Linear Response and Plasmon Decay in Hot Gluonic Matter

Sudhir Nadkarni
Center for Computational Sciences and Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506
(Received 25 March 1988)

The current controversy over the damping of color oscillations in a quark-gluon plasma is resolved through a gauge-invariant linear-response analysis of relevant physical processes. The plasmon decay rate is shown to be $\gamma = -11 N_c g^2 T/24\pi$ to lowest perturbative order, but this result does not survive at higher orders. A self-consistent estimate suggests that plasmon decay is in fact a nonperturbative phenomenon governed by static screening effects.

PACS numbers: 12.38.Mh

What makes a plasma more than just a simple aggregate of particles is that apart from the single-particle properties of the underlying fields it also displays *collective behavior* characteristic of the statistical ensemble. The simplest such quasiparticle excitation, the *plasmon*, is the quantum of coherent fluctuations in the charge or color density of the plasma. Plasmons propagate via normal modes, typically plane waves of amplitude $\sim \exp\{i[\mathbf{k}\cdot\mathbf{x} - \omega(\mathbf{k})t] - \gamma(\mathbf{k})t\}$. Here $\omega(\mathbf{k})$, the energy of the plasmon, and $\gamma(\mathbf{k})$, its decay rate, must meet the following criteria: (i) $\gamma \geq 0$, on grounds of stability of the system, and (ii) $\gamma \ll |\omega|$, or else the mode is overdamped and of little interest.

With quark-gluon–plasma physics rapidly gaining ground as an experimental science,[1] its theoretical understanding has become important. However, in quantum chromodynamics (QCD) even concepts as basic as plasmons are poorly understood and a consistent scheme for the calculation of quantities such as ω and γ is needed. To set the stage, let us first review how this is done in a simpler theory, quantum electrodynamics (QED).

In any field theory, the response of a plasma to small external perturbations is described by a *linear-response function* (LRF). For perturbations which couple to a Heisenberg operator O, the LRF is given by the retarded commutator[2]

$$iD^R(\mathbf{x},t;\mathbf{x}',t') \equiv \theta(t-t')\langle[O(\mathbf{x},t),O(\mathbf{x}',t')]\rangle,$$

where $\langle \cdots \rangle$ denotes averaging over the unperturbed ensemble. In Fourier space, $D^R(\mathbf{k},\omega)$ is an analytical continuation of the imaginary-time propagator $D^R(\mathbf{k},i\omega_n \to \omega + i\eta)$, the correct continuation being uniquely fixed by the requirement $D^R(\mathbf{k},\omega) \sim 1/\omega$ for $|\omega| \to \infty$.[2] I shall continue this discussion in the much simpler imaginary-time formalism.

In QED, we wish to study fluctuations in the charge density and therefore apply an electromagnetic source field (generated by an external current) which couples locally to the charge current $j_\mu(x)$. The plasma responds to the perturbation, producing a disturbance in the current at the detector, and we identify $\langle j_\mu(x) j_\nu(y)\rangle$ as the LRF [see Fig. 1(a)]. Since $j_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$ is conserved ($\partial_\mu j_\mu = 0$) and gauge invariant, the LRF is both transverse and physical.

At $T \neq 0$, an arbitrary transverse tensor $\Pi_{\mu\nu}$ has two independent components,[3]

$$\Pi_E \equiv \frac{k^2}{\mathbf{k}^2}\Pi_{44}, \quad \Pi_M \equiv \frac{1}{2}\left(\Pi_{ii} - \frac{k_4^2}{\mathbf{k}^2}\Pi_{44}\right),$$

corresponding to two independent transverse projectors, P^E and P^M, the latter also being transverse to the three-vector direction of propagation $\mathbf{k}$. Taking $\Pi_{\mu\nu}$ to be the one-particle-irreducible polarization tensor, the LRF in QED can be written as

$$\langle j_\mu j_\nu \rangle_k \sim ik^2 \left(\frac{\Pi_{44} P^E_{\mu\nu}}{\mathbf{k}^2 - \Pi_{44}} + \frac{\Pi_M P^M_{\mu\nu}}{k^2 - \Pi_M} \right).$$

Analyzing the linear response of a QED plasma thus boils down to the computation of the physical transverse polarization tensor $\Pi_{\mu\nu}$. From its components one obtains the poles of the LRF at the complex values $\omega_{E,M}(\mathbf{k}) - i\gamma_{E,M}(\mathbf{k})$, which are just the E and M dispersion relations that characterize the corresponding normal modes of the plasma.

