Upper Bound on the Higgs-Boson Mass in the Standard Model

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The electroweak model with one Higgs doublet is investigated when the mass of the Higgs particle is larger than the weak-interaction scale. It is shown that the SU(2) Higgs sector with spontaneously broken O(4) symmetry and perturbative gauge coupling becomes a trivial field theory at infinite cutoff. Around the trivial Gaussian fixed point, for finite cutoff, a low-energy effective theory is found with nontrivial couplings and mass generation from spontaneous symmetry breaking. We find an upper bound of $M_H \approx 640$ GeV on the mass of the Higgs particle with a lattice momentum cutoff 2π times larger than the Higgs-boson mass.

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In the standard electroweak model with $SU(2)_L \times U(1)$ symmetry the Higgs sector is described by a complex scalar doublet Φ with quartic self-interaction. The Euclidean action is given by

$$S=\int d^4x\left[\tfrac{1}{2}(\partial_\mu+\tfrac{1}{2}ig\boldsymbol{\tau}\cdot\mathbf{W}_\mu)\Phi^\dagger(\partial_\mu-\tfrac{1}{2}ig\boldsymbol{\tau}\cdot\mathbf{W}_\mu)\Phi + V(\Phi^\dagger\Phi)\right]+S_{\text{gauge}}+S_{\text{fermion}}, \quad (1)$$

where S_{gauge} designates the gauge part of the action with an SU(2)-triplet vector field $\mathbf{W}_\mu$; $\boldsymbol{\tau}$ stands for the three isospin Pauli matrices, and g is the gauge coupling constant ($g^2 \approx 0.4$). The gauge coupling is expected to generate small corrections to the dynamics of the Higgs sector. The fermion masses are generated by S_{fermion} through Yukawa couplings. We will assume here that the Yukawa couplings are small and play no role in the dynamics of the Higgs field (light fermions). The O(4)-symmetric Higgs potential has the form

$$V(\Phi^\dagger\Phi)=-\tfrac{1}{2}m^2\Phi^\dagger\Phi+\lambda(\Phi^\dagger\Phi)^2, \quad (2)$$

where m^2 is a mass parameter and λ designates the quartic coupling constant.

Consider the Lagrangean for the Higgs sector before the gauge coupling is turned on. Shifting the origin of the field Φ in the spontaneously broken phase by the vacuum expectation value v we find that Eq. (1) describes three massless Goldstone bosons (w^+,w^-,z^0) and one neutral Higgs particle h with the tree-level mass $M_H=\sqrt{2}m$. The Fermi coupling constant G_F and the Higgs-boson mass M_H, in tree approximation, are related to v and λ by $1/v^2=\sqrt{2}G_F$ and $\lambda=G_F M_H^2/4\sqrt{2}$, respectively. The vacuum expectation value $v\approx 250$ GeV is determined by the Fermi coupling constant and λ depends explicitly on the Higgs-boson mass.

When the gauge coupling is turned on, the Goldstone bosons will acquire gauge-dependent masses through mixing with the longitudinal W bosons. In 't Hooft-Feynman gauge where the masses of $w^\pm$ and z become M_W and M_Z, respectively, the following high-energy theorem[1] holds: The scattering amplitude $T(W_L^+,W_L^-,Z_L,H)$ for longitudinal vector bosons and physical Higgs particles, at center of mass energies $\sqrt{s}$ much larger than M_W and M_Z, is identical [apart form $O(M_W/\sqrt{s})$ corrections] to the equivalent scattering amplitude $T(w^+;w^-,z,h)$ of the O(4)-symmetric Higgs model at zero gauge coupling.

Unitarity constraint on the partial-wave scattering amplitudes leads to the upper bound $M_H/v \leq (16\pi/3)^{1/2}$ in tree-level approximation[1] with the numerical value $M_H \leq 1.02$ TeV. The tree-level relation $\lambda=M_H^2/8v^2$ implies a perturbative unitarity bound $\lambda \leq 2\pi/3$ for the coupling constant. At the $M_H=1$ TeV saturation point $\lambda=2\pi/3$ is rather strong coupling and perturbation theory is expected to break down. It has been also suggested that Higgs-boson contributions to radiative corrections for M_H larger than the weak-interaction scale $1/\sqrt{G_F}$ may lead to the breakdown of the perturbative expansion for weak interactions.[2]

In the language of modern quantum field theory and critical phenomena it is believed that the complex Higgs doublet of the SU(2) sector in the standard electroweak model is defined on a trivial Gaussian fixed point in the continuum limit. The renormalized coupling λ_R has to vanish in the infinite cutoff limit and the allowed range of λ_R at finite and large cutoff has to be determined in the low-energy effective theory with correct renormalization and scaling properties. The limited range of λ_R implies an upper bound on the mass of the Higgs particle.[3] The purpose of our work is to investigate this problem in a nonperturbative fashion.

