Fluctuations in the Shape Transitions of Hot Nuclei

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(Received 2 February 1988)

The effect of quantal and thermal quadrupole shape fluctuations in the giant dipole response function of hot nuclei at high spin is studied within the Landau theory of phase transitions. The effects are found to be important in the relation of the nuclear shape to the experimental findings, and in the identification of shape phase transitions.

PACS numbers: 24.60.Ky, 24.30.Cz, 24.60.Dr, 27.70.+q

In a recent paper,[1] the Landau theory of phase transitions[2,3] was applied to the study of quadrupole shape changes in hot nuclei, and the universal features of such transitions derived. These results are likely to play an important role in the study of nuclei at finite temperature. However, fluctuations of the shape in a small system like the nucleus are expected to be of such importance that their effect must be taken into account before the results of the model can be compared with experiment.[4-7]

In the present paper we introduce fluctuations in the Landau model for nuclear phase transitions of Ref. 1, and discuss some of the effects associated with this phenomenon. It will be concluded that fluctuations of the shape are, as a rule, as important as the average deformation values in the determination of the effective nuclear shape.

The calculations have been carried out as a function of temperature and angular momentum, making use of the parametrization adequate for the nucleus ^{166}Er, in keeping with the example discussed in Ref. 1. The thermodynamic potential[2,3] $\Phi(\phi)$ describing the system as a function of the order parameters $\phi \equiv \{\beta,\gamma\}$ is written as

$$\Phi(\beta,\gamma)=\Phi_0+A\beta^2-B\beta^3\cos 3\gamma+C\beta^4-\tfrac{1}{2}\mathcal{J}\omega^2,$$

where ω is the rotational frequency. The quantities Φ_0, A, B, and C, as well as the moment of inertia $\mathcal{J}$ are universal functions of temperature and are taken from Ref. 1, while β and γ measure, in the rotating body-fixed frame of reference, the size of the quadrupole moment and the departure of the spheroid from axial symmetry.

The general theory of thermodynamic fluctuations states that, for a given temperature and pressure, the probability $\rho(\phi)$ of a definite configuration $\phi \equiv \{\beta,\gamma\}$ of the order parameters is given by[2,3]

$$\rho(\beta,\gamma)=Z^{-1}e^{-\Phi(\beta,\gamma)/T}, \tag{1}$$

where Z is the partition function

$$Z=\int d\tau\, e^{-\Phi(\beta,\gamma)/T}, \tag{2}$$

and $d\tau$ is the volume element associated with the β and γ parameters. Following the arguments presented in Ref. 4 (cf. also Ref. 6) in connection with the analysis of the γ deexcitation of hot nuclei with fixed total angular momentum and projection,

$$d\tau=\beta\, d\beta\, d\gamma. \tag{3}$$

Employing Eqs. (1)-(3), one can calculate the expectation value

$$\langle F\rangle=\int d\tau\, \rho(\beta,\gamma)F(\beta,\gamma), \tag{4}$$

of any operator $F(\beta,\gamma)$ that can be expressed in terms of the collective coordinates β and γ.

In Fig. 1, we display the average values

$$\bar{\beta}=\langle\beta\rangle, \quad \bar{\gamma}=\langle\gamma\rangle, \tag{5}$$

corresponding to the nucleus ^{166}Er, as a function of temperature and for zero rotational frequency. It is noted that the associated thermodynamic potential given in Ref. 1 was determined by making use of a cranked Nilsson Hamiltonian including standard Strutinsky shell corrections. To the extent that the resulting function $\Phi(\beta,\gamma)$ provides an accurate description of the microscopic calculations, one is confronted with a situation similar to that discussed, e.g., in Refs. 4 and 5. In keeping with these references, we carry out the averaging (4) numerically over the (β,γ) plane. We also show in Fig. 1 the value β_0 of the deformation parameter at the minimum of the potential-energy surface for each temperature.[1] The effect of the fluctuations is apparent and, as previously shown elsewhere[8] washes out the first-order phase transition predicted at $T \approx 1.7$ MeV. In this con-