Attempts have been made to apply the above formalism to a high-temperature QCD plasma ($T \gg \Lambda_{\rm QCD}$), where $g(T) \ll 1$ and perturbative calculations are possible. As often happens, what works for the Abelian theory does not translate readily to the non-Abelian case, and a serious disagreement presently exists concerning

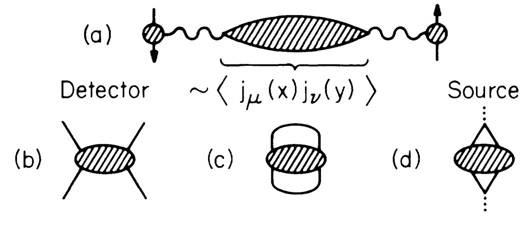

FIG. 1. The linear-response function for (a) a generic current-current interaction, and typical physical processes dominated by current-current interactions: (b) on-shell amplitude, (c) Wilson loop, and (d) meson self-energy.

the plasmon decay rate γ at the one-bare-loop level. At this lowest perturbative order, where the dominant damping mechanism is purported to be the decay of plasmons into gluon pairs, even the sign of γ, not to mention its magnitude, is controversial.

Early computations,[4,5] focusing on $\Pi_{\mu\nu}$ at the one-bare-loop level in covariant ξ gauges, yielded a ξ-dependent $\gamma = -O(g^2T)$, with the negative sign persisting for all values of ξ. This result has been interpreted in various ways. Kalashnikov and Klimov[4] deemed this value to be too small in magnitude and concluded that within the limits of validity of their one-bare-loop approximation, γ should be taken to be identically zero. They therefore attached no special significance to the negative sign. Gross, Pisarski, and Yaffe[5] interpreted the negative sign as being an artifact of the bare calculation; a self-consistent calculation (i.e., with full propagators replacing bare ones) would, they argued, make $\gamma = 0$ at the one-loop level. More recently, Lopez, Parikh, and Siemens[6] have conjectured that the negative sign is related to asymptotic freedom and should therefore persist in a gauge-invariant calculation, signaling the instability of the perturbative vacuum at nonzero temperature. In the first genuine attempt at gauge invariance,[7] quantum fluctuations of the plasma are integrated out in the presence of a background field to obtain an effective action which is invariant under gauge transformations of the background but depends parametrically on the gauge of the fluctuations. Following Weldon,[3] quantities such as ω and γ can then be extracted from the background-field effective action. For an SU(N) theory with a covariant ξ-gauge fixing of the quantum fields, the decay rate at the one-bare-loop level is found to be

$$\gamma_B^{(1)} = -[11 + \tfrac{1}{4}(\xi - 1)^2] N g^2 T / 24\pi,$$

which is negative for all values of ξ. Hansson and Zahed[7] argue that for a specific fixing of the gauge, which they determine at the one-bare-loop level to be Landau gauge, $\xi = 0$, the effective action can be given a gauge-invariant interpretation.

In direct opposition to the above are the temporal ($A_0 = 0$) gauge calculations,[8,9] which yield a *positive* value for γ. Kajantie and Kapusta[8] considered perturbations which couple to the color field rather than the current, leading to the correlation $\langle \mathbf{E}^a(x) \mathbf{E}^b(y) \rangle$, which in temporal gauge is simply related to the polarization tensor. Dispersion relations based on the latter yield for the plasmon decay rate the formula

$$\gamma_B^{(1)} = +N g^2 T / 24\pi.$$

In response to questions about its gauge invariance, this result has been reproduced in Coulomb gauge (where the calculation is considerably more involved) by Heinz, Kajantie, and Toimela.[9] Needless to say, a positive sign is very satisfying and causes no problems of interpretation.

These conflicting results raise several questions, which this work seeks to answer: (i) What is the correct *physical* definition of γ (and other plasma parameters) for a non-Abelian gauge theory, and is the corresponding $\gamma_B^{(1)}$ positive or negative? (ii) How is $\gamma_B^{(1)}$ modified by self-consistency, and what does this imply for the dominant damping mechanism? (iii) What would a self-consistently negative sign for γ imply?

