Frequency Up-Conversion of Electromagnetic Radiation with Use of an Overdense Plasma

S. C. Wilks, J. M. Dawson, and W. B. Mori

Department of Physics, University of California, Los Angeles, California 90024

(Received 8 February 1988)

We investigate the effects of quickly creating a plasma around a monochromatic electromagnetic source wave, on time scales on the order of a cycle of the wave. It is found that this results in an upward shifting of the wave frequency, which can be varied by changing of the plasma density. It is also found that a substantial fraction of the magnetic field associated with the initial wave can be sustained in the plasma as a time-independent magnetic field. Computer simulations have been used to study this process in detail, including the effects of finite ionization time. For long ionization times, strong plasma heating results.

PACS numbers: 52.40.Nk, 42.65.Ky, 52.25.Rv, 52.35.Mw

Recent advances in high-power lasers capable of producing 10–100-fs-long pulses[1] of photons with energies of between 2 and 4 eV have made possible the ionization of small (1 mm^3) regions of gas in a time on the order of the pulse duration. In this Letter, we consider what happens to an electromagnetic wave, which we will refer to as the source wave (which might be a CO_2 laser beam, for example), propagating through a gas which is ionized during a single cycle of this wave. It is predicted from linear theory that the frequency of the resultant radiation is shifted upward past the plasma frequency and that a fraction of the original wave energy remains in the plasma as a steady-state magnetic field. These predictions, which are based on the ideal case of instantaneous plasma creation, are found to agree with results given by 1D and 2D particle-in-cell computer simulations. We also use the simulations to investigate how the quality of the resulting radiation is altered when the gas is slowly ionized over a number of cycles (1 to 10) of the source wave. It is also observed that for longer ionization times, the plasma is heated to between 1 and 100 kV, depending on the initial wave intensity and gas density, because of the different phases of the electric field at the time of ionization.

The linear analysis begins with a purely right-going laser pulse, propagating in the as yet un-ionized gas. Experimentally, it will be required that the source laser power be sufficiently low such that this laser does not ionize the gas; the gas at this point has an index of refraction of $\simeq 1$, for a laser of this frequency. As a first approximation, the source laser is represented by plane-polarized-wave solutions to Maxwell's equations in vacuum:

$$\mathbf{E} = E_0 \cos(k_0 x - \omega_0 t)\hat{\mathbf{e}}_y, \tag{1}$$

$$\mathbf{B} = B_0 \cos(k_0 x - \omega_0 t)\hat{\mathbf{e}}_z, \tag{2}$$

where $E_0 = B_0$ and $\omega_0 = k_0 c = 2\pi\nu_0$ is the frequency of the source laser. Suddenly, in a time interval short compared with ν_0^{-1}, we create a plasma around a portion of this laser pulse at $t=0$. Details regarding the creation of this plasma with existing lasers will be discussed shortly. We now consider what effect the introduction of this plasma has on the frequency spectrum of the pulse. At time $t=0$, inside the plasma the fields have the form of Eqs. (1) and (2) evaluated at $t=0$. At subsequent times, the fields still have the same spatial periodicity. However, the field now evolves in time with a final frequency ω_f given by the dispersion relation

$$\omega_f^2 = k_0^2 c^2 + \omega_p^2. \tag{3}$$

Note that although this is true for any plasma density, a large upward shift in frequency is possible if the density is chosen to be greater than the critical density, $n_{\text{crit}} = \omega_0^2 \times m/4\pi e^2$. What is unusual about this situation is that electromagnetic waves of the original frequency do not ordinarily exist inside an overdense plasma. The more typical case considered is that of a vacuum-plasma boundary; and if a laser is fired from the vacuum at the overdense plasma, it penetrates only a skin depth ($\sim c/\omega_p$) and is reflected back into vacuum. This arises from the fact that the discontinuity in space (the vacuum-plasma interface) keeps ω fixed (time independence of the medium) but allows for a change in k. However, for the case of flash ionization, we are now creating a discontinuity in time, which means that ω can change, but the wavelength must remain the same before and after the ionization takes place. Therefore, the frequency of the wave shifts to satisfy Eq. (3), keeping k at its original value, k_0. Mathematically, the initial-value problem allows these solutions inside the plasma, whereas the boundary-value problem does not.