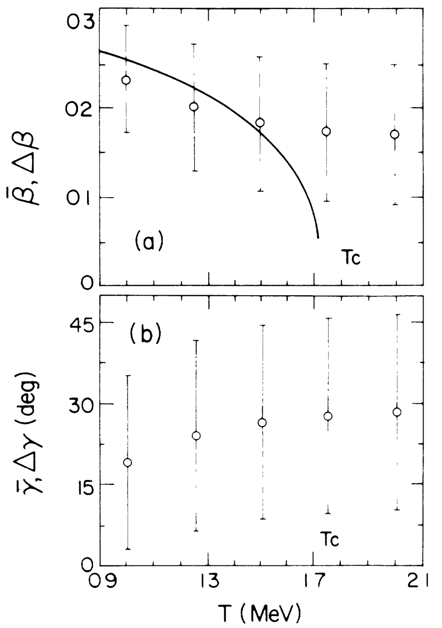

FIG. 1. The average value [Eq. (5)] of the deformation parameters β and γ and standard deviations $\Delta\beta$ and $\Delta\gamma$ for ^{166}Er as a function of temperature at zero rotational frequency. The continuous line in (a) represents the equilibrium values β_0 extracted from the phase diagram of Ref. 1. The circles and error bars are results based on Eqs. (4) and (5) of the present calculation. The quantity T_c is the critical temperature for the first-order transition according to Ref. 1.

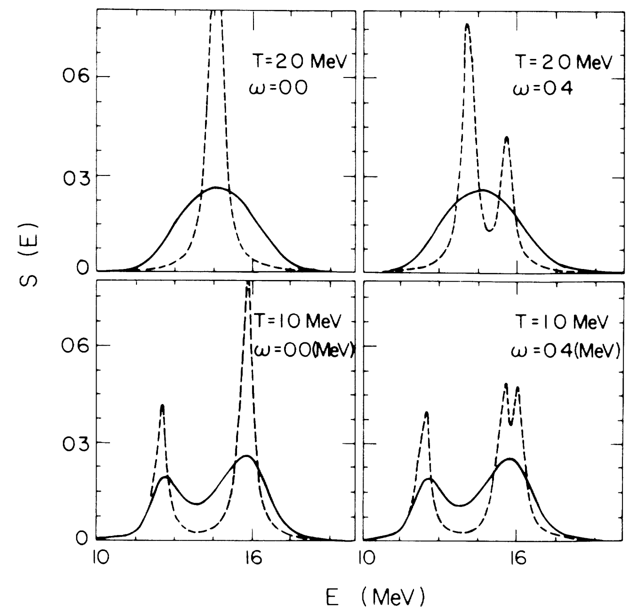

FIG. 2. Strength functions associated with the GDR in ^{166}Er as a function of the excitation energy and of the rotational frequency. The results displayed with a continuous curve were calculated making use of Eq. (6), while those displayed with a dashed curve do not contain fluctuations [Eq. (7)], and correspond to the strength functions associated with the equilibrium values (β_0, γ_0) predicted in Ref. 1. An averaging parameter $\Gamma = 0.5$ MeV has been used throughout.

text, note the large values of the standard deviations, $\Delta\beta = \langle \beta^2 - \bar{\beta}^2 \rangle^{1/2}$ and $\Delta\gamma = \langle \gamma^2 - \bar{\gamma}^2 \rangle^{1/2}$, also displayed in Fig. 1. Similar results have been found in Ref. 6.

The giant dipole resonance (GDR) couples directly to the quadrupole deformation of the nuclear surface leading to a breaking of the dipole strength in three components[9] according to

$$\omega_\kappa = \omega_{\text{dip}} \left[1 - \left(\frac{5}{4\pi} \right)^{1/2} \beta \cos \left(\gamma - \frac{2\pi}{3}\kappa \right) \right], \quad \kappa = 1,2,3,$$

where $\hbar\omega_{\text{dip}} \approx 80/A^{1/3}$ MeV. Consequently, the study of the dipole strength function in the γ decay of hot nuclei is expected to provide detailed information on the quadrupole order parameters as a function of temperature and angular momentum.[10-12]

In an axially symmetric—either prolate or oblate—nucleus, the dipole is split into two components. In the former case, the low-frequency component arises from oscillations along the major axis of the nucleus that coincides with the symmetry axis. The high component is built out of oscillations along a plane perpendicular to the symmetry axis.