In the following, I outline the construction of a gauge-invariant formalism for analyzing the response of non-Abelian systems, and use it to compute the correct QCD plasmon dispersion relations. From these I obtain the plasmon energy and decay rate at the one-bare-loop level and estimate the changes that a self-consistent calculation would bring about. I end with some conclusions.

Controversies in gauge-theoretical calculations can often be traced to conflicting ways of extracting physical information from the theory. Not all such approaches are infallible, and among the works just discussed one finds the following: (i) A gauge-dependent quantity (e.g., $\Pi_{\mu\nu}$) is computed in a ghost-free gauge. It is believed that in certain limits this may give physical results. (ii) A gauge-dependent quantity is computed in a variety of gauges. In an expansion of that quantity in certain limits, some terms may turn out to be gauge independent; these are ascribed a physical meaning. Although (ii) is somewhat more reliable than (i), both can give misleading results in exploratory calculations (these and other issues will be discussed more completely in a lengthier article[10]).

The present gauge-invariant linear-response formalism for QCD is based on the only *foolproof* method that I know for extracting physical information from a gauge theory. It consists of three steps: (i) Select a physical process of relevance to the observable that one wishes to compute. (The chosen process need be neither practical nor unique, but computationally simple.) (ii) Write down the gauge-invariant amplitude corresponding to that process. (This will typically be a closed Wilson loop or an on-shell amplitude.) (iii) Compute the amplitude in any convenient gauge.

In QCD, we wish to study fluctuations in the *color* density of the plasma, and so, keeping the similarity to QED as close as possible, we couple the color current to source and detector fields [see Fig. 1(a)]. The color current, which is of the form

$$j_\mu^a = \bar{\psi} \gamma_\mu T^a \psi + (\text{Yang-Mills pieces}),$$

is neither conserved nor gauge invariant. Therein lies the crux of the problem.

The remedy is to consider Fig. 1(a) as originating from physical processes dominated by current-current interactions, illustrated generically in Figs. 1(b)–1(d). In the linear-response approximation, the source is weak and one needs to consider only two-point exchanges of the form $\hat{\Gamma}_{\text{detector}} \hat{D} \hat{\Gamma}_{\text{source}}$, where $\hat{\Gamma}$ is the effective vertex and $\hat{D}$ the effective gluon propagator. Now note that $\hat{\Gamma}$ is *process dependent:* It knows about the nature of the source and detector (spin and color representations, mass, etc.) and the details of the process. On the other hand, $\hat{D}$ is characterized only by the momentum transfer and knows nothing about the details of the process or the nature of the source and detector. Clearly, $\hat{D}$ is nothing but the conventional gluon propagator D in some chosen gauge augmented by additional gauge-dependent terms δD arising from vertex corrections.

Cornwall[11] has discussed an elegant and economical scheme for extracting δD from any physical process dominated by current-current exchanges. The idea is to use Ward identities to "pinch out" internal source and detector propagators from certain vertex diagrams, yielding one-gluon-exchange parts which are indistinguishable from propagator corrections. When added to the usual propagator, a *gauge-invariant, transverse* effective polarization tensor $\hat{\Pi}$ results. Accordingly, $\hat{D}$ can be written as the sum of two parts. The longitudinal part is gauge dependent and not quite uniquely defined since it gives a vanishing contribution to physical processes; it serves only to make the effective propagator invertible and contains no physics. On the other hand, the transverse part is gauge invariant and uniquely defined; it occurs in every physical process involving a current-current interaction.

Since all physical information about the plasma in the linear-response approximation is contained in $\hat{\Pi}$, all we need to do to obtain the gauge-invariant linear-response function for QCD is to transcribe the QED result, replacing Π by $\delta^{ab}\hat{\Pi}$ everywhere:

$$\langle j_\mu^a j_\nu^b \rangle_k^{\text{phys}} \sim i\delta^{ab} k^2 \left(\frac{\hat{\Pi}_{44} P_{\mu\nu}^E}{\mathbf{k}^2 - \hat{\Pi}_{44}} + \frac{\hat{\Pi}_M P_{\mu\nu}^M}{k^2 - \hat{\Pi}_M} \right).$$

