Transparency in Nuclear Quasiexclusive Processes with Large Momentum Transfer

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According to perturbative QCD, nuclear quasiexclusive reactions should be $\propto A^1$ at asymptotically large momentum transfer. Here we analyze how the asymptotic prediction is modified at finite energies by the expansion of the quark systems as they become hadrons and exit from the nucleus. We find that the phenomenon of nuclear transparency should be apparent in presently feasible experiments, but the degree of transparency is sensitive to the expansion model.

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Large-momentum-transfer exclusive hadron scattering is experimentally observed to obey the scaling behavior $d\sigma/dt - S^{2-n}f(t/s)$, when n is the total number of pointlike constituents in the final and initial hadrons, as predicted by perturbative QCD (PQCD), neglecting logarithms.[1,2] This behavior can be understood heuristically as follows. The initial hadrons A and B fluctuate with some amplitudes f_A and f_B to their minimal Fock-space component ($q\bar{q}$ for a meson and qqq for a baryon). In order for the exclusive large-momentum-transfer scattering to occur, both of these hadrons must occupy a region of transverse dimension $\sim 1/\sqrt{|t|} \equiv 1/q$. The amplitude for such a fluctuation to occur is in general $(m/q)^{k-1}$, where k is the number of constituents of the hadron and m^{-1} is the transverse dimension of the typical configuration of the hadron. Similarly, the final quarks have amplitudes $f_C(m/q)^{k_C-1}$ and $f_D(m/q)^{k_D-1}$ to be small-size fluctuations of hadrons C and D. The combination of the above factors leads to the predicted scaling behavior.

The fixed-c.m.-angle energy dependence of the exclusive cross sections is the best evidence for the relevance of PQCD to exclusive hadron scattering. Calculations of the magnitudes of form factors have been criticized[3] as being too sensitive to soft regions of internal momenta to be legitimately described to PQCD, while violation of hadronic helicity conservation[4] and the oscillations of the pp differential cross sections about the simple s^{-10} energy dependence[5] are further evidence that leading-twist PQCD is not the whole story.[6] In this circumstance it is very useful to have some direct evidence of the relevance of PQCD. Mueller and Brodsky[7] have suggested studying the A dependence of large-momentum-transfer reactions such as $h+A \rightarrow h+N+A'$, where A' is a nucleus consisting of $A-1$ nucleons, in order to probe the transverse-size description of large-t exclusive scattering described above. The requirement that there be no soft pions produced ensures that the fundamental process is just elastic hN scattering, with the nucleus as a source of nucleons.[8,9] (In general, of course, the target nucleon is not at rest due to the nuclear-binding effect, nor is it precisely on mass shell.) If, indeed, large-t exclusive scattering involves hadrons which have fluctuated to be of transverse size $\sim 1/\sqrt{|t|}$, they must also be small for some time before or after the interaction with the target nucleon, and therefore have reduced total cross section with the nuclear medium and a better chance to travel out of the nucleus without breaking up the nucleus and/or producing soft pions. Thus asymptotically the cross section for this reaction $\propto A^1$.

Our purpose in this Letter is to examine quantitatively whether one can expect to observe the nuclear-transparency effect in the presently accessible regime of energy.

Nuclear-binding effects are neglected here and we treat them elsewhere.[8] Under this approximation we have for the nuclear transparency A_{eff}/A in the semiclassical approximation:

$$\frac{A_{\text{eff}}(\mathbf{p}_0,\mathbf{p}_1,\mathbf{p}_2)}{A} = \frac{1}{A}\int d^3r \rho_A(\mathbf{r})\mathcal{P}_0(\mathbf{p}_0,\mathbf{r})\mathcal{P}_1(\mathbf{p}_1,\mathbf{r})\mathcal{P}_2(\mathbf{p}_2,\mathbf{r}),$$

where $\mathbf{p}_0$, $\mathbf{p}_1$, and $\mathbf{p}_2$ are the momenta for the incoming hadron, outgoing hadron, and the knocked-out nucleon, respectively; ρ_A is the nuclear density. The probabilities that no interaction occurs to the entering and exiting hadrons are

$$\mathcal{P}_i(\mathbf{p}_i,r) = \exp\left[-\int_{\text{path}} dz\, \sigma^{\text{eff}}(\mathbf{p}_i,z)\rho_A(z)\right].$$

The integration $\int_{\text{path}}$ is along the physical path of the hadron between the interaction point and the nuclear surface (see Fig. 1), and σ^{eff} is its effective cross section which is a function of its momentum and distance, z, from the interaction point.