A further difference from the standard vacuum-plasma interface solution can be found from the wave equation for the field B. In addition to the two solutions (the right- and left-going waves) implied in Eq. (3), there exists a solution for which $\omega = 0$ and $k = k_0$. This additional solution is required because introduction of the plasma adds an additional degree of freedom (the motion of the electrons) and the additional wave amplitude is required to solve the initial-value problem with given **E**, **B**, and **v**. This stationary, sinusoidally varying magnetic field remains in the plasma, even after the up-shifted light has been radiated out of the plasma. All three solutions are shown in the ω-k diagram in Fig. 1. These solutions can be understood from the electrons' response as follows. When the electrons are suddenly set free and are accelerated, they radiate in all directions such that the initially right-going wave is now broken into both right- and left-going components. In addition, by following the electric field of the initial wave, the electrons immediately create a transverse current in the plasma, allowing for the static-B-field, or zero-frequency, solution. In fact, it is easy to see why the wavelength of the static B field is the original laser wavelength from this argument.

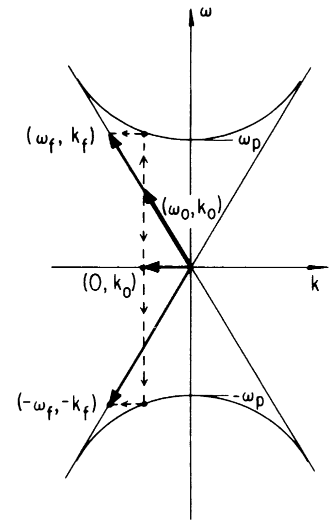

FIG. 1. Dispersion relation for electromagnetic waves in an instantaneously created overdense plasma. Note that the discontinuity in time (caused by ionization of the plasma) not only converts the original wave (ω_0, k_0) into left- and right-going waves $(\pm\omega_f, k_0)$, but also allows for a zero-frequency solution which corresponds to the trapped B field.

We now consider such a laser-plasma interaction occurring in a plasma of length L with well-defined vacuum-plasma boundaries at $\pm L/2$. We choose L to be a large number of wavelengths such that we can neglect edge effects, and concentrate on the response of the bulk of the plasma. At times $t>0$, when the plasma is present, the fields are given by

$$\mathbf{E}_{t>0}=[E_+\cos(k_0x-\omega_f t)+E_-\cos(k_0x+\omega_f t)]\hat{\mathbf{e}}_y, \tag{4}$$

$$\mathbf{B}_{t>0}=(1-\omega_p^2/\omega_f^2)^{1/2}[E_+\cos(k_0x-\omega_f t)-E_-\cos(k_0x+\omega_f t)]\hat{\mathbf{e}}_z+B_s\cos(k_0x)\hat{\mathbf{e}}_z, \tag{5}$$

where we have broken up the resulting electromagnetic field into left- and right-going components. To complete the set of equations describing our model, we include the velocity of the plasma,

$$\mathbf{v}_{t>0}=(-ie/m\omega_f)[E_+\cos(k_0x-\omega_f t)-E_-\cos(k_0x+\omega_f t)]\hat{\mathbf{e}}_y+v_s\cos(k_0x)\hat{\mathbf{e}}_y, \tag{6}$$

where we relate v_s and B_s which are the magnitudes of the static transverse velocity and static magnetic field, using Amperes law,

$$ik_0B_s=4\pi n_0ev_s/c, \tag{7}$$

and have assumed that the plasma temperature at $t=0$ is negligible. To solve for E_+, E_-, and B_s, we must match the initial fields in Eqs. (1) and (2) with the solutions of the wave equation with the plasma present. The amplitudes of the left- and right-going components of the fields are found to be

