# Quantized Hall Effect in the Presence of Backscattering

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The quantized Hall effect comprises edge states flowing in opposite directions on opposite edges of a two-dimensional electron gas. In a narrow wire, the presence of a potential barrier in the wire can cause the state to be reflected back into the oppositely directed state at the outer edge of the sample. Experiments on narrow ($\sim 1\ \mu$m) GaAs-$Al_xGa_{1-x}$As wires with short cross gates confirm that backscattering is caused by the barrier drawn up by the gate. The barrier, by reflecting some fraction of the states, causes quantized plateaus in any four-probe measurement of resistance.

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The quantized Hall effect, in which the Hall resistance is quantized *exactly* in units of $h/e^2$, occurs in any sufficiently mobile two-dimensional electron gas. It has been a subject of intense investigation for some years now, and it offers a practical metrological standard for resistance and for the fine-structure constant.[1] In the past year, the quantized Hall effect has been studied theoretically by the application of Landauer resistance formulas.[2-7] The underlying picture in this approach is that there are two or more reservoirs of particles which feed the two-dimensional electron gas, and that the carriers remain in the Landau levels while in the two-dimensional electron gas. The spatial location of the Landau levels and their direction of flow are illustrated by the thin solid lines in Fig. 1 and a four-probe sample.[6] The results from the theory[4-7] suggested that obstructions in a narrow two-dimensional electron gas could induce "tunneling" between the states on opposite sides of the channel. This tunneling between the Landau levels results in the inner pair of paths being broken and reflected back towards the directions whence they came as illustrated by the dotted lines in Fig. 1. If this "backscattering" occurs, then[6] both the longitudinal resistance and the Hall resistance increase by a rational fraction of $h/e^2$. The predictions by Jain and Kivelson are centered on the effect of localized impurities, which, because of the bound states which encircle them, can give rise to resonant spikes in the conductance. Such resonances have been observed in very narrow GaAs-$Al_xGa_{1-x}$As wires at low temperatures in the vicinity of the Hall plateaus.[8]

Büttiker has extended the theory to more realistic situations in which inelastic scattering appears explicitly, and in which the measurements are made with voltage probes as well as current probes.[6] In a phase-coherent wire with four probes (the average phase coherence length $L_\phi \gtrsim L$, where $L$ is the probe separation), the two resistances one can measure are $R_{12,43}$ and $R_{13,42}$ (and topologically equivalent variants). These resistances are defined as the ratio of voltage measured across a pair of the leads to the current through the sample: $R_{ij,kl} = V_{kl}/I_{ij}$, where $V_{kl}$ refers to the voltage drop measured from lead $k$ to lead $l$. $R_{12,43}$ and $R_{13,42} - R_{12,43}$ also bear a resemblance to $\rho_{xx}$ and $\rho_{xy}$, which are the longitudinal and transverse resistivities in large samples.[6] In the absence of backscattering, $R_{12,43}$ will be zero and $R_{13,42}$ will be quantized to $h/e^2 i$, where $i$ is the number of occupied Landau levels. If backscattering occurs, then one

