Length Scales at the Metal-Insulator Transition in Compensated GaAs

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We present results on the magnetic-field–induced metal-insulator transition in compensated GaAs. The electrical conductivity was found to obey $\sigma=a+bT^{1/3}$ down to 60 mK, on both sides of the transition, where a and b were constant for a given magnetic field. This indicates that the conductivity is determined by the shortest length scale, in this case the interaction length, on both the metallic *and* insulating sides of the transition. Thus, the behavior of the conductivity is essentially unchanged as the transition is scanned, as long as one length scale dominates.

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It is well known that the temperature dependence of the conductivity, σ, of a disordered system can be specified in terms of a length scale.[1-3] For example, in the weakly disordered regime σ depends on the interaction and phase-coherence lengths,[4] and in the variable-range–hopping regime,[5] the hopping length determines σ. If the inelastic length is less than the localization length, ξ, in an insulator, the system will behave as if the states were extended, and localization will only be apparent when ξ is the shortest length.[6]

In the work described here, we show that the interaction length, $L_{\rm int}$, can determine the conductivity on both the metallic and insulating sides of the metal-insulator transition (MIT). Localization will only be apparent when $\xi < L_{\rm int}/G_c$ where G_c is the critical dimensionless conductance. The system used was compensated n-type GaAs and the MIT has been induced by the application of a strong magnetic field which shrinks the electronic wave functions.

Considerable work[2,7,8] has shown that the zero-temperature conductivity, $\sigma(T=0)$, falls continuously to zero at the MIT:

$$\sigma(T=0)=\sigma_c(a_\perp^2 a_\parallel/a_{\perp c}^2 a_{\parallel c}-1)^\nu. \qquad (1)$$

$a_\perp$ and $a_\parallel$ are the effective Bohr radii perpendicular and parallel to the applied magnetic field, respectively, and $a_{\perp c}$ and $a_{\parallel c}$ are their values at the transition. σ_c is a constant and ν is a constant known as the critical exponent. This behavior has been studied in a variety of heavily doped semiconductors,[8-13] and the value of ν has received considerable interest.[14-16] For an interacting disordered system in a magnetic field large enough to quench quantum interference, theory[17] predicts $\nu=1$.

In order to test Eq. (1) and subsequently to obtain ν, $\sigma(T=0)$ must be found by extrapolation of finite-temperature data on the metallic side. Quantum interference[18,19] and electron-electron interaction[20] corrections to the conductivity have been calculated, the latter giving $\delta\sigma \propto T^{1/2}$. Many different forms have been used to fit the conductivity near the MIT, including functions of the forms of these corrections. However, they have only been calculated for low disorder and thus are not valid very close to the MIT.

Altshuler and Aronov[21] considered the MIT by integrating the lowest-order one-parameter scaling equation for 3D to get $G=G_c+L/\xi$. Here G is the dimensionless conductance, G_c its critical value, L the relevant length, and ξ the correlation length. The corresponding conductivity is $\sigma=Ge^2/\hbar L$, or

$$\sigma=G_c e^2/\hbar L+e^2/\hbar\xi. \qquad (2)$$

Altshuler and Aronov argue that as the transition approaches, the Fermi-liquid description of electrons breaks down and the only temperature-dependent length scale that remains is the interaction length $L_{\rm int}=(D\hbar/k_{\rm B}T)^{1/2}$. Substituting $L=L_{\rm int}$ into Eq. (2), eliminating D via the Einstein equation $\sigma=(\partial n/\partial\mu)e^2D$, and solving for the conductivity very close to the MIT (where ξ diverges) gives

$$\sigma=\frac{2}{3}\frac{e^2}{\hbar\xi}+\frac{e^2}{\hbar}G_c^{2/3}\left(\frac{\partial n}{\partial\mu}k_{\rm B}T\right)^{1/3}. \qquad (3)$$

That is,

$$\sigma=a+bT^{1/3} \text{ for } \xi \gg L_{\rm int}. \qquad (4)$$

$\partial n/\partial\mu$ is the density of states at the Fermi level. This suggests that in a system where interactions are important, the form $\sigma=a+bT^{1/2}$ for low disorder becomes $\sigma=a+bT^{1/3}$ as the MIT is approached, and at the transition $\sigma=bT^{1/3}$. Ovadyahu[22] has used Eq. (2) with the inelastic length $L_{\rm in}$ as the length scale L, as developed by

TABLE I. Sample properties.

