calculate how close to the quantum limit one may approach in a realistic experimental configuration. We use the following physical parameters: $I_{\rm dc} = 10^{-7}$ A, $R = 10^7$ Ω, $E = 10^9$ V/m, $C = 10^{-15}$ F, and $p_{\rm F} = 1.4\times10^{-24}$ kg-m/sec. The mechanical oscillator is assumed to be a cantilever or similar structure which could be fabricated from silicon with anisotropic etching techniques to dimensions on the order of 10 μm. We assume $m = 10^{-11}$ kg, $\omega_0 = 2\pi\times10^3$ rad/sec, $T = 1$ K, and $Q = 10^6$. The amplifier is assumed to have $S_{V_I} = 1.4\times10^{-21}$ V^2/Hz and $S_{I_I} = 3.2\times10^{-28}$ A^2/Hz which corresponds to an amplifier noise number of $n_A = 10^6$. We calculate that $S_{V_T} = 3.2\times10^{-12}$ V^2/Hz. These values give $n_I = 25$. In this case the back action of the stray capacitance accounts for the noise in excess of the quantum limit. If the capacitance were 100 times less, say 10^{-17} F, it should be possible to reach the quantum limit. Point-contact junctions have been reported with capacitances in the 10^{-18} F range.[14]

We emphasize that a conventional transducer which is reciprocal would have obeyed the limit $n_I = n_A = 10^6$. Our calculations show that the reduced back action of the nonreciprocal tunneling probe allows a reduction of the noise by a factor of 40000. The near elimination of the transducer back action accounts for this improvement and the small remaining back action of the tunneling probe combined with the shot noise enforces the quantum limit for the measurement of the harmonic-oscillator position.

We have shown that the tunneling probe, which is already in wide use, has the remarkable feature that it is nonreciprocal in much the same way as a transistor, both being examples of active two-port devices. We have also shown that nonreciprocity allows one to greatly exceed the amplifier limit obeyed by conventional transducers and that the quantum limit is enforced by the momentum shot noise associated with the tunneling current. If the practical difficulty of mechanically impedance matching a tunneling-probe transducer to a massive gravitational-wave antenna can be solved it may offer a new way to reach the quantum limit for gravitational-wave detectors. We also note that since the tunneling probe is so well suited for the monitoring of small masses, in which back-action noise is a dominant factor, it may become an important strategy for miniaturized integrated electromechanical sensors.

We would like to thank Professor Edward Jacobsen, Professor Michael Wengler, and Professor Alan Kadin at the University of Rochester and Dr. Rolf Landauer at IBM for helpful discussions. This work was supported by the U.S. Office of Naval Research.

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