Using light-cone gauges and the process depicted in Fig. 1(d), Cornwall, Hou, and King[11] have shown that $\hat{\Pi}$ at the one-bare-loop level is given by the expression

$$\hat{\Pi}_{\alpha\beta} = 4Ng^2(k^2\delta_{\alpha\beta} - k_\alpha k_\beta) \int \frac{d_4 l}{l^2(l+k)^2} + Ng^2 \int d_4 l \left[\frac{4l_\alpha l_\beta - k_\alpha k_\beta}{l^2(l+k)^2} - \frac{2\delta_{\alpha\beta}}{l^2} \right],$$

where $d_n l \equiv d^n l/(2\pi)^n$ and the integrals are to be evaluated at finite temperature. (The uncontroversial quark-loop contribution has been omitted for simplicity.) I have rederived this result in covariant ξ gauges for other processes in order to verify both its gauge and process independence.[10]

Let us briefly revisit the background-field method.[7] In a very useful and instructive calculation, Elze *et al.*[12] have derived the relationship between the polarization tensors in the ordinary and background covariant ξ gauges. If we compare their results for $\xi = 1$ with the above expression for $\hat{\Pi}$, an important relationship emerges:

$$\hat{\Pi}_{\alpha\beta} = \Pi_{\alpha\beta}^{\text{FBG}} = \Pi_{\alpha\beta}^{\xi=1} + 2Ng^2(k^2\delta_{\alpha\beta} - k_\alpha k_\beta) \int \frac{d_4 l}{l^2(l+k)^2},$$

where FBG stands for Feynman background gauge. This verifies the assertion of Hansson and Zahed that in a specific gauge, the background-field effective action has a gauge-invariant meaning. However, that gauge is seen to be *Feynman* gauge and not Landau gauge as they have claimed; I do not yet understand the reason for this discrepancy.

The poles of the gauge-invariant LRF for QCD give E- and M-type dispersion relations $\omega(\mathbf{k}) - i\gamma(\mathbf{k})$ in terms of the components of $\hat{\Pi}$. At the one-bare-loop level, one gets for ω the uncontroversial result $\omega^2 = \omega_p^2 + O(\mathbf{k}^2)$, where ω_p is the $O(gT)$ plasma frequency and the subleading terms differ for E and M modes.[3-5,7-9] γ at this level is the same for both modes and is given by

$$\gamma_B^{(1)}(\mathbf{k}) = -\frac{11Ng^2T|\omega|}{48\pi^2} \int \frac{d^3 l\, \delta(l^2 - \omega^2)}{(\mathbf{l}+\mathbf{k})^2} \approx -11Ng^2T/24\pi,$$

which is of course the result of Hansson and Zahed in Feynman gauge. The coefficient (-11) is precisely the one that characterizes the gluonic contribution to the gauge-invariant, asymptotically free β function, which verifies the gauge invariance of our formalism and confirms the conjecture of Lopez, Parikh, and Siemens.[6] We can write this coefficient in an instructive way,[12] viz., $-11 = 1 - 6 - 6$. Temporal gauge, which is an incomplete gauge at $T \neq 0$ since it excludes certain physical field configurations,[13] contributes only the 1, leading to the erroneous positive sign for γ. The Feynman-gauge propagator alone contributes $1-6$; the extra -6 comes from vertex corrections which make the total gauge invariant.

It is obvious from the preceding expression for $\gamma_B^{(1)}$ that γ depends crucially on the poles of the internal propagators and therefore the one-bare-loop result has little significance.[4,5] A self-consistent calculation is mandatory and results in the bare expression being replaced by one of the form

$$\gamma_{SC}^{(1)}(\mathbf{k}) = -\frac{11Ng^2T|\omega|}{48\pi^2} \sum_{AB} c_{AB} \int \frac{d^3 l\, \delta[l^2 - \omega^2 - \text{Re}\hat{\Pi}_B(\omega, \mathbf{l})]}{(\mathbf{l}+\mathbf{k})^2 - \hat{\Pi}_A(0, \mathbf{l}+\mathbf{k})},$$

