Longitudinal Magnetic Resonance in Superfluid ^{3}He-B at High Fields

D. Candela,[a] D. O. Edwards, A. Heff, N. Masuhara,[b] Y. Oda,[c] and D. S. Sherrill[d]
Physics Department, The Ohio State University, Columbus, Ohio 43210
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We have measured the longitudinal resonance frequency ν_l in superfluid ^{3}He-B in a magnetic field strong enough to distort the superfluid energy gap. At low temperatures, the resonance is narrower than in zero field, and the measurements are precise enough to determine both the quadratic and quartic dependences of ν_l on the field. The quadratic term has been fitted by the quasiclassical theory to give the $l=3$ pairing interaction. At extremely low temperatures ($T \lesssim 0.2T_c$), an unexplained structure in the resonance line is observed.

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We present the first measurements of the magnetic field dependence of ν_l, the longitudinal nuclear-magnetic-resonance frequency in superfluid ^{3}He-B, and compare the results with the "quasiclassical" (QC) theory.[1] The QC theory, which is applicable to liquid ^{3}He and to superconductors, treats Fermi liquids on scales of time and distance which are large compared to atomic scales but short compared to those of the hydrodynamic approximation. Consequently it can deal with a broad range of complicated phenomena without the intractable calculations of the microscopic theory.

The field dependence of ν_l arises[2] from the distortion of the superfluid energy gap which occurs when the Zeeman energy is comparable with the gap. In the QC theory the field dependence of ν_l depends on the Fermi-liquid parameters and the $l=3$ (f wave) coupling. For most properties of superfluid ^{3}He the $l=3$ coupling is masked by the $l=1$ coupling which causes the superfluid ordering, although it does affect the field dependence of the B-phase susceptibility,[3] the AB transition,[4] and the collective modes.[5,6] Efforts to determine the f-wave coupling experimentally have hitherto been inconclusive.[5-7] However, with the QC theory, the field dependence of the longitudinal resonance gives a quite unambiguous result for low pressures.

The experiment indicates that, at low pressure where the so-called "nontrivial" strong-coupling effects are small, the $l=3$ coupling is insignificant. Quantitatively we find that $\ln(T_c/T_{c3}) \gtrsim 10$, where T_c is the actual transition temperature into the $l=1$ state and T_{c3} would be the transition temperature if f-wave pairing occurred.

The longitudinal resonance in superfluid ^{3}He-B is due to oscillations of the nuclear magnetization which are parallel to the static field H_0 and the rf field H_1. At low temperatures the oscillation is due entirely to the Cooper pairs and is damped by quasiparticle collisions. Since there are few quasiparticles and the collision time is long, the longitudinal resonance is very sharp, allowing very precise measurements of ν_l. On the other hand, since H_0 and H_1 are parallel, there are strong mechanical resonances of the rf coil, which makes the resonance of the ^{3}He difficult to detect. This problem was overcome in our experiments by making the mechanical Q of the coil high, so that the ^{3}He signal can be resolved quite easily, as shown in Fig. 1. (For more details, see the work of Sherrill and co-workers.[8] The field-induced frequency shift is a large effect, amounting to roughly 20 kHz, or 150 times the minimum observed linewidth, for an applied field H_0 of about 5 kG.

The measurements were made over a wide range of magnetic fields, at temperatures from $0.18T_c$ to $0.45T_c$, and pressures P of 3.0, 6.1, 12.3, 21.0, and 33.0 bars. These pressures, measured with a low-temperature gauge calibrated against a dead-weight tester, are accurate to ± 0.05 bar. The reduced temperature T/T_c was measured by means of the susceptibility of a lanthanum cerium magnesium nitrate thermometer calibrated against the nuclear susceptibility of platinum. The superfluid transition at each pressure was located from cooling and warming curves. The ^{3}He was cooled by a nuclear demagnetization refrigerator and a superconducting solenoid provided the static field.[9] The ^{3}He sample was

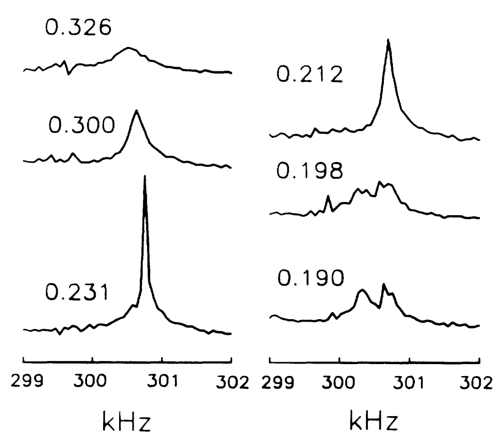

FIG. 1. Longitudinal NMR lines at 33 bars in field H_0 of 3.09 kG ($\gamma H_0/2\pi = 10$ MHz). The magnitude of the Fourier transform of the free-induction decay is plotted for six values of T/T_c, with $T_c \approx 2.5$ mK.

