Observation of Spatial Quantum Beating with Separated Photodetectors

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When signal and idler photons produced in the process of parametric down-conversion are mixed together, and directed to two photodetectors that respond to nonoverlapping optical frequencies centered at ω_1 and ω_2, it is found that the joint probability of two-photon detection exhibits a modulation of the form $\cos(\omega_1-\omega_2)\tau$, where $c\tau$ is the path difference. The experimental results are well described by a simple quantum-mechanical analysis.

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Recently a number of optical interference experiments have been reported in which nonclassical effects were observed.[1-4] Although interference is generally regarded as a classical wave phenomenon, quantum effects in which classical probability is violated can show up, particularly when correlation measurements are made with two detectors and when the number of photons is small. For example, when the signal and idler photons produced together in the process of parametric down-conversion are allowed to interfere, the visibility and shape of the resulting interference pattern can be very different from what classical optics predicts.[1,4] Still, in the previous interference experiments[1-4] the observed phenomenon was readily describable by a simple classical model of two incoherent waves, even if the classical predictions were wrong in detail.

We wish to report an experiment in which a new interference effect in the form of spatial beating is observed in a photon coincidence counting measurement. In this experiment one detector responds to light of frequency ω_1 and the other to light frequency ω_2, and the two-photon coincidence counting rate is found to exhibit a modulation of the form $\cos(\omega_1-\omega_2)\tau$, where $c\tau$ is the optical-path difference. The beat frequency $\omega_1-\omega_2$ is of order 2×10^{14} sec^{-1}, yet the beats can be detected with photodetectors whose response is thousands of times slower.

Experiment.— Figure 1 shows an outline of the experimental setup, which has a good deal in common with an experiment reported previously.[2] Broad-band signal and idler photons of wavelengths centered around 700 nm are produced simultaneously in the process of parametric down-conversion by a uv laser beam of wavelength 351.1 nm interacting with a nonlinear crystal of potassium dihydrogen phosphate. The photons are directed to a beam splitter (BS) from opposite sides by mirrors M1 and M2. Mixed and carefully aligned signal and idler photons emerge from both sides of the beam splitter, and pass through two apertures and through two different interference filters IF1 and IF2 to two photodetectors D1 and D2. The detector pulses, after amplification and pulse shaping, are counted and are also fed to the start and stop inputs of a computer-controlled time-to-digital converter (TDC), which functions as a coincidence counter. After subtraction of accidental coincidences, the number of pulse pairs arriving within the resolving time $T_R=7.5$ nsec in some long measurement interval provides a measure of the joint probability P_{12} for the "simultaneous" detection of photons at both detectors within T_R. We treat all pulse pairs in the delay range 50 to 95 nsec as accidentals, and multiply this number by 7.5/45 to arrive at the number of accidental coincidences within T_R. By making small displacements of the beam splitter BS towards one or the other mirror M1 or M2, as indicated, we vary the time delay or overlap between signal and idler photons and investigate the resulting interference effects.

What principally distinguishes this experiment from the previously reported one[2] is the fact that the pass bands of the two interference filters IF1 and IF2 are centered on different frequencies ω_1 and ω_2 and *do not overlap*, so that correlations between different Fourier components of the optical field are being measured. In practice the two pass bands were centered on the conjugate wavelengths 680 and 725 nm, which means that the filters differed by 45 nm in wavelength or by $(\omega_1-\omega_2)/2\pi=0.27\times10^{14}$ Hz in frequency.

Figure 2 shows the observed two-photon coincidence

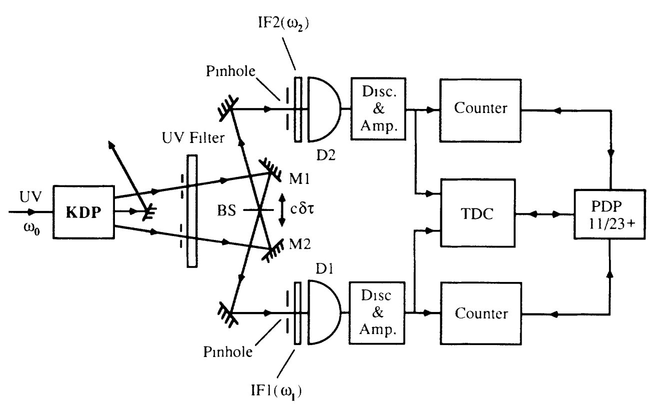

FIG. 1. Outline of the experiment.