where $A,B=E,M$ and $\sum_{AB} c_{AB}=1$. Calculation of the coefficients c_{AB} is straightforward but algebraically tedious and for the present we must content ourselves with estimating the expression as it stands. Observe that the δ function in the above integral peaks on those values of $\mathbf{l}$ which satisfy the dispersion relation for $\omega_{E,M}(\mathbf{l})$, and that in the long-wavelength limit the denominator is dominated by the electric or magnetic screening masses. With $c_{\rm el}, c_{\rm mag}$ denoting simple combinations of c_{AB}, we obtain the estimate

$$\gamma_{\rm SC}^{(1)}(\mathbf{k}) \approx -\left(\frac{c_{\rm el}}{m_{\rm el}^2}+\frac{c_{\rm mag}}{m_{\rm mag}^2}\right) N g^2 T \omega_p |\mathbf{k}| \quad (|\mathbf{k}| \ll m_{\rm el}, m_{\rm mag}).$$

The self-consistent analysis thus suggests that the dominant damping mechanism is not the decay of a plasmon into gluon pairs but rather its scattering off static color fields. Nonvanishing screening masses appear to be necessary for a finite γ; this is reminiscent of the divergence of the total scattering cross section for an unscreened Coulomb potential in elementary quantum mechanics. The magnitude and sign of $c_{\rm el}$ and $c_{\rm mag}$, in relation to the strengths of the screening masses, will determine the overall sign of γ. Should the negative sign survive the self-consistent calculation, it would be yet another indication of the instability of the high-T perturbative vacuum.[14,15]

In conclusion, I have constructed a gauge-invariant linear-response formalism for QCD with a structure very similar to that of QED and thereby resolved the controversy over the sign of the plasmon decay rate at lowest order. I have further shown that in a self-consistent calculation, γ depends nontrivially on the electric and magnetic screening masses, neither of which is perturbative.[14] In the spirit of the original calculations,[4,5] we are therefore obliged to take $\gamma=0$ up to small, but now nonperturbative, corrections.

Finally, consider real-life quark-gluon plasmas of the type that might be produced in relativistic heavy-ion collisions.[1] For such plasmas, the coupling constant $g(T,\mu)$ is large and it is doubtful whether linear-response theory would be of much use. To obtain meaningful results one may have to invent a gauge-invariant *nonlinear*-response theory and treat it nonperturbatively, for example, on a lattice, to obtain meaningful results. Here again, the "physical process" approach should prove useful.

I thank U. Heinz and I. Zahed for informative discussions of their results. This work was supported in part by the National Science Foundation under Grant No. NSF-RII-8610671 and by the U.S. Department of Energy under Grant No. DE-FG05-84ER40154.

[1]See "Search and Discovery," Phys. Today **41**, No. 3, 17 (1988); "Quark Matter '87," Z. Phys. C **38**, 1–370 (1988).

[2]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[3]H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).

[4]O. K. Kalashnikov and V. V. Klimov, Yad. Fiz. **31**, 1357 (1980) [Sov. J. Nucl. Phys. **31**, 699 (1980)].

[5]D. J. Gross, R. D. Pisarski, and L. G. Yaffe, Rev. Mod. Phys. **53**, 43 (1981).

[6]J. A. Lopez, J. C. Parikh, and P. J. Siemens, unpublished.

[7]T. H. Hansson and I. Zahed, Phys. Rev. Lett. **58**, 2397 (1987), and Nucl. Phys. **B292**, 725 (1987).

[8]K. Kajantie and J. Kapusta, Ann. Phys. (N.Y.) **160**, 477 (1985).

[9]U. Heinz, K. Kajantie, and T. Toimela, Phys. Lett. B **183**, 96 (1987), and Ann. Phys. (N.Y.) **176**, 218 (1987).

[10]S. Nadkarni, to be published.

[11]J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982); J. M. Cornwall, W.-S. Hou, and J. E. King, Phys. Lett. **153B**, 173 (1985).

[12]H.-T. Elze, U. Heinz, K. Kajantie, and T. Toimela, Z. Phys. C **37**, 305 (1988).

[13]S. Nadkarni, Phys. Rev. D **33**, 3738 (1986).

[14]S. Nadkarni, Phys. Rev. Lett. **60**, 491 (1988).

[15]J. E. Mandula and M. Ogilvie, Brookhaven National Laboratory Report No. BNL-40440, 1987 (unpublished).