Evidently, those parts of the integration which contribute most to the nuclear absorption are the parts where σ^{eff} is relatively large. However, in that case one cannot deduce the exact form of σ^{eff} from perturbative QCD and some model for the shrinkage-expansion mechanism has to be invented. In this Letter, we investigate the sensitivity of the predictions for A_{eff}/A to the expansion model by taking three different models of the expansion.

We estimate the dependence of σ_{hN}^{eff} on the distance z from the point where the hard interaction occurs, as follows. We suppose the effective cross section is simply scaled by the transverse size of the quark system relative to the average size $\langle x_t \rangle$ of the hadron, so that $\sigma_{hN}^{\text{eff}} = [x_t^2(z)/\langle x_t^2 \rangle]\sigma_{hN}$.[10] We take $\sigma_{pp}=40$ mb and $\sigma_{p\pi}=25$ mb in our calculation. To find the dependence of x_t on z, we imagine that the quarks occupy a transverse area $\sim(n^2\langle k_t^2\rangle/t)\sigma_{hN}$ at the point of interaction, where $n=2$ for pion, $n=3$ for nucleon, and $\langle k_t^2\rangle^{1/2} \simeq 0.35$ GeV/c is the average transverse momentum of a parton in a hadron. They then expand until they reach their normal hadronic size in a distance we shall denote l_h. To describe the expansion, we consider two extreme models.

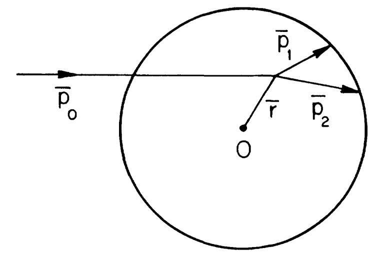

FIG. 1. Nuclear quasiexclusive scattering.

In a naive picture of partons separating at the velocity of light, $x_t \sim t \sim (E/m)^{-1}z$, where t is the hadron-rest-frame time, z is the laboratory longitudinal coordinate, and E/m is the time-dilation factor. This suggests $x_t^2 \sim z^2$. The alternate picture we consider is inspired by perturbative QCD, but is rather generically a behavior which could be called "quantum diffusion." The asymptotically most important energy denominator in PQCD diagrams describing the evolution of the quark system $\sim\sum_i(m_i^2/\alpha_i + k_{ti}^2)/2P_h$. This suggests $x_t^2 \sim 1/\langle k_t^2\rangle \sim z$. It is important that gluon radiation, which is gauge dependent, does not change this result, because of its coherence.

Based on the above reasoning, we take the following form for the effective cross section:

$$\sigma_h^{\text{eff}} = \sigma_{hN}^{\text{tot}}\left(\left\{\left[\left(\frac{z}{l_h}\right)^\tau + \frac{\langle n^2k_t^2\rangle}{t}\left[1-\left(\frac{z}{l_h}\right)^\tau\right]\right]\right\}\theta(l_h - z) + \theta(z - l_h)\right).$$

$\tau=0$ corresponds to a "non-PQCD" picture, in which there is no reduction in the effective cross section of the hadrons which participated in the large-momentum-transfer exclusive scattering. $\tau=1$ corresponds to quantum diffusion, and $\tau=2$ to the naive parton case.