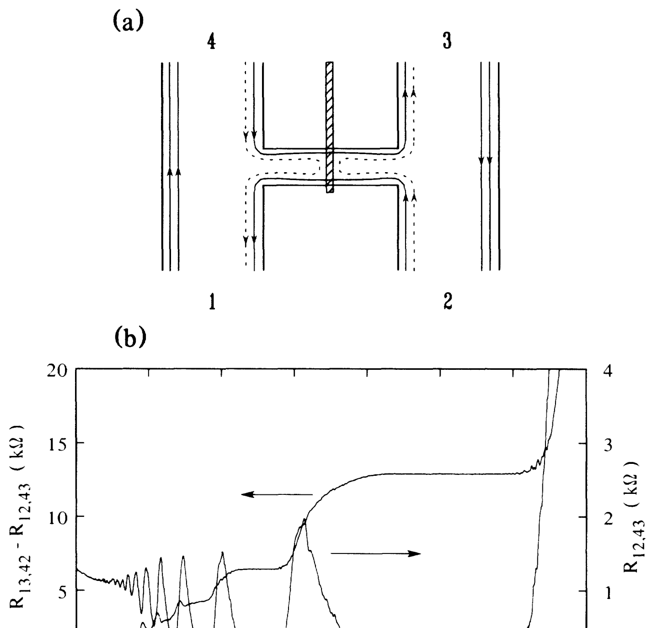


FIG. 1. (a) A schematic view (not to scale) of the four-probe GaAs-$Al_xGa_{1-x}$As wire with a narrow cross gate (hatched). The thin solid lines runing around the edges of the device represent the Landau-level paths and their directions. If the gate voltage is large enough, then the barrier raised by the gate will induce breakdown of the quantized Hall effect in which some the paths are reflected back (dotted lines). (b) $R_{12,43}$ and $R_{13,42} - R_{12,43}$ at $V_g$ as functions of magnetic field.

or more of the channels will be reflected back, and $R_{13,42}$ will increase[6] by some rational fraction of $h/e^2$. $R_{12,43}$ will increase at the same time to keep the difference $R_{13,42}-R_{12,43}$ constant. The theoretical predictions[3-6] rely on the assumption that transport occurs entirely through the edge states and not through the interior of the sample, and so, strictly speaking, the predictions are valid only in the plateaus. In situations where the Fermi level is not among the localized regions in the density of states, there are qualitative predictions that conductance fluctuations[9] and resonances[4] should occur.

We have manufactured short-gate GaAs-$Al_xGa_{1-x}As$ transistors ($x=0.3$) to study the effects of narrow barriers on one-dimensional transport.[10] Figure 1 contains a schematic plan view of the device. In this paper we will describe experiments at high magnetic fields which illuminate the backscattering physics discussed in the above theoretical work. The sample has a nominal (lithographically defined) width $w=2$ $\mu$m and length $L=10$ $\mu$m. The carrier density in the channel is $n_s=2.60\times10^{15}/\text{m}^2$ and a mobility $\mu\simeq10$ $\text{m}^2/\text{V}\cdot\text{sec}$; these parameters have been determined from Shubnikov–de Haas and zero-magnetic-field resistance measurements. A negative voltage applied to the gate serves to deplete the 0.1-$\mu$m-long region beneath the gate, and eventually to shut off any current flow once the barrier is higher than the Fermi level in the wire.[11] The purpose of the experiments is to study the controlled breakdown of the quantized Hall effect as the barrier is raised and the Landau levels are successively backscattered. The experiment consists simply of measuring $R_{12,43}$ and $R_{13,42}$ as functions of gate voltage at various magnetic fields. For all measurements reported herein, the sample temperature is 0.9 K.

In Fig. 1(b) we display $R_{12,43}$ and $R_{13,42}-R_{12,43}$ as functions of magnetic field $H$ at $V_g=0$. These combinations are the nearest correspondence to the classical resistivities $\rho_{xx}$ and $\rho_{xy}$ which are the parameters usually studied in large samples. The curves are the ratios of the outputs of phase-sensitive detectors which measured the voltage between some pairs of probes and the current through the sample. The sample is driven with a constant-amplitude current $I\lesssim1$ nA, which is chosen to prevent heating of the carriers, but experiments with a constant-voltage source yield the same results. $R_{12,43}$ approaches zero over wide regions of field ($4.9<H<5.5$ T and $8.5<H<10$ T), and in the same ranges $R_{13,42}$ is quantized[12] to $h/4e^2$ and $h/2e^2$, respectively. The spin splitting between the $i=3$ and $i=4$ plateaus is not resolved at all at this temperature, but the $i=1$ plateau has developed nearly to completion as deduced from measuring $R_{13,42}-R_{12,43}$ at $H=15$ T. The peaks in $R_{12,43}$ are not symmetrical about the half-filling of the Landau level. The asymmetry is common in narrow two-dimensional electron gases, and it has been characterized by Zheng *et al.*[13] The sample, in the absence of voltage applied to the gate, thus exhibits the canonical quantized Hall effect, and satisfies Büttiker's scheme for the case of no backscattering in a four-probe, phase-coherent sample. There is no direct measure of the phase-coherence length in these samples, but from the resistivity alone, we obtain $L_\phi\gtrsim l\gtrsim1$ $\mu$m, where $l$ is the Drude mean-free-path length. Conductance fluctuations[10] in these samples (at $H=0$ and at $H>0$) are of order $e^2/h$ in amplitude implying that $L_\phi\simeq L$.