confined within an open-ended quartz tube 20 mm long with a 4-mm inside diameter. The static field was determined from the Larmor frequency ν_0 in a neighboring transverse NMR coil.[9] The longitudinal NMR was excited by an 11-mm-long silver coil wound around the outside of the quartz tube, with use of an H_1 pulse of 7 to 20 cycles with an amplitude of 10–70 mG. The oscillation of the ^{3}He magnetization was detected by the same coil in an untuned circuit ($Q \approx 2$). After amplification, the signal was digitized and signal averaged 10^2–10^4 times.

Textures in ^{3}He-B are characterized by the vector field $\hat{\mathbf{n}}$ which specifies the local axis of spin-orbit rotation. In our experiments $\hat{\mathbf{n}}$ was aligned along the resonance-coil axis by the strong field H_0.[10] (Earlier zero-field experiments of Bloyet *et al.*[11] used a series of closely spaced plates perpendicular to the coil axis to align $\hat{\mathbf{n}}$.) At the walls of our coil, the angle θ between $\hat{\mathbf{n}}$ and $\mathbf{H}_0$ is 63.4° as a result of the surface-field term in the free energy.[10] Towards the center of the coil θ tends to zero over a bending length ξ_H which is inversely proportional to H_0. We estimate ξ_H to be less than one-tenth the sample diameter for $\nu_0 = \gamma H_0/2\pi > 5$ MHz at any pressure. The local longitudinal resonance frequency of the ^{3}He near the coil wall is reduced from the bulk value by a factor $\cos\theta$,[12] but probably the response of the sample is coherent over distances greater then ξ_H (i.e., longitudinal spin waves[13] are excited, as discussed below).

As the magnetization oscillates along $\mathbf{H}_0$, the vector $\mathbf{d}$ (the spin part of the superfluid order parameter) executes angular oscillations. The dipolar restoring force on $\mathbf{d}$ is nonlinear if the amplitude $\delta\phi$ of these oscillations is too large. In our experiment, $\delta\phi$ was between 0.004 and 0.04 rad. An increase of $\delta\phi$ did not affect the results, verifying linearity.

The longitudinal resonance appears as a sharp peak in the Fourier transform of the signal (Fig. 1). The frequency and decay time for the resonance were determined by subtracting the background from the main peak and then fitting it with the Fourier transform of an exponentially decaying oscillation.

According to theory,[14] the decay is due to relaxation between the superfluid component and the quasiparticles with a time constant of the same order as the quasiparticle collision time. As $T \rightarrow 0$, the number density of quasiparticles decreases exponentially. This gives a temperature dependence of the decay time of $\exp(2\Delta/k_B T)$, probably changing to $\exp(\Delta/k_B T)$ at extremely low temperatures where the free path becomes comparable to the sample size.

Our experimental decay time has little field dependence for Larmor frequencies $\nu_0 \geq 3$ MHz but it decreases rapidly at lower fields, possibly because of misalignment of the texture. Figure 2 compares the decay time we measure in a high field and open geometry with some zero-field results from Bloyet *et al.*[11] At high temperatures, both sets of data show the expected $\exp(2\Delta/k_B T)$ temperature dependence with a small difference in absolute values. When the quasiparticle mean free path is comparable to the sample size, the data have a maximum instead of the $\exp(\Delta/k_B T)$ behavior predicted by theory. The origin of this size effect is not understood.

Our maximum decay times are much longer than the dephasing time due to the gradient in H_0, which is about 1 msec. This suggests that the response is nonlocal and due to longitudinal spin waves. Spin waves trapped by the texture against the outer wall of the cell should appear as weak satellites on the low-frequency side of the line. For many combinations of field and temperature, there is a small subsidiary peak 500–1000 Hz below the main resonance. However the theory[13] does not include gradients in H_0 and we are unable to explain the observed satellites quantitatively. Since the main signal comes from modes which extend far from the walls, the spin waves should not affect the accuracy of ν_l.