$$E_\pm=(E_0/2)(1\pm\omega_0/\omega_f). \tag{8}$$

Additionally, we find from Eqs. (4)–(7) that the stationary B field is given as

$$B_s\cos(k_0x)=\frac{\omega_p^2E_0}{\omega_p^2+\omega_0^2}\cos(k_0x). \tag{9}$$

Note that as the plasma density is increased, such that $\omega_p\gg\omega_0$, the amplitude of the field approaches the value of the original field, which can be on the order of a megagauss at the focal spot of a CO_2 laser with an intensity of 2.5×10^{14} W/cm^2. For this laser, the magnetic field will vary sinusiodally in space with a wavelength of 10 μm. This field may have applications as an undulator; in fact, tapering could easily be achieved by variation of the neutral gas pressure.

In order to test this model, we have performed a series of computer simulations using the plasma code WAVE.[2] For the case of instantaneous turnon (top of Fig. 2, $\Delta t=0$), the frequency and amplitude of the output radiation agree with what the above model predicts. An important question that can now be asked is the following: How is this phenomenon affected when finite ionization time is considered? Although this new problem is analytically solvable in certain limits, the simulations have allowed us to determine what the effects of slowly creating the plasma, over periods of 1 to 10 cycles of the incident wave, are on the quality of the resulting radiation. Figure 2 compares the power spectrum of the ideal, instantaneously created plasma case with two other cases where density was ramped linearly over times of $1\nu_0^{-1}$ and $10\nu_0^{-1}$. As might be expected, an increase of the ionization time still results in up-shifted light albeit at progressively lower powers. However, the longer ionization times may provide for a method of controlled continuum generation, as seen in Fig. 2(c).

The dependence of the static B field on ionization time

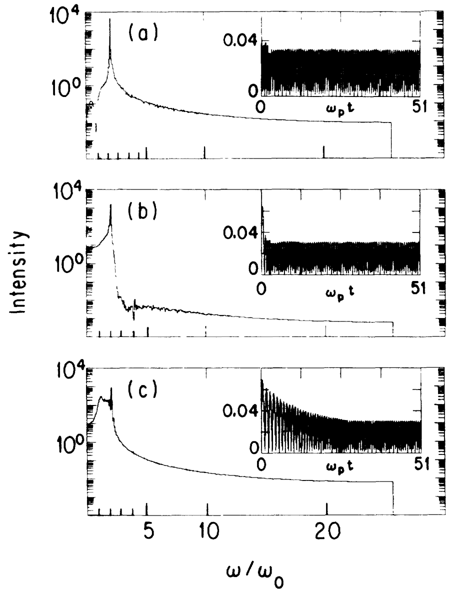

FIG. 2. Power spectrum for radiation leaving the right-hand boundary of the plasma for increasing ionization times, Δt. The plasma (20 source-wavelengths long) was created instantaneously in (a); over a time interval ν_0^{-1} for (b); and over a time interval $10\nu_0^{-1}$ in (c). Plasma density is $2n_{\text{crit}}$ for each run; thus, the peak frequency has been shifted up according to Eq. (3). Insets: Time history of $|E|$ at the boundary.