In the naive parton model, $l_h \approx (E/m)(\sigma_{hN}/\pi)^{1/2}$, leading to $l_h \approx E/m_h$ fm. In the PQCD-inspired model, l_h is determined by the average value of the dominant energy denominator:

$$l_h \simeq \langle 1/(E_n - E_h)\rangle \simeq 2P_h\langle 1/(M_n^2 - M_h^2)\rangle,$$

where M_n^2 is the mass squared of the typical intermediate state, n, of the hadron. In a constituent quark model which is tuned to correctly describe nucleon electromagnetic form factors, and in the multiperipheral model, $\langle 1/(M_n^2 - M_N^2)\rangle \simeq 1/(0.7\ \text{GeV}^2)$. Estimating l_π is much more difficult because of the Goldstone nature of the pion. Mueller has suggested[7] $\langle 1/(M_n^2 - M_\pi^2)\rangle \simeq 1/(0.25\ \text{GeV}^2)$ on semiclassical grounds. Clearly, the above estimates are educated guesses at best.

We approximate the nuclear density as $\rho(\mathbf{r}) = c/(1+e^{-R+r}/b)$ with $b=0.56$ fm and $R=1.1A^{1/3}$ fm. The overall normalization is guaranteed by our choosing c such that $\int\rho(\mathbf{r})d^3r = A$. This should be a good approximation for medium and heavy nuclei.

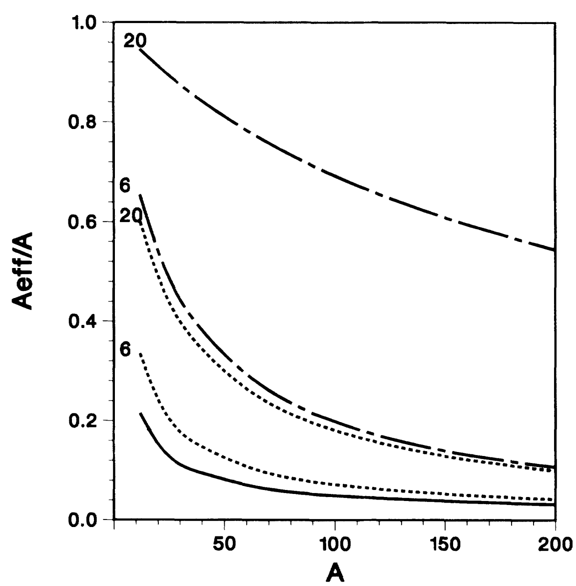

FIG. 2. A_{eff}/A for pA scattering (in this and the following figures all reactions are 90° in the c.m. system) as a function of A for $\tau=0$ model (solid line), $\tau=1$ model (dashed line) with $p_{\text{lab}}=6$ GeV/c (marked 6) and $p_{\text{lab}}=20$ GeV/c (marked 20), and $\tau=2$ model (long-short-dashed line) with $p_{\text{lab}}=6$ GeV/c (marked 6) and $p_{\text{lab}}=20$ GeV/c (marked 20).

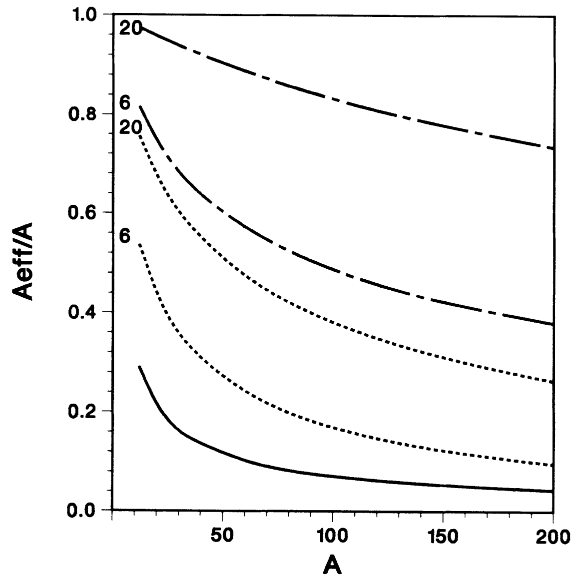

FIG. 3. As in Fig. 2, but for πA scattering.