Supercomputer analysis of the effective lattice action.—For a realistic estimate of an upper bound on the Higgs mass we will study the O(4) limit of the lattice SU(2) Higgs model with the Euclidean lattice action

$$S=\tfrac{1}{2}\sum_i\left[\sum_{\hat\mu}(\vec\Phi_{i+\hat\mu}-\vec\Phi_i)^2-m^2\vec\Phi_i^2\right]+\lambda\sum_i\vec\Phi_i^4, \quad (3)$$

where m^2 and λ are bare parameters and the field variables $\vec\Phi_i$ are O(4) vectors on lattice sites labeled by i. The unit vector $\hat\mu$ points along the four positive lattice directions (the lattice spacing is set to unity in our calculations).

Consider the phase diagram of the O(4)-symmetric Euclidean lattice action of Eq. (3) in the bare parameter space of λ and m. The critical line where the inverse Higgs-boson mass diverges in lattice spacing units will separate the spontaneously broken Higgs phase from the symmetric phase. The trivial Gaussian fixed point implies $\lambda_R \to 0$ as we approach the critical line for any fixed bare λ.

For any reasonable definition of the renormalized coupling, the relation $M_H^2/v^2 = 8\lambda + O(\lambda_R^2)$ holds in perturbation theory. In leading order of the gauge coupling g the relation $M_W = gv/2$ between the renormalized vector boson mass and the renormalized Higgs-field expectation value holds so that M_H/M_W is expected to vanish on the critical line (continuum limit) which is the dynamical origin of the triviality upper bound on the Higgs-boson mass. We will use a nonperturbative technique[4] to study the effective lattice action for the verification of the conjectured triviality scenario and the related upper bound.

The effective action when restricted to constant classical field configurations is identical to the effective potential $U_\Omega(\vec{\Phi}_c)$ defined by[4-6]

$$e^{-\Omega U_\Omega(\vec{\Phi}_c)} = \int D[\vec{\Phi}]\delta\left(\vec{\Phi}_c - \Omega^{-1}\sum_i \vec{\Phi}_i\right) e^{-S[\vec{\Phi}]}, \quad (4)$$

where Ω designates the finite lattice volume (it is equal to the number of lattice sites in our units) and the integration is over the field variables $\vec{\Phi}_i$. The effective potential $U_\Omega(\vec{\Phi}_c)$ is nonconvex in the broken-symmetry phase and has a direct physical interpretation: The probability density $P(\vec{\Phi}_c)$ to find the system in a state of "magnetization" $\vec{\Phi}_c$ is given by $P(\vec{\Phi}_c) = \text{const} \times \exp[-\Omega U_\Omega(\vec{\Phi}_c)]$, in close analogy with statistical physics. With this unique feature we can develop a direct and visual physical picture of spontaneous symmetry breaking.

The critical line in the bare parameter space is observed at the crossover points of the effective potential from convex to nonconvex shape. Although there is no conventional order parameter for spontaneous breaking of the continuous O(4) symmetry in a finite volume, with the effective potential a clean signal is provided for the transition. The O(3) symmetry of the nonconvex effective potential in the broken phase accounts for the three Goldstone particles associated with the symmetry breaking.

We first determined the critical points on the phase transition line for several values of the bare coupling constant as a function of the bare mass m^2. Results for $\lambda = 10$, which is strong bare coupling with our normalization of the quartic interaction, are reported here. We calculated the effective potential at seven different values of m^2 within a narrow range on both sides of the critical point. At $\lambda = 10$ the critical point on the second-order phase transition line was found at $m_c^2 = 27.10(5)$.

In unconstrained runs using the hybrid Monte Carlo algorithm[7] we also studied the two-point function which is a sum of a longitudinal and transverse part,

$$[G^{-1}]_{ij}^{\alpha\beta} = n_\alpha n_\beta (G_L)_{ij}^{-1} + (\delta_{\alpha\beta} - n_\alpha n_\beta)(G_T)_{ij}^{-1}, \quad (5)$$

where i,j label the lattice sites whose correlation is measured and n_α is the four-component unit vector along the "magnetization" $M^\alpha = (1/\Omega)\sum_i \Phi_i^\alpha$. The two-point function $\tilde{G}_L^{-1}(p)$ in momentum space was measured at every value of m^2 for an independent determination of the renormalized mass M_H and the field renormalization constant Z_Φ.