Sample	N_D-N_A (cm^{-3})	N_A/N_D	B_c (T)	ν	σ_c ($\Omega^{-1}\,cm^{-1}$)	Theoretical $T^{1/3}$ slope ($\Omega^{-1}\,m^{-1}\,K^{-1/3}$)	Experimental $T^{1/3}$ slope ($\Omega^{-1}\,m^{-1}\,K^{-1/3}$)
X	3.5×10^{16}	0.2	9.78	0.99 ± 0.03	12.1 ± 0.3	173	91
Y	3.1×10^{16}	0.4	5.26	0.97 ± 0.05	15.2 ± 0.4	168	329
Z	1.8×10^{16}	0.37	2.98	1.01 ± 0.02	14.5 ± 0.5	160	320

Imry,[4] to explain data near the MIT in In_2O_3. In the present work negative magnetoresistance was only seen at fields ($B<0.8$ T) below those needed to produce the MIT. Thus the quantum interference was clearly suppressed and electron-electron interactions will dominate at the transition.

In this work the magnetic-field–induced MIT has been investigated in three compensated GaAs samples, whose properties are given in Table I. All three samples were grown on semi-insulating GaAs substrates, X and Y by liquid-phase epitaxy, and Z by metal-organic chemical-vapor deposition. The samples were etched Hall bars 2.75×0.25 mm^2 with evaporated Au-Ni-Ge contacts.

The measurements were done in a top-loading dilution refrigerator down to 60 mK in magnetic fields of over 10 T. Four-terminal low-frequency measurements were made to obtain the conductivity perpendicular to the magnetic field.

The Mott criterion gives a critical concentration in zero field for GaAs of $n_c=1.56\times10^{16}$ cm^{-3}. From Table I it can be seen that all three samples were doped so that $|N_D-N_A|>n_c$, and at $B=0$ all three had conductivities that extrapolated to finite values at $T=0$. Sample X is sufficiently metallic that the low-disorder theories of weak localization and interactions should apply at $B=0$. As shown in Fig. 1, a $T^{1/2}$ correction to the conductivity was found at $B=0$, with a negative slope indicating that electron-electron interactions are dominating. Also shown in Fig. 1 is σ vs $T^{1/2}$ for $B=8.00$ and 9.00 T, illustrating that the slope of the $T^{1/2}$ correction changed sign and increased as the MIT was approached. Very close to the MIT on both sides, however, the data are fitted better by $T^{1/3}$ than by $T^{1/2}$. In Fig. 2 the conductivity is plotted versus $T^{1/3}$ for magnetic fields on either side of the critical field, the lowest field for which $\sigma(T=0)=0$. The solid lines are the least-squares best fits of $\sigma=a+bT^{1/3}$ to the data. The value of b changes only very slightly with field. Deeper in the insulating region the data deviate above the $a+bT^{1/3}$ fit at the lowest temperatures. This must be so since the conductivity extrapolates to zero on the insulating side.