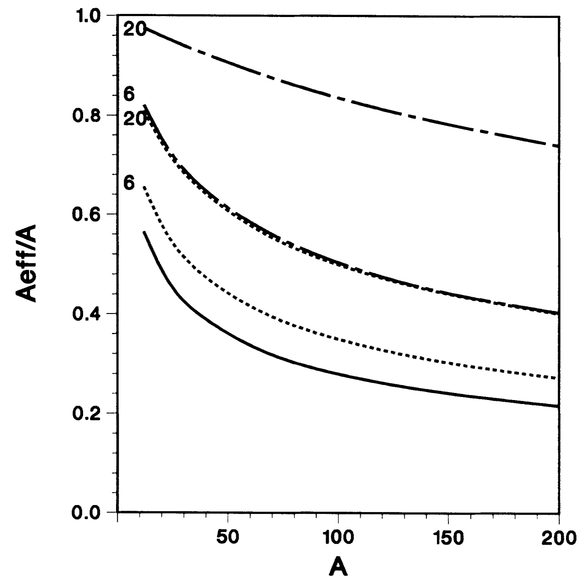

FIG. 4. As in Fig. 2, but for eA scattering.

We have calculated the nuclear transparency, $A_{\rm eff}/A$, as a function of A for $p+A \rightarrow p+N+(A-1)$ (Fig. 2), $\pi+A \rightarrow \pi+N+(A-1)$ (Fig. 3), and $e+A \rightarrow e+N+(A-1)$ (Fig. 4). The beam momenta considered are 6 and 20 GeV/c; throughout, we take the scattering to be at 90° in the center of mass. For each energy we plot two curves for $\tau=1$ and $\tau=2$, respectively. The case where $\tau=0$, which is energy independent, is also plotted in each of the three figures. The $\tau=0$ are consistent with those given in the less sophisticated calculation reported in Ref. 7. As discussed above, $\langle \Delta M^2 \rangle$ is taken to be 0.7 GeV^2 for a proton and 0.25 GeV^2 for a pion, for the $\tau=1$ model.

As can be seen from the figures, the transparency predictions are model dependent even for 20-GeV/c incident momentum. The absorption is less for the "naive parton" model of expansion, both because $\sigma_{\rm eff}$ has a lower average value when it grows quadratically with the distance, and because the naive parton values of l_h are a factor of 3 to 5 larger than the corresponding quantum diffusion values. The important point is that both models predict a significant departure from what would be expected in the absence of the transparency effect, even at low energies.

In Fig. 5 we plot the nuclear transparency $A_{\rm eff}/A$ for 90° pA scattering as a function of the incoming momentum in order to show the energy dependence more directly. To illustrate the sensitivity to the parameter $\langle \Delta M^2 \rangle$, we plot results of the $\tau=1$ model, using two values of $\langle \Delta M^2 \rangle$, 0.7 and 0.5 GeV^2. Note that the $p_{\rm inc}=0$ intercept is also the value of the $\tau=0$ (no transverse shrinkage) prediction, which is energy independent.

Similar results for eA scattering are given in Fig. 6. However, this time we plot results from both $\tau=1$ and $\tau=2$ models, with $\langle \Delta M^2 \rangle = 0.7$ GeV^2 for $\tau=1$. From the results displayed in the figures, we conclude that experimental investigation of nuclear transparency is worthwhile even at present energies ($t \gtrsim$ a few GeV^2 and $p_{\rm lab} \sim 10$ GeV/c). In spite of the model dependence, perturbative QCD predicts rather unambiguously that $A_{\rm eff}/A$ should be greater than ~ 0.3 for hadron-induced reactions, and that it should increase with increasing $p_{\rm lab}$.