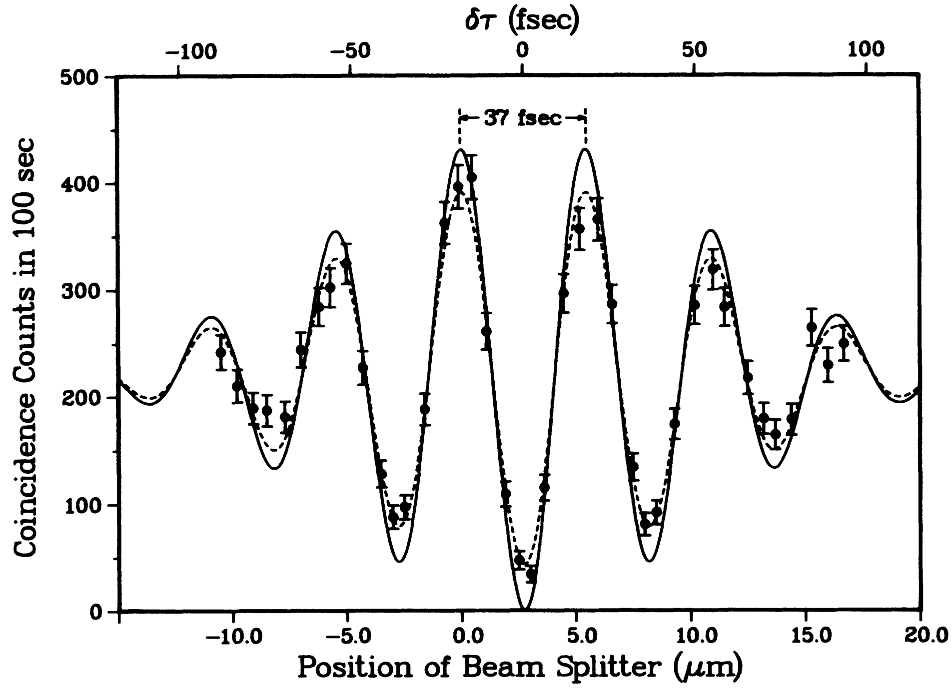

FIG. 2. Measured number of photon coincidences in 100 sec as a function of the beam splitter position or the time delay $\delta\tau$ between signal and idler photons. The solid curve is theoretical and is based on Eq. (10) with $\sigma = 1.85\times 10^{13}$ sec^{-1}, $\omega_1 - \omega_2 = 1.70\times 10^{14}$ sec^{-1}. The dashed curve is obtained when the interference term in Eq. (10) is multiplied by 0.8.

counting rate as a function of the beam-splitter position or the time delay $\delta\tau$ between signal and idler photons. 1 μm of BS displacement corresponds to about 6.67 fsec of time delay, because the image seen in the BS mirror moves through twice the displacement of BS. It will be seen that the coincidence rate exhibits an interference pattern with periodicity 5.5 μm or 37 fsec, which corresponds almost exactly to the period $2\pi/(\omega_1 - \omega_2)$ of the beat frequency $\omega_1 - \omega_2$. But in this experiment neither of the two photodetectors registers any beat note directly.

Theory.—In order to account for these results theoretically, let us represent the two-photon state resulting from the down-conversion at the two mirrors M1 and M2 by

$$|\psi\rangle = \int_0^\infty d\omega\, \phi(\omega)\, |\omega\rangle_s\, |\omega_0 - \omega\rangle_i. \tag{1}$$

Here ω_0 is the pump frequency, ω the signal, $\omega_0 - \omega$ the conjugate idler frequency, and $\phi(\omega)$ is a symmetric weight function representing the substantial frequency spread of signal (s) and idler (i) photons, which is peaked at $\omega = \frac{1}{2}\omega_0$. In the absence of the two interference filters IF1 and IF2, the electric fields $\hat{E}_1$ and $\hat{E}_2$ at the two photodetectors D1 and D2, respectively, would be related to the fields $\hat{E}_{01}$ and $\hat{E}_{02}$ at the mirrors M1 and M2, respectively, by

$$\hat{E}_1^{(+)}(t) = \sqrt{T}\hat{E}_{01}^{(+)}(t-\tau_1) + i\sqrt{R}\hat{E}_{02}^{(+)}(t-\tau_1+\delta\tau), \quad \hat{E}_2^{(+)}(t) = \sqrt{T}\hat{E}_{02}^{(+)}(t-\tau_1) + i\sqrt{R}\hat{E}_{01}^{(+)}(t-\tau_1-\delta\tau). \tag{2}$$