At very low temperatures ($T \lesssim 0.2 T_c$) the Lorentzian line shape, which corresponds to exponential decay of the longitudinal ringing, breaks down very dramatically (Fig. 1). Over a narrow temperature interval, the line assumes a complicated structure with multiple peaks, which is not reproducible from one measurement to the next (~30 min apart). The structure generally appears on the low-frequency side of the resonance. This differs from an overdriving of $\delta\phi$ at high temperatures, which merely flattens the Lorentzian line shape. A recent vibrating-wire viscometry study of ^{3}He-B shows unexplained, highly nonlinear behavior in roughly the same temperature range.[15]

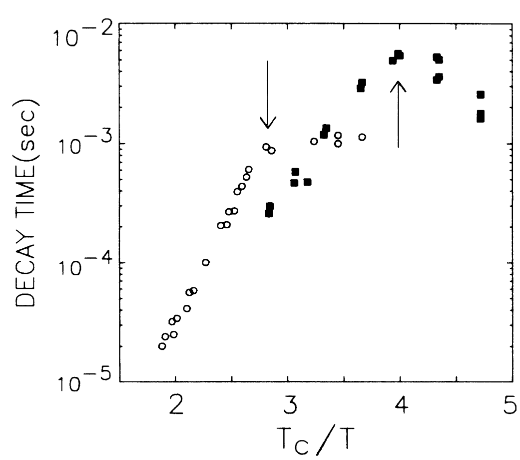

FIG. 2. Decay time of the longitudinal resonance as a function of inverse temperature. Filled squares: our data at 33 bars in a field with $\gamma H_0/2\pi = 10$ MHz. Open circles: data of Bloyet *et al.* (Ref. 11) at 29.3 bars in zero field. The arrows mark the temperatures at which the quasiparticle mean free path equals the smallest sample dimension.

We now discuss the longitudinal frequency ν_l. To summarize the measurements precisely and to compare with theory,[1] we fit the data at each pressure with a polynomial in $\nu_0^2=(\gamma H_0/2\pi)^2$, the square of the Larmor frequency:

$$\nu_l(H_0,T)=\nu_l(0,T)+\alpha(T)\nu_0^2+\beta\nu_0^4. \quad (1)$$

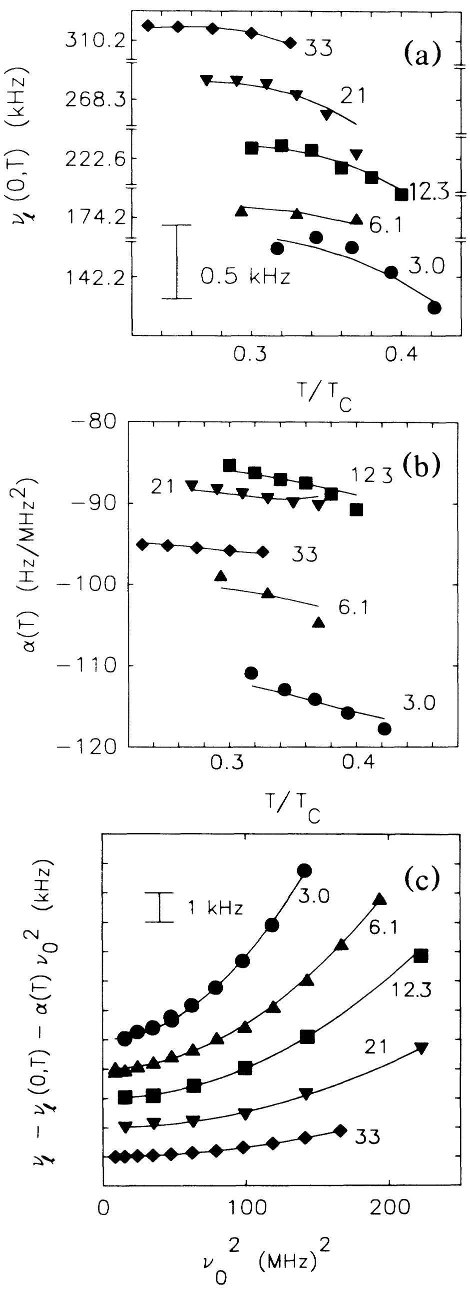

FIG. 3. (a) The zero-field longitudinal frequency $\nu_l(0,T)$ as a function of T/T_c. The vertical scale is offset for each pressure. (b) The coefficient of the quadratic dependence on field $\alpha(T)$, from Eq. (1) fitted to the measurements, as a function of T/T_c. In both (a) and (b), the curves, labeled with the pressure in bars, are the fit of the QC theory to the data. (c) The fit between the data and the quartic, temperature-independent term in Eq. (1), after the first two terms are subtracted.