The longitudinal two-point function $\tilde{G}_L^{-1}(p)$ was fitted against the inverse of the free and massless longitudinal propagator given by $\tilde{G}_{0L}^{-1}(p) = 4\sum_\mu \sin^2(p_\mu/2)$ on the finite lattice with discrete momentum components p_μ appropriate for helical boundary conditions. The plot in Fig. 1 shows a linear dependence on $\tilde{G}_{0L}^{-1}(p)$ indicating small contributions to the spectral function of the propagator from higher-mass intermediate states besides the dominant pole term. The slope of the fit determines Z_Φ and the intercept at zero momentum corresponds to M_H^2/Z_Φ.

The infrared behavior of the two-point function and the effective potential in finite volume, together with a detailed account of our work, will be discussed elsewhere.[8] Here we can only briefly outline how the infrared problem is handled in the calculation.

The longitudinal two-point function in the infinite volume limit has a branch cut starting at zero momentum in the broken-symmetry phase as a result of the presence of Goldstone modes. The massive Higgs parti-

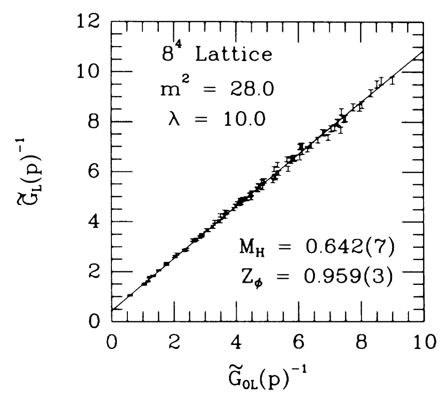

FIG. 1. The longitudinal part of the inverse momentum-space lattice propagator is plotted against the longitudinal part of the inverse free momentum-space propagator (massless) on the finite lattice for $m^2 = 28.0$ in the O(4) model; $1/Z_\Phi$ is the slope of the curve and M_H is obtained from the intercept. The errors on M_H and Z_Φ are determined from the linear fit of a typical Hybrid Monte Carlo run with 2×10^4 momentum refreshes and ten microcanonical steps between refreshes. Errors from averages of several runs are given in Figs. 2, 3, and 4.

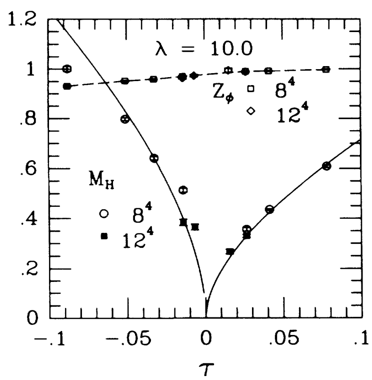

FIG. 2. The renormalized mass M_H and the field renormalization constant Z_Φ are plotted against $\tau = 1 - m^2/m_c^2$. The solid line indicates the logarithmically corrected scaling law for $M_H \sim (|\tau|)^{1/2} |\ln|\tau||^{-1/4}$. The dashed line for Z_Φ is only drawn to guide the eye and has no other significance.

cle appears as a complex pole in the complex momentum square plane. The real part of the pole defines the mass of the Higgs particle and the decay width into Goldstone particles is given by the imaginary part of the pole. Even for a 600-GeV Higgs-boson mass the width is expected to be less than 20% of the mass and a narrow peak in the propagator spectral function should be a very good approximation

To avoid the infrared branch point at zero momentum in the propagator, the wave-function and mass renormalization are carried out first at some nonzero Euclidean point $p^2 = \mu^2$. The inverse propagator will deviate from an approximate straight line only at very low momenta $p^2 \ll \mu^2$ which are avoided in our finite lattice simulation. The intercept of the straight-line inverse propagator gives the mass defined at the Euclidean renormalization point and it differs from the physical mass by a calculable small amount. We estimate that the procedure leaves only a few percent undetermined correction in the mass of the Higgs particle which is comparable, or smaller than the error from the zero-width approximation. The infrared regularization for the effective potential works very similarly to the method we just outlined for the longitudinal propagator.

From the analysis of the longitudinal part of the momentum-space propagator and from the effective potential we determined the renormalized mass M_H and the field renormalization constant Z_Φ in the symmetric and broken phases, the vacuum expectation value of the renormalized field operator in the broken-symmetry phase, and the renormalized coupling constant λ_R in the symmetric phase. The renormalized mass M_H and the field renormalization constant Z_Φ are plotted against $\tau = 1 - m^2/m_c^2$ in Fig. 2; the vacuum expectation value $\langle |\vec{\Phi}_R| \rangle$ and the ratio $M_H / \langle |\vec{\Phi}_R| \rangle$ are shown in Fig. 3 against the τ variable.