R and T are the reflectivity and transmissivity, respectively, of the beam splitter BS, τ_1 is the propagation time between mirror and detector, and $\pm c\delta\tau$ is the small displacement of the beam splitter from the symmetric position. In the presence of the interference filters IF1 and IF2 with complex frequency responses $G_1(\omega)$ and $G_2(\omega)$, we make a Fourier decomposition of $\hat{E}_{01}^{(+)}(t)$ and $\hat{E}_{02}^{(+)}(t)$ and multiply each Fourier component $\hat{a}_1(\omega)$ and $\hat{a}_2(\omega)$ by $G_1(\omega)$ and $G_2(\omega)$, respectively. Then we have in place of Eqs. (2)

$$\hat{E}_1^{(+)}(t) = \frac{1}{2\pi}\int_0^\infty d\omega [\sqrt{T}\hat{a}_1(\omega) e^{-i\omega(t-\tau_1)} + i\sqrt{R}\hat{a}_2(\omega) 6e^{-i\omega(t-\tau_1+\delta\tau)}] G_1(\omega), \tag{3}$$

$$\hat{E}_2^{(+)}(t) = \frac{1}{2\pi}\int_0^\infty d\omega [\sqrt{T}\hat{a}_2(\omega) e^{-i\omega(t-\tau_1)} + i\sqrt{R}\hat{a}_1(\omega) e^{-i\omega(t-\tau_1-\delta\tau)}] G_2(\omega). \tag{4}$$

Now the joint probability $\mathcal{P}_{12}(\tau)$ of photodetection by D1 at time t and by D2 at time $t+\tau$ is given by the normally ordered expectation[5]

$$\mathcal{P}_{12}(\tau)=K\langle\psi|\hat{E}_1^{(-)}(t)\hat{E}_2^{(-)}(t+\tau)\hat{E}_2^{(+)}(t+\tau)\hat{E}_1^{(+)}(t)|\psi\rangle, \quad (5)$$

in which K is a constant characteristic of the detector and the measurement interval. If we substitute Eqs. (1), (3), and (4) in Eq. (5) and make use of the fact that $G_1(\omega)$ and $G_2(\omega)$ do not overlap, we obtain

$$\mathcal{P}_{12}(\tau)=K[|g_{12}(-\tau)|^2T^2+|g_{12}(2\delta\tau-\tau)|^2R^2-2RTg_{12}^*(-\tau)g_{12}(2\delta\tau-\tau)e^{i\omega_0\delta\tau}+\text{c.c.}], \quad (6)$$

in which $g_{12}(\tau)$ is the Fourier transform

$$g_{12}(\tau)=\frac{1}{2\pi}\int_0^\infty d\omega\,\phi(\omega)G_1(\omega)G_2(\omega_0-\omega)e^{-i\omega\tau}. \quad (7)$$

In practice the apparatus measures the integral of $\mathcal{P}_{12}(\tau)$ over the resolving time T_R of the coincidence detector. But as this time T_R is in the range of nanoseconds and very long compared with the range of $g_{12}(\tau)$, we may effectively integrate each term in Eq. (6) from $-\infty$ to ∞. Thus we obtain for the measured coincidence detection probability

$$P_{12}=K\int_{-\infty}^{\infty}|g_{12}(-\tau)|^2d\tau \times\left[T^2+R^2-2TR\,\mathrm{Re}\left(e^{i\omega_0\delta\tau}\int_{-\infty}^{\infty}d\tau\,g_{12}^*(-\tau)g_{12}(2\delta\tau-\tau)\right)\left(\int_{-\infty}^{\infty}d\tau\,|g_{12}(-\tau)|^2\right)^{-1}\right]. \quad (8)$$

In the special case in which $G_1(\omega)$ and $G_2(\omega)$ can be well approximated by Gaussian functions with rms widths σ,

$$G_1(\omega)=(2\pi)^{-1/2}\sigma^{-1}\exp[-(\omega-\omega_1)^2/2\sigma^2],\quad G_2(\omega)=(2\pi)^{-1/2}\sigma^{-1}\exp[-(\omega-\omega_2)^2/2\sigma^2], \quad (9)$$

and σ is much narrower than the width of $\phi(\omega)$, which can be treated as approximately constant under the integral in Eq. (7), Eq; (8) reduces to

$$P_{12}=K(|\phi|^2\mathrm{Z}\sqrt{2}\pi\sigma^3)e^{-(\omega_1+\omega_2-\omega_0)^2/2\sigma^2}[T^2+R^2-2TRe^{-\sigma^2\delta\tau^2/2}\cos(\omega_1-\omega_2)\delta\tau]. \quad (10)$$

Evidently the probability P_{12} is greatest when the two center frequencies ω_1 and ω_2 are conjugates satisfying the condition $\omega_1+\omega_2=\omega_0$.