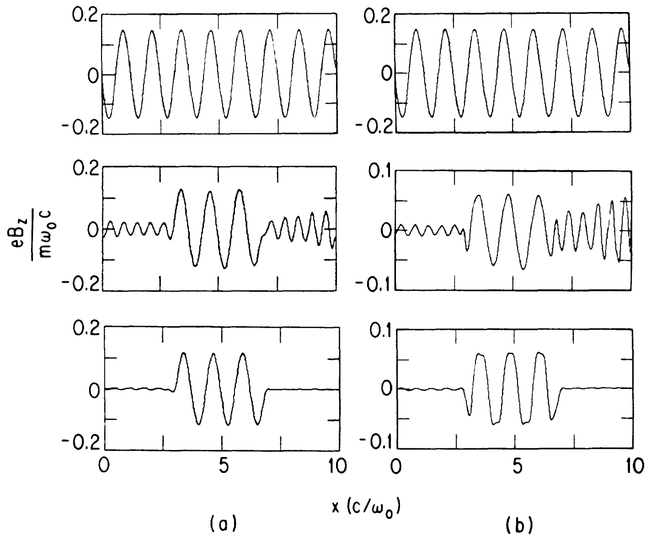

FIG. 3. Time history of magnetic field in real space for (a) instantaneously created and (b) slowly ($\sim 1\nu_0^{-1}$) created plasma.

was also studied, and the results compared with the instantaneously created plasma case. Figure 3(a) shows the results of a simulation where $L=3\times 2\pi c/\omega_0$, and the plasma density was chosen to be $2n_{\text{crit}}$. The amplitude of the field is in agreement with Eq. (9). As the time taken to fully ionize the gas increases, the simulations show both a decrease in the magnitude of this field and a slight degradation from the sinusoidal structure of the field. This would obviously affect the performance of this device as an undulator. However, as shown in Fig. 3(b), the residual magnetic field can still be as high as half the original field for creation times as long as $1\nu_0^{-1}$. Because this static magnetic field is maintained by a transverse current, and up to this point neither the linear analysis nor the 1D simulations have taken into account the transverse position of the electrons carrying this current, a question arises as to the effect of including the transverse dimension. In order to answer this question, several 2D simulations were performed. The preliminary result is that current vortices developed so as to sustain the trapped magnetic field, as long as the transverse dimension of the source wave is greater than a wavelength, and the transverse dimension of the plasma is at least this large.

The linear analysis neglects the longitudinal motion of the plasma electrons and, ultimately, that of the ions. However $\mathbf{v}\times\mathbf{B}$ forces arising from the static magnetic field and the counterpropagating electromagnetic waves cause motion of the plasma in the longitudinal direction. The first of these is a high-frequency $(\omega_f, 2k_0)$ force attributed to the beating of the static B field with the right- and left-going electromagnetic waves. Since the electrons have a natural frequency of ω_p, this longitudinal force evidently becomes important when densities are chosen such that ω_f approaches ω_p. Additionally, there are two low-frequency terms $(\omega=0, 2k_0)$ due to (i) the stationary magnetic field pressure gradient, and (ii) the beating of the two counterpropagating waves with each other. Thus, electrons are forced out of high-field regions and bunch up in the low-field regions; the resulting space-charge force then causes the ions to move. However, simulations including ion motion have been done for times greater than $10\nu_{pi}^{-1}$, with little effect on the resultant radiation, although the magnetic field is again slightly degraded because of this motion. This is due to the fact that because the ions are massive, they move on a time scale of ν_{pi}^{-1}, and therefore do not significantly enter the problem until well after a large fraction of the radiation has escaped the plasma (for the plasma volumes discussed here).

Another interesting effect that has been noted in the simulations is that if the ionization time is much longer than ν_0^{-1}, the transverse oscillations are sufficiently out of phase so that instead of a coherent radiator, strong transverse heating of the plasma results. This heating differs from other laser heating methods[3] in that it occurs in the absence of collisions or parametric instabilities. Physically, what has happened is that because the ionization takes place over a number of cycles of the source wave, electrons are born at different phases of the electric field associated with the wave. Therefore, some of the energy of the electric field of the source wave, which would have gone into a coherent transverse current to sustain the magnetic field in the case of instantaneous ionization, ends up as random thermal motion of the electrons for long turnon times.