In Fig. 2, we display the averaged[4,5] strength function

$$S = S_0 \textstyle\sum_\kappa \int d\tau \, \rho(\beta,\gamma) F_\kappa(\beta,\gamma), \tag{6}$$

where

$$F_\kappa(\beta,\gamma) = \Gamma_\kappa \{ [E - \hbar\omega_\kappa(\beta,\gamma)]^2 + \Gamma_\kappa^2/4 \}^{-1},$$

as a function of the excitation energy and the rotational frequency of the compounds nucleus ^{166}Er*. A common value of the widths Γ_κ ($=\Gamma$) has been used. Also plotted in Fig. 2 is the dipole strength function

$$S = S_0 \textstyle\sum_\kappa \Gamma \{ [E - \hbar\omega_\kappa(\beta_0,\gamma_0)]^2 + \Gamma^2/4 \}^{-1}, \tag{7}$$

associated with the equilibrium values β_0 and γ_0 of the deformation parameters.[1] The effects of fluctuations are, as expected from the previous results, very important at all temperatures, and can totally mask the value of the deformation parameters at the minimum of the potential.[13]

The γ decay of the compound nucleus ^{166}Er* has been carried out at a variety of temperatures and angular momenta.[14,15] The main conclusion reached by Gosset *et al.*[14] is that the shape of the GDR strength function based on the compound state of ^{166}Er with $T = 1.2$ MeV and $I \sim 15\hbar$ ($\omega \sim 0.2$ MeV) is quite similar to that based on the ground state.

To check this result, we have calculated the GDR strength function at $T = \omega = 0$, taking now into account the quantal fluctuations associated with the β,γ degrees of freedom. For this purpose we have calculated the

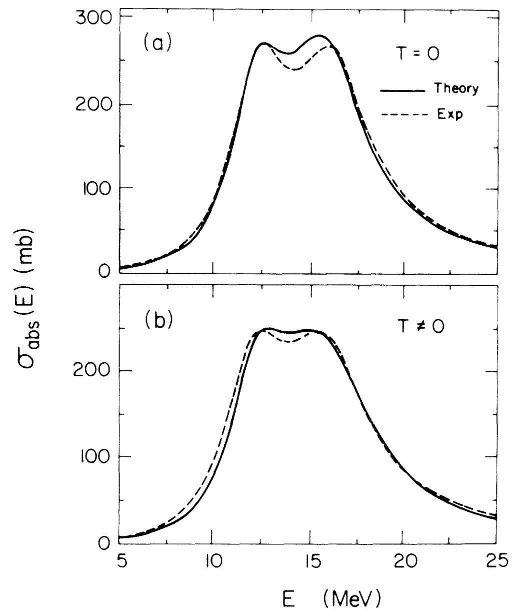

FIG. 3. The experimental results at (a) $T=0$ and (b) $T\neq 0$ were taken from Refs. 14 and 19, and are displayed as a dashed curve. The full line in (a) gives the results of the present calculation for the ground state of ^{166}Er. The full line in (b) gives the results of the present calculation for the same nucleus and for $T\neq 0$ (cf. Ref. 20) and $\omega=0.2$ MeV, which corresponds to an angular momentum $I=15\hbar$. The rotational frequency ω and angular momentum I are related according to Ref. 1.

ground-state wave function $\Psi_0(\beta,\gamma)$ of the Bohr Hamiltonian[16]

$$H=T_{\beta,\gamma}+[\Phi(\beta,\gamma)]_{T=0,\omega=0}\,,$$

where

$$T_{\beta,\gamma}=-\frac{\hbar^2}{2D}\left(\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}\right)$$

are the kinetic-energy terms associated with the β and γ degrees of freedom. The quantity D is the inertial parameter which, for simplicity, will be assumed to be constant. The results that we are going to present hardly depend on the actual value of D, which we have set equal to $(126\ \mathrm{MeV}^{-1})\hbar^2$.[17]