When $T=R=\frac{1}{2}$, Eq. (10) describes an interference pattern with 100% visibility at the center, but falling off exponentially on either side. At the position of symmetry where $\delta\tau=0$, $P_{12}=0$, which is reminiscent of a previous experiment.[2] Asymmetric frequency response functions $G_1(\omega)$ and $G_2(\omega)$ in general result in a phase-shifted modulation term $\cos[(\omega_1-\omega_2)\delta\tau+\phi]$, so that P_{12} does not vanish when $\delta\tau=0$.

From a direct measurement of the transmissivity curves for the interference filters supplied by the manufacturer, we find that $G_1(\omega)$ and $G_2(\omega)$ are very approximately Gaussian with standard deviations $\sigma_1\approx1.70\times10^{13}$ sec^{-1} for IF1 and $\sigma_2\approx2.06\times10^{13}$ sec^{-1} for IF2. We can combine these values to form an average σ defined by $2/\sigma^2=1/\sigma_1^2+1/\sigma_2^2$, and obtain $\sigma\approx1.85\times10^{13}$ sec^{-1}. The full curve shown in Fig. 2 is a plot of the function P_{12} given by Eq. (10) with the experimental values $\sigma=1.85\times10^{13}$ sec^{-1} and $\omega_1-\omega_2=1.70\times10^{14}$ sec^{-1}, and with the origin of position and the vertical scale adjusted for best agreement with the experimental data. It will be seen that there is reasonable agreement between theory and experiment, except that the observed visibility is somewhat smaller than ideal, probably because of less than perfect alignment, and because the bandwidths of the interference filters were not identical. A similar effect was observed in a previous experiment.[2] The dashed-line curve in Fig. 2 is obtained from Eq. (10) when the interference term is multiplied by 0.8 to allow for the reduced visibility.

There remain questions as to how it is possible for us to observe beating with a period that is thousands of times shorter than the resolving time of the detectors, and about the extent to which the phenomenon can be modeled in terms of classical light waves. The answer to the first question is of course that we do not directly observe beating in time, but only somewhat indirectly through the two-photon coincidence rate, which varies with path difference $c\delta\tau$ according to $\cos(\omega_1-\omega_2)\delta\tau$. The dependence on path difference $c\delta\tau$ is impressed on the correlation function $g_{12}(2\delta\tau-\tau)$, and in the product $g_{12}^*(-\tau)g_{12}(2\delta\tau-\tau)$ its effect persists even after integration over τ in Eq. (8).

The question of to what extent the phenomenon can be treated classically is an interesting one, which differentiates this experiment from earlier fourth-order interference experiments with photon pairs.[1,2] In those earlier experiments the phenomenon could be modeled classically to an extent by our treating the signal and idler as mutually incoherent light waves of equal intensities, although the quantitative details were incorrect.[6-9] The mutual incoherence is required because the signal and idler photons do not exhibit ordinary second-order interference.[1] This model is not adequate for the present experiment, because nonvanishing fourth-order cross-spectral densities of the type $\langle a_1^*(\omega)a_2^*(\omega')a_1(\omega')\times a_2(\omega)\rangle$ play an essential role here, which requires

Fourier components of different frequencies to be correlated. Even then it can be shown that a classical field can exhibit a relative modulation or visibility no greater than 50%, unlike that described by Eq. (10). In this respect the experiment violates classical probability rather like the previous ones.[1,2] However, a full classical wave calculation would require an explicit model of the optical field produced in down-conversion, which really does not exist.

However, lest it be thought that correlations between the frequencies of signal and idler photons play the essential role here, we point out that the beating phenomenon can show up, in principle, even when the two photons are completely independent. For example, if we take the initial two-photon state to be a direct product state of the form

$$|\psi\rangle = \frac{1}{(2\pi)^2}\int\int d\omega' d\omega''\,\phi_1(\omega')\phi_2(\omega'')\,|\omega'\rangle\,|\omega''\rangle$$

in which $\phi_1(\omega)$ and $\phi_2(\omega)$ may or may not be identical weight functions, the joint detection probability again exhibits beating after we integrate $P_{12}(t,t+\tau)$ with respect to both t and τ over the resolving time of the detectors.

It is noteworthy that the interference technique in principle allows beating at optical frequencies $|\omega_1-\omega_2|$ to be detected with photodetectors whose response times are thousands of times slower.

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