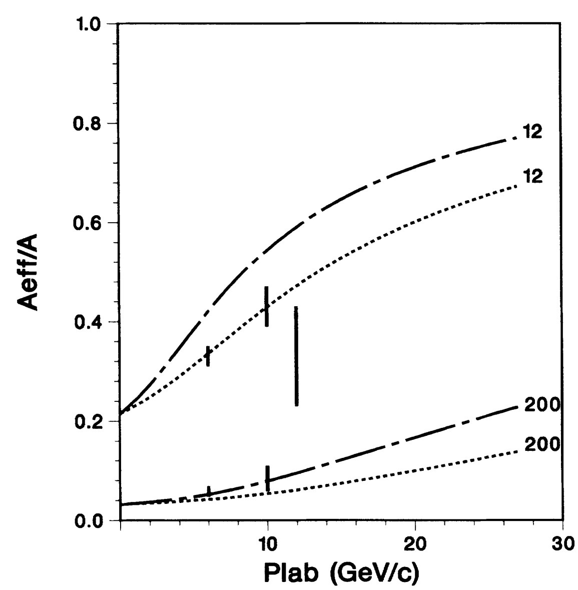

FIG. 5. $A_{\rm eff}/A$ for pA scattering as a function of beam energy for the $\tau=1$ model for different $\langle \Delta M^2 \rangle$'s compared with experimental data points (vertical bars) from Ref. 11 with dashed lines for $\langle \Delta M^2 \rangle = 0.7$ GeV^2 and dash-chain lines for 0.5 GeV^2 for $A=12$ and 200 as explicitly marked in the figure.

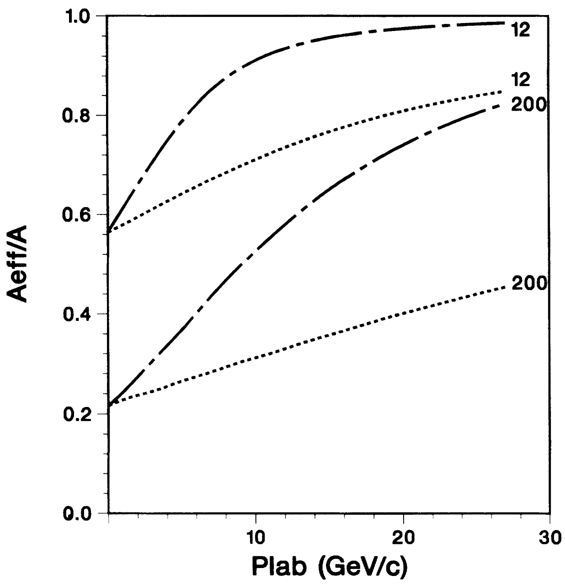

FIG. 6. A_{eff}/A for eA scattering as a function of beam energy, for $\tau=1$ model (dashed line) and $\tau=2$ model (dash-chain line) for $A=12$ and $A=200$ as marked in the figure.

A BNL experiment[11] has recently reported a level of transparency which is significantly larger than would be expected in the absence of a reduction in σ_{eff}; thus agreeing with the qualitative PQCD expectation. However, the momentum dependence they find is in apparent disagreement with the perturbative QCD prediction: Although A_{eff}/A increases with increasing momentum up to about 10 GeV/c, it then decreases. The meaning of this result is unclear. It has been interpreted as a charm-threshold effect[12] and as evidence for interference effects.[13] The experiment does not measure the three-momenta of both final particles, so that the target-particle four-momentum cannot be determined without some additional assumption about the mass or energy of the bound target nucleon. Thus s of the elementary scattering process is not known with certainty, even if the three-momentum of the target nucleon is small. Since the elementary scattering cross section is an extremely sensitive function of s, altering their assumption that the energy of the target nucleon is equal to the proton mass could significantly alter the A_{eff}/A which they extract from the data.[14] (Monte Carlo studies suggest that their conclusion is not very sensitive to this problem.[15]) We are currently studying the expected energy spectrum of the target nucleons in harmonic-oscillator and mean-field models, to estimate the extent of the effect. Another issue which deserves closer scrutiny is the excitation of the "$A-1$" nucleus, and its implications for the experimental acceptance of the event. Thus far, the theoretical treatment of this aspect of the problem has been very superficial. Clearly, independent verification of the BNL results is desirable, ideally in an experiment capable of measuring both final three-momenta.

We have benefitted very much from discussions with A. Mueller; he suggested the $\tau=1$ model in leading-logarithmic approximation. We also wish to acknowledge the BNL experimental group, especially A. Carroll and S. Heppelmann, for piquing our interest in this subject. One of us (G.R.F.) was supported in part by the National Science Foundation under Grant No. NSF-PHY-84-15535-05 and another (H.L.) was supported by National Science Foundation Grant No. NSF-PHY-86-05380.

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