If we plot $\sigma(T=0)=a$ as a function of field it can easily be seen (Fig. 3) that $\sigma(T=0)$ falls linearly to zero with field at a critical value $B_c=9.78$ T. Performing a least-squares best fit to $\ln\sigma(T=0)$ vs $\ln(a_\perp^2 a_\parallel/a_{\perp c}^2 a_{\parallel c}$

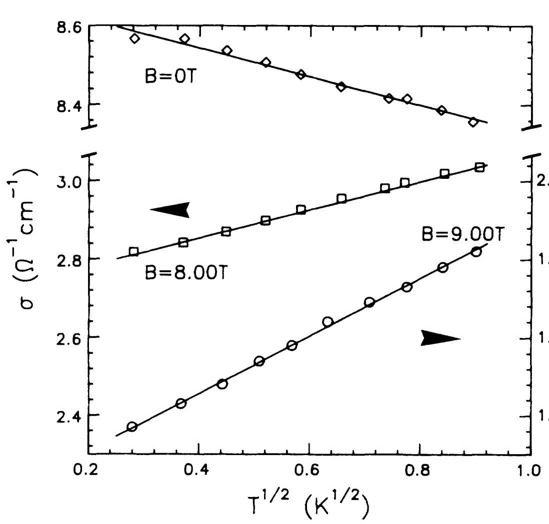

FIG. 1. σ vs $T^{1/2}$ for sample X for $B=0$, 8, and 9 T. The negative slope at $B=0$ indicates that electron interactions are important. As B increases the slope changes sign and increases. The lines are least-squares fits to the data.

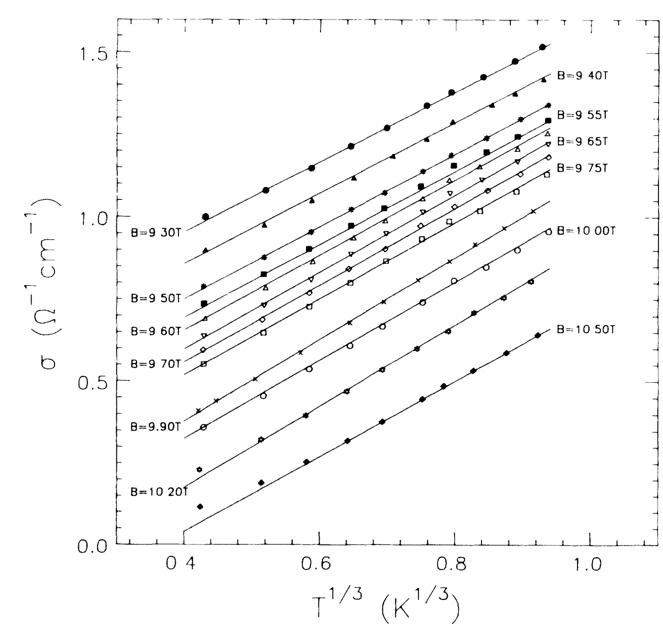

FIG. 2. σ vs $T^{1/3}$ for sample X for several fields on both sides of the critical field, 9.78 T. The lines are the best fits by $\sigma=a+bT^{1/3}$. Note that b changes little with field.

-1) gave a slope of 0.99 ± 0.03. Thus the data are consistent with Eq. (1) with a critical exponent of 1. Figure 4 is a plot of $\sigma(T=0)$ vs $a_{\perp}^2 a_{\parallel}/a_{\perp c}^2 a_{\parallel c} - 1$ for all three samples. The solid line for X represents the best fit by $\sigma(T=0) = \sigma_c(a_{\perp}^2 a_{\parallel}/a_{\perp c}^2 a_{\parallel c} - 1)^{\nu}$ with $\nu = 1$ and $\sigma_c = 12.1\ \Omega^{-1}\,\mathrm{cm}^{-1}$.