To show that creation of plasmas with densities near $2n_{crit}$ for CO_2 in tens of femtoseconds may be possible with existing technology, consider that to create a xenon plasma of density 2×10^{19} cm^{-3} in a volume of dimensions $100\times50\times50$ μm^3 with a 0.3-μm laser requires only about 10^{-2} mJ of energy in the laser pulse. Here we have taken multiphoton ionization to be the mechanism responsible for plasma creation, and have assumed that complete ionization takes place only over the length of the pulse (i.e., no cascading). Currently, XeCl excimer lasers are capable of 3.5 mJ in 100 fs (about three oscillations of 10-μm radiation) which, when focused into a 50-μm^2 area, gives an intensity of 10^{15} W/cm^2. Such a pulse both exceeds the multiphoton ionization threshold and contains enough energy to create a sizable region of plasma adequate for production of a significant amount of up-shifted radiation. One possible scenario would be to direct a single ionizing laser perpendicular to the source wave. In this case, an ionization front propagates across the source wave. Preliminary 2D simulations show that the only modification to our analytical results is that the wave fronts become curved. Our studies also show that with two or more time-tailored laser pulses, it is possible to (approximately) instantaneously ionize a finite region of gas without violation of causality.

In conclusion, we have suggested a possible method for the upward shifting of the light of existing lasers, provided that one can quickly create a plasma with a higher-power, shorter-wavelength laser. This up-shifted light is also tunable in the sense that by varying the plasma density, one is actually varying the output frequency given in Eq. (3). In addition, the model discussed above also predicts a residual, static magnetic field, given by Eq. (9). 1D and 2D computer simulations of this phenomena have shown all of these effects to occur as predicted in the linear regime. In addition, we have investigated the effects of finite ionization time, and nonlinear longitudinal motion, with the help of computer modeling. Although we have commented on only three applications of this phenomenon (conversion of a CO_2 pulse to tunable, up-shifted radiation, the use of the trapped, static magnetic field as the wiggler field for a free-electron laser, and the production of kilovolt plasmas) other possible applications are currently being investigated. For instance, by our embedding the wave in a gas of nonuniform pressure before ionization, any desired frequency time history for the radiation can be generated.

We would like to thank Dr. D. Umstadter for helpful discussions concerning short-pulse, high-power lasers, and also Dr. F. F. Chen, Dr. C. Joshi, and Dr. T. Katsouleas for their useful comments on this work. This work was supported by U.S. Department of Energy Contract No. DE-FG03-87ER13752, and the computing was done at the San Diego Super Computer.

[1]A. Migus, A. Antonetti, J. Etchepare, D. Hulin, and A. Orszag, J. Opt. Soc. Am. B **2**, 584 (1984), and references therein.

[2]R. L. Morse and C. W. Neilson, Phys. Fluids **14**, 830 (1971).

[3]C. E. Max, LLNL Report No. UCRL-53107, 1981 (unpublished).

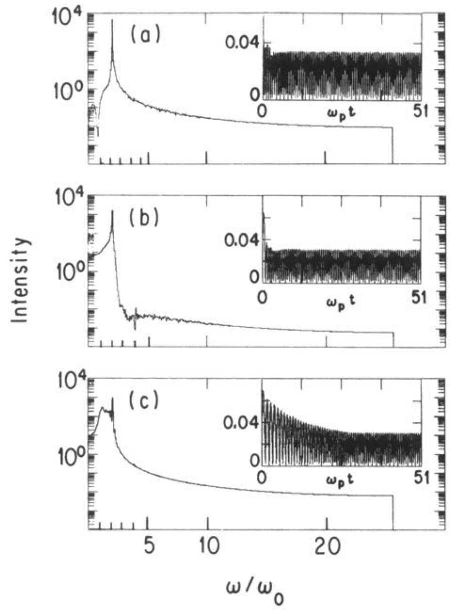

FIG. 2. Power spectrum for radiation leaving the right-hand boundary of the plasma for increasing ionization times, Δt. The plasma (20 source-wavelengths long) was created instantaneously in (a); over a time interval ν_0^{-1} for (b); and over a time interval $10\nu_0^{-1}$ in (c). Plasma density is $2n_{\text{crit}}$ for each run; thus, the peak frequency has been shifted up according to Eq. (3). Insets: Time history of $|E|$ at the boundary.