The calculation of the photoabsorption cross section of ^{166}Er based on the ground state and the taking into account of quantal fluctuations was carried out with Eq. (6), substituting $\rho(\beta,\gamma)$ by $|\Psi_0(\beta,\gamma)|^2$, $F_\kappa(\beta,\gamma)$ by Lorentzian functions, and $d\tau$ by $d\tau_Q$ [cf. Eq. (6-282) of Ref. 16]. The widths Γ_κ were adjusted to fit the data. Following Carlos *et al.*[18] (cf. also Ref. 5) we have used

TABLE I. Comparison between the parameters reported in Refs. 14 (cf. Table I) and 19, determined from a fitting of the data (Exp.) with a static deformed nucleus, and those obtained in the present paper (Th.) including fluctuations. The labels h and l indicate high and low (peak).

			Γ (Mev)		E (Mev)	
			Th	Exp	Th	Exp
T=0	h	x	4.6 (0.3)	5.3	15.3 (0.6)	16.1
		y	5.2 (0.3)		16.2 (0.6)	
	ℓ	z	3.1 (0.1)	3.4	12.3 (0.5)	12.4
T≠0	h	x	4.8 (0.8)	5.8	14.9 (1.6)	15.8
		y	5.4 (0.4)		15.9 (0.7)	
	ℓ	z	3.8 (0.8)	3.7	13.2 (0.4)	12.2

the parametrization

$$\Gamma_\kappa=\Gamma_0(\hbar\omega_\kappa)^{1.9},$$

where $\Gamma_0=0.026$, extracted from a systematic analysis of photoabsorption cross sections of giant resonances at zero temperature. The results are shown in Fig. 3(a) in comparison[14] with the experimental data.

Because of the overall agreement obtained, one feels confident to proceed to the calculation of the finite-temperature cross sections. The results, which reproduce the main findings (cf., however, Ref. 20), are shown in Fig. 3(b) in comparison with the data. In this case the best fit was obtained by $\Gamma_0=0.028$.

In Table I we collect the quantal ($T=0$) and thermodynamic ($T\neq 0$) average values of the parameters Γ_κ and $\hbar\omega_\kappa$ as well as the corresponding standard deviations, associated with the three Lorentzian functions contributing to the photoabsorption cross section. Also given are the empirical values of the parameters obtained with the assumption of static deformations.[14,19] Although the results of the two analyses display overall compatibility, fluctuations can change the values of the different quantities by up to 20%. The reason for this visible, but modest effect is that, in strongly deformed nuclei, fluctuations mix with similar weight both smaller and larger deformations; larger effects are expected both at higher temperatures and in the case of nuclei that are spherical in their ground state (cf. Ref. 5).

We conclude that surface fluctuations play a central role in the changes of shapes that the nucleus undergoes under the influence of temperature and spin, and that they influence profoundly the structural modifications associated with nuclear shape phase transitions. In fact, it is likely that the study of nuclear structure at finite temperature and spins gives a unique opportunity to study fluctuations in finite systems.

Discussions with G. F. Bertsch, P. F. Bortignon, B. R. Mottelson, and S. Levit are gratefully acknowledged. One of us (J.M.P.) acknowledges financial support from Fundação Caloustre Gulbenkian. One of us (C.Y.) is a NATO fellow for the 1987-1988 academic year.

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[20]With use of the potential Φ of Ref. 1 corresponding to $T = 1.2$ MeV and $\omega = 0.2$ MeV, a poor fitting of the data shown in Fig. 3(b) is obtained, even if the parameters Γ_κ and $\hbar\omega_\kappa$ are freely adjusted. If one, however, uses the potential corresponding to $T = 0$ or, for that matter, that associated with $T = 0.6$ MeV and $\omega = 0.2$ MeV, which was the one used in the calculation shown in Fig. 3(b), one obtains a good reproduction of the data, implying that the parametrization proposed in Ref. 1 for the thermodynamic potential—although displaying an overall correct behavior with temperature—is slightly too soft. This problem, which deserves a detailed study by itself, does not affect the qualitative aspects of the discussion of fluctuations carried out in this paper.