The other two samples, Y and Z, were doped closer to the MIT and their data did not follow the perturbative theories for weak localization and interaction. Near the transition the lowest-temperature data did follow $\sigma = a + bT^{1/3}$, but only below $\simeq 450$ mK as compared with $\simeq 800$ mK for X. Again the slope of the $T^{1/3}$ term was nearly constant with field and deviations above the $T^{1/3}$ law came at the lowest temperatures on the insulating side. $\sigma(T=0)$ fell linearly to zero at $B_c = 5.26$ and 2.98 T for Y and Z, respectively. Thus all three samples showed $\sigma(T=0) \propto B_c - B$. However, the slope was 1.0 $(\Omega\ \mathrm{cm}\ \mathrm{T})^{-1}$ for X ($K = N_A/N_D \simeq 0.2$) compared with 1.9 for Y and Z (both $K \simeq 0.4$), indicating that compensation is a factor in determining the sharpness of the transition. Least-squares best fits give $\nu = 0.97 \pm 0.05$ for Y and $\nu = 1.01 \pm 0.02$ for Z; all three samples gave a critical exponent of 1. The value of σ_c was found to be slightly higher than for X, but about equal for these two samples, 15.2 ± 0.4 and $14.5 \pm 0.5\ \Omega^{-1}\,\mathrm{cm}^{-1}$ for Y and Z, respectively, indicating that σ_c depends upon the compensation. Figure 4 shows $\sigma(T=0)$ vs $a_{\perp}^2 a_{\parallel}/a_{\perp c}^2 a_{\parallel c} - 1$ for all three samples.

Equation (3) indicates that the slope of the $T^{1/3}$ term is expected to be $(e^2/\hbar)G_c^{2.3}[(\partial n/\partial\mu)k_B]^{1/3}$. Experimentally the slope changed very little with field near the transition, implying that $\partial n/\partial\mu$ changes little with field in this region. Using $G_c = 2/3\pi^3$ as calculated by Kawabata[23] and the free-electron density of states, we can theoretically estimate the value of the slope of the $T^{1/3}$ term. We obtain a slope of 173 $(\Omega\ \mathrm{m})^{-1}\,\mathrm{K}^{-1/3}$ for X while experimentally the gradient was 91 $(\Omega\ \mathrm{m})^{-1}\,\mathrm{K}^{-1/3}$, implying that $\partial n/\partial\mu$ is 7 times smaller than for free electrons. For samples Y and Z the theoretical values of the slope are 168 and 160 $(\Omega\ \mathrm{m})^{-1}\,\mathrm{K}^{-1/3}$, respectively, while experiment yielded 329 and 320 $(\Omega\ \mathrm{m})^{-1}\,\mathrm{K}^{-1/3}$. This implies that for the more compensated samples $\partial n/\partial\mu$ is 8 times larger than the free-electron value. It is unlikely that $\partial n/\partial\mu$ actually changes so drastically with compensation. Either G_c depends upon the compensation or another compensation-dependent factor is missing from the theory. This agrees with previous results[10] which showed that the $T^{1/3}$ gradient in the metallic regime for a nominally uncompensated GaAs sample gave excellent agreement with theory, while the gradient for a compensated InSb sample was a factor of 3 out.

The behavior of the conductivity in the vicinity of the MIT can be understood in terms of length scales. From Eq. (2) it follows that the conductivity on the metallic side is determined by the shortest length scale, and in this case it is the interaction length:

$$\sigma = G_c e^2/\hbar L_{\mathrm{int}} + e^2/\hbar\xi. \tag{5}$$

Near the transition $L_{\mathrm{int}} \ll \xi$, and this leads to $\sigma = a + bT^{1/3}$. Our data imply that the interaction length determines the conductivity just on the insulating side of the MIT as well, with the conductivity given by

$$\sigma = G_c e^2/\hbar L_{\mathrm{int}} - e^2/\hbar\xi. \tag{6}$$

For $L_{\mathrm{int}} \ll \xi$ this leads to $\sigma = a + bT^{1/3}$, $a < 0$. It is im-