The triviality bound.—With the assumption of a Gaussian fixed point at $\lambda_R = 0$, the logarithmic corrections to mean-field critical behavior at the higher critical dimension $d = 4$ can be calculated in perturbation theory for the O(n) lattice model,[9]

$$\langle |\vec{\Phi}_R| \rangle \approx (-\tau)^{1/2} |\ln(-\tau)|^{3/(n+8)},$$
$$m_R \approx |\tau|^{1/2} |\ln|\tau||^{-(n+2)/2(n+8)}, \qquad (6)$$
$$\lambda_R \approx 1/\ln|\tau|.$$

The logarithmic scaling corrections of Eq. (6) are consistent with the data points as illustrated in Fig. 4 for the renormalized field vacuum expectation value. The trivial Gaussian fixed point at $\lambda_R = 0$ is also demonstrated in Fig. 4. For strong bare coupling, renormalization effects make λ_R small and logarithmically decreasing as we ap-

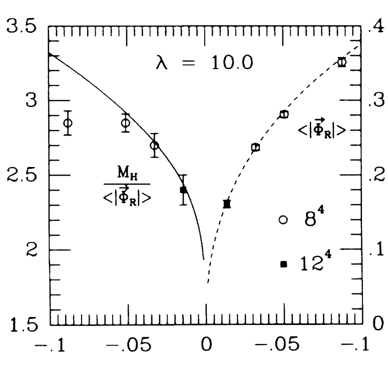

FIG. 3. The dashed line is the logarithmically corrected scaling law of the vacuum expected value $\langle |\vec{\Phi}_R| \rangle \sim (|\tau|)^{1/2} |\ln|\tau||^{1/4}$. The solid line corresponds to $M_H / \langle |\vec{\Phi}_R| \rangle \sim |\ln|\tau||^{1/2}$ with the same scaling behavior as $\sqrt{\lambda_R}$. Both sides of the figure represent the broken-symmetry phase.

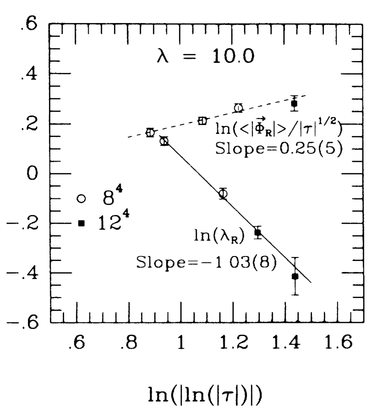

FIG. 4. Logarithmic plot to test the leading scaling correction for the renormalized field vacuum expectation value as given by Eq. (6) (the theoretical value of the slope is $\frac{1}{4}$). The same test is also shown for the renormalized coupling constant λ_R in the symmetric phase with a slope of -1 from Eq. (6).

proach the critical line from the unbroken phase.

Inside the scaling region we have a reasonable effective theory with nonvanishing renormalized coupling λ_R. The ratio $M_H/\langle|\vec{\Phi}_R|\rangle$ in Fig. 3 cannot grow very large before scaling will break down. Since explicit cutoff dependence begins to show in physical observables outside the scaling region, a bound exists on the ratio of the renormalized Higgs-boson mass and the renormalized vacuum expectation value of the Higgs field at fixed bare coupling.

At $\tau=-0.02$ which corresponds to a Higgs correlation length $M_H^{-1}=2$ we find $M_H/v=2.6$. At a correlation length of 5 we estimate from our fit that the bound would change to 2.2. With the value $M_H/v=2.2$ we find the upper bound $M_H \approx 550$ GeV on the mass of the Higgs particle. If scaling can be extended without noticable cutoff artifacts to $M_H^{-1}=2$ then the upper bound on the Higgs-boson mass would change to $M_H \approx 640$ GeV, a logarithmically slow change $\ln^{1/2}(\tau_1/\tau_2)$ between two points. We do not expect the bound on the ratio M_H/v to change significantly in the $\lambda \to \infty$ limit.

After we completed our work[10] we received a preprint where a Monte Carlo calculation of the triviality bound on the Higgs-boson mass was reported with some results similar to ours.[11] We should note that an upper bound on the mass of the Higgs particle was reported earlier in the O(4) approximation using approximate renormalization-group equations in momentum space.[12] We felt, however, that it was necessary to study the model beyond approximate analytic techniques. Some results on the upper bound were also reported in the full SU(2) Higgs model with an inverse Higgs-boson mass less than one lattice unit and therefore outside the range of the effective scaling theory.[13,14] Recently, a report was called to our attention with the determination of the effective potential from histograms of field distributions.[15] Our numerical results disagree with Ref. 15.

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