Comment on "Is There a Breakdown of Quantum Electrodynamics?"

Recently Samuel[1] stated that a prediction of quantum electrodynamics and experiment[2] disagree by 4 standard deviations, implying its possible breakdown. This is based on his calculation of the light-by-light scattering contribution $a_{\gamma\gamma}$ to the electron anomaly,

$$\frac{a_{\gamma\gamma}}{(\alpha/\pi)^3}=0.398(5)\,, \tag{1}$$

where the numerical enclosed in parentheses represents the uncertainty in the last digit. What I want to point out is that this calculation is not accurate enough to warrant such a drastic conclusion.

By far the most accurate evaluation of $a_{\gamma\gamma}$ to date is that of Engelmann and Levine[3]:

$$\frac{a_{\gamma\gamma}}{(\alpha/\pi)^3}=0.370\,986(20)\,. \tag{2}$$

It was obtained by the analytical reduction of a twelve-dimensional integral to a three-dimensional one, then evaluating it numerically. Other results[1,4] are based on numerical evaluations of seven-dimensional Feynman-parameter integrals. They are easier to write down but require a lot of computing to achieve high precision.

The result (1) is clearly at odds with (2) and earlier results.[4] To claim the superiority of (1), one must show what is wrong with other calculations. This has not been done. Since it is just a matter of evaluating a well-defined integral, I offered to exchange programs with Samuel for mutual inspection. Subsequently he sent me his program which consists of the integrand and the integration subroutine SPCINT. I examined and found his integrand to be identical with our formula in Ref. 4. Then I integrated it using VEGAS.[5] After eighteen iterations (5 h on IBM 3090) of 10^7 sampling points each, I obtained

$$\frac{a_{\gamma\gamma}}{(\alpha/\pi)^3}=0.3704(24)\,, \tag{3}$$

the χ^2 per iteration being 0.83. It is in good agreement with (2) and in clear disagreement with (1). In view of the high reliability of VEGAS, which has been tested on hundreds of integrals, I am certain that the error estimate in (3) is accurate. On the other hand, judging from extensive communications I have had with Samuel, it appears that his iteration procedure is not working as well as mine. Thus the most plausible cause for the discrepancy between (1) and all other results is the overly optimistic, perhaps by a factor of 5 or 6, estimate of errors in (1).

As a further support of (2), I present a new evaluation, obtained by VEGAS, of $a_{\gamma\gamma}$:

$$\frac{a_{\gamma\gamma}}{(\alpha/\pi)^3}=0.371\,12(28)\,. \tag{4}$$

The integrand for this calculation was derived with use of a transformation based on the Ward-Takahashi identity,[6] and is independent of all previous works. The number of sampling points per iteration is 10^8. It is iterated 30 times with a very good χ^2.

In addition, I have reevaluated the light-by-light contribution to the muon anomaly to check the value 21.32(5) reported by Samuel and Chlouber.[7] The new result is

$$\frac{a^{\mu}_{\gamma\gamma}}{(\alpha/\pi)^3}=20.9471(29)\,. \tag{5}$$

The number of sampling points is 1.4×10^8 for the first ten iterations and 2.8×10^8 for the next twenty iterations. This confirms and improves the earlier result.[8] Meanwhile, the error in Ref. 7 must be multiplied by 7 in order to bring it into agreement with (5). Thus Ref. 7 seems to suffer from the same disease as (1).

This work is supported in part by the National Science Foundation. The author acknowledges the hospitality of the National Laboratory for High Energy Physics (KEK), Japan, where the results (4) and (5) were obtained with the HITAC S-810 computer. The rest of the work was carried out at the Cornell National Supercomputing Facility, a resource of the Center for Theory and Simulation in Science and Engineering, which is funded by the National Science Foundation and the IBM Corporation with additional support from New York State and members of the Corporate Research Institute.

Toichiro Kinoshita
Newman Laboratory
Cornell University
Ithaca, New York 14853

Received 23 November 1987
PACS numbers: 12.20.Ds, 14.60.Cd

[1]M. A. Samuel, Phys. Rev. Lett. **57**, 3133 (1986).

[2]R. S. Van Dyck, Jr., P. B. Schwinberg, and H. G. Dehmelt, Phys. Rev. Lett. **59**, 26 (1987).

[3]T. Engelmann and M. J. Levine, unpublished [as quoted in M. J. Levine, H. Y. Park, and R. Z. Roskies, Phys. Rev. D **25**, 2205 (1982)].

[4]J. Aldins, S. J. Brodsky, A. Dufner, and T. Kinoshita, Phys. Rev. D **1**, 2378 (1970); J. Calmet and A. Peterman, Phys. Lett. **47B**, 369 (1973); C. T. Chang and M. J. Levine, unpublished.

[5]G. P. Lepage, J. Comput. Phys. **27**, 192 (1978).

[6]P. Cvitanović and T. Kinoshita, Phys. Rev. D **10**, 4007 (1974).

[7]M. A. Samuel and C. Chlouber, Phys. Rev. Lett. **36**, 442 (1976).

[8]T. Kinoshita, B. Nizic, and Y. Okamoto, Phys. Rev. Lett. **52**, 717 (1984).