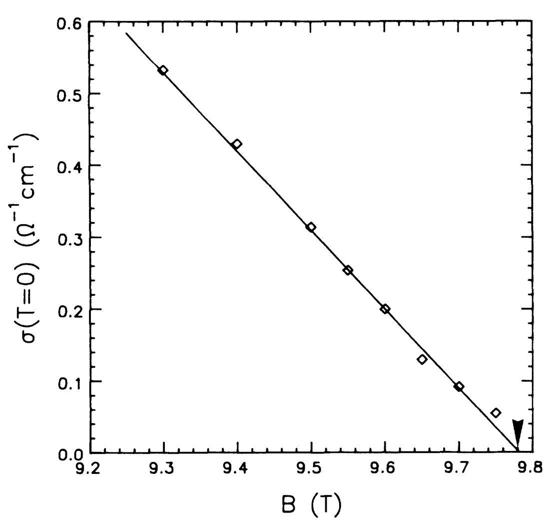

FIG. 3. $\sigma(T=0)$ vs B for sample X. $\sigma(T=0)$ decreases linearly to zero at the critical field, marked with an arrow, of 9.78 T. The line is a least-squares fit to the data.

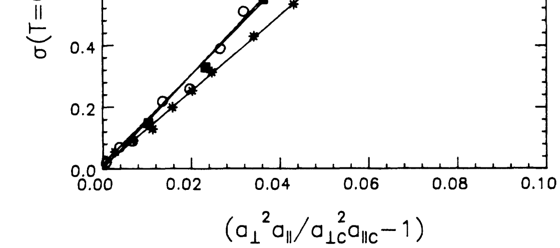

FIG. 4. $\sigma(T=0)$ vs $a_{\perp}^2 a_{\parallel}/a_{\perp c}^2 a_{\parallel c} - 1$ for X (asterisks), Y (circles), and Z (squares). The lines are best fits with $\sigma_c = 12.1$, 15.2, and 14.5 $\Omega^{-1}\,\mathrm{cm}^{-1}$ for X, Y, and Z, respectively. The good linear fits indicate a critical exponent of 1 for all three samples. $N_A/N_D = K$.

portant to note that the form of the conductivity does not change through the MIT, and that it is determined by the shortest length scale.

On the insulating side ξ is interpreted as the localization length. It is infinite at the transition, but it decreases in the insulator until the two terms in Eq. (6) become comparable (i.e., $L_{\rm int}/G_c \sim \xi$) at an obtainable temperature and the behavior deviates from $a+bT^{1/3}$. Further into the insulator the deviation should occur at a higher temperature, as was seen. For $\xi \ll L_{\rm int}$ hopping should dominate. With the assumption that $|a| \sim e^3/h\xi$ on the insulating side, the data for X imply $\xi^{-1} \sim 3 \times 10^5 (B-B_c)$ m^{-1}. Since $\sigma(T=0) \propto (B_c - B)$ it follows that ξ^{-1} has the same critical exponent as $\sigma(T=0)$.

We can check our interpretation by seeing if $L_{\rm int}/G_c \sim \xi$ where the data deviate from the $\sigma = a+bT^{1/3}$ fit. For X at $B=10.50$ T the data deviate where $\sigma = 25$ Ω^{-1} m^{-1} and $T=0.2$K. $L_{\rm int}$ can be estimated for this point by use of $L_{\rm int} = (\hbar D/k_B T)^{1/2} = [\hbar\sigma/k_B T e^2 (\partial n/\partial\mu)]^{1/2}$, with the assumption of a free-electron density of states. This gives $L_{\rm int}/G_c \sim 1.2 \times 10^{-6}$ m to compare with $\xi \sim [3\times 10^5 (B-B_c)]^{-1} \sim 4.6 \times 10^{-6}$ m. In consideration of the assumptions involved, the agreement is reasonable.

In conclusion, we have found that the conductivity of compensated GaAs follows the law $\sigma = a+bT^{1/3}$ near the transition on both sides of magnetic-field–induced metal-insulator transition. This can be explained in terms of the conductivity being determined by the shortest length scale, in this case the interaction length. The zero-temperature conductivity was found to have a critical exponent of 1 in agreement with recent theory on interaction-dominated transitions in a strong magnetic field.

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