The data at low fields and very low temperatures, which had anomalously short decay times, were not used. Note that $\nu_l(0,T)$ and $\alpha(T)$ are temperature dependent but the data are adequately represented with β, the coefficient of the quartic term, independent of temperature. The fitted zero-field intercept $\nu_l(0,T)$ and the quadratic field shift $\alpha(T)$ are shown in Figs. 3(a) and 3(b). Figure 3(c) shows the fit by the quartic term after the subtraction of the intercept and quadratic terms. The scatter in ν_l is less than 100 Hz. Our values of $\nu_l(0,T)$ agree well with the zero-field measurements of Bloyet *et al.*[11] and the transverse NMR measurements of Ahonen, Krusius, and Paalenen.[12] A quadratic fit[16] of $\nu_l(0,T)$ vs P is within $\simeq 500$ Hz of the three sets of data. All other measurements[17] are in the hydrodynamic regime and at higher temperatures than the present experiment.

The QC theory for the longitudinal resonance in a strong field[1] gives $\nu_l(0,T)$ and $\alpha(T)$ but the quartic term has not been worked out. The quantities $\nu_l(0,T)$ and $\alpha(T)$ depend on the transition temperature T_c, the Fermi-liquid parameters F_0^a and F_2^a, the dipolar interaction g_D, and the cutoff-independent measure of the $l=3$ coupling strength $x_3^{-1}\equiv(V_1^{-1}-V_3^{-1})^{-1}$, where V_1 and V_3 are the dimensionless $l=1$ and $l=3$ interactions.[1] When V_1 and V_3 are positive (attractive), $x_3=\ln(T_{c3}/T_c)$. The QC formulas are too long to reproduce here, but the dependence on the parameters is roughly as follows: The zero-field frequency $\nu_l(0,T)$ is proportional to g_D. The quadratic shift α depends strongly on T_c, F_0^a, and x_3^{-1}, while F_2^a mainly affects the temperature dependence of α. According to the theory,[2] the weak temperature dependence of α which we observe places strong

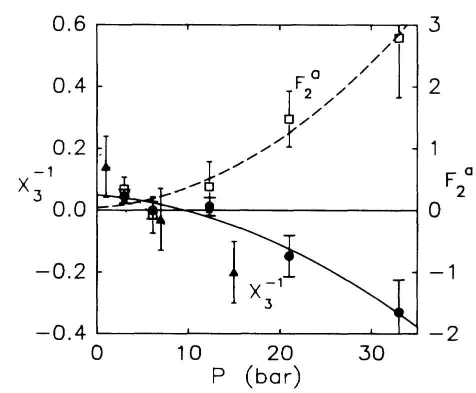

FIG. 4. The $l=3$ coupling strength x_3^{-1} (circles) and the Fermi-liquid parameter F_2^a (squares) as a function of pressure, from our data fitted by the QC theory. The dashed and solid curves are quadratic fits to the points. The triangles and dotted curves are estimates of x_3^{-1} from Refs. 5 and 6.

limits on F_2^a.

Figures 3(a) and 3(b) show that the QC theory adequately describes the data within the experimental uncertainty of $\simeq 100$ Hz. Figure 4 shows the fitted values of F_2^a and x_3^{-1}. The parameters F_0^a and T_c were taken from Greywall.[18] "Trivial" strong-coupling effects were included by use of the temperature-dependent strong-coupling gap predicted by the "weak-coupling plus" theory.[19] The error bars in Fig. 4 reflect the experimental uncertainties, which are comparatively unimportant, the uncertainties in the gap, assumed to be half the difference between BCS and weak-coupling plus, and the uncertainties in the input parameters, F_0^a and T_c. We assumed an unrealistically large uncertainty of 10% in T_c with a correlated uncertainty in $1+F_0^a$ of 20%. The values of x_3^{-1} and F_2^a are remarkably insensitive to the absolute temperature scale.

The fit indicates that both F_2^a and x_3^{-1} are small low pressure. Ultrasound studies of collective modes by Shivaram *et al.*[5] give considerably more negative values for F_2^a. On the other hand, our determinations of x_3^{-1} are in fairly good agreement with the collective-mode data.[5,6] Results for x_3^{-1} from the susceptibility[3] are inconclusive.[7] At high pressure, where nontrivial strong coupling may invalidate the theory, the fitted F_2^a becomes large and positive while x_3^{-1} approaches -0.3, indicating a large and attractive $l=3$ pairing interaction.

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[a]Present address: Department of Physics and Astronomy, University of Massachusetts, Amherst, MA 01003.

[b]Present address: Department of Physics and Center for Materials Science, Massachussets Institute of Technology, Cambridge, MA 02139.

[c]Present address: Faculty of Engineering Science, Osaka University, Japan.

[d]Present address: Siemens Medical Systems, Inc., Alexandria, VA 22314.

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