Data from gate-voltage sweeps at $H=5.16$ T are displayed in Fig. 2(a). $R_{12,43}$ starts near 0 at $V_g=0$ and tends to increase as $V_g$ is lowered toward the threshold voltage $V_t=-0.45$ V. $R_{13,42}$ starts near $h/4e^2$ and increases, more or less tracking the increase in $R_{12,43}$. For gate voltage $-0.32<V_g<-0.25$, there are plateaus in both $R_{12,43}$ and $R_{13,42}$, which are quantized at $h/4e^2$ and $h/2e^2$, respectively. The presence of a plateau in $R_{12,43}$ and the plateau in $R_{13,42}$ when $R_{12,43}$ is not zero are new experimental observations, but all of these features have been anticipated in the theory—at least qualitatively.[5,6] More precisely,[5,6]

$$R_{12,43}=\frac{h}{e^2}\frac{j}{i(i-j)} \tag{1}$$

and

$$R_{13,42}=\frac{h}{e^2}\frac{1}{i-j}, \tag{2}$$

where $j$ is the number of backscattered channels, and $i$ is the total number of occupied channels. [For instance in Fig. 1(a), $i=2$ and $j=1$.] Since in our sample there are

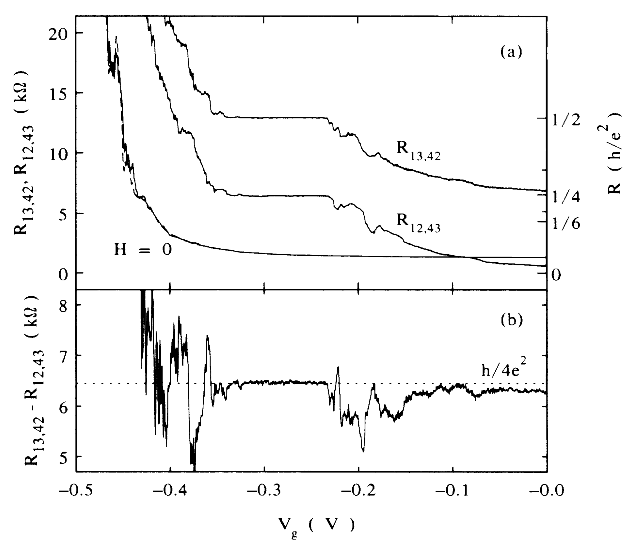


FIG. 2. (a) $R_{12,43}$ and $R_{13,42}$ (two upper solid curves) as functions of gate voltage at $H=5.16$ T. For reference, the behaviors of $R_{12,43}$ (lower solid curve) and $R_{13,42}$ (dashed curve) at $H=0$ are displayed as well. (b) The difference $R_{13,42}-R_{12,43}$ as a function of gate voltage at $H=5.16$ T.

no plateaus with $i>4$ and the spin splitting is not resolved ($j$ increases by 2 each time another channel is reflected), the rational fractions of $h/e^2$ allowed by the theory reduce to the integers seen in the experiment.

Plateaus similar to those that we find in $R_{12,43}$ have been observed in some circumstances from much larger samples (with $\simeq 100$ $\mu$m and gate length 1–20 $\mu$m).[7] There is no mention of higher plateaus in $R_{13,42}$ in those experiments. In some of the samples, the gate length is longer than reasonable estimates of the phase-coherence length, and in all of the samples the voltage probes are many $L_\phi$ from the gates region. At this time there is no clear indication of qualitative differences among the experiments on $R_{12,43}$. It appears that, because of the spatial separation of the Landau levels on opposite edges of the samples, phase loss (inelastic scattering) affects the quantized Hall effect very differently than it affects simple diffusive transport.

As the paths are either scattered back or not, any excess resistance in $R_{13,42}$—beyond the value at $V_g=0$—appears in $R_{12,43}$. It follows from this argument[6] that the difference between the two measurements should be $\simeq h/e^2 i$, where $i$ is the number of channels occupied before the barrier is raised. A test of this claim is given in Fig. 2(b) which plots the difference between the two measurements $R_{13,42}-R_{12,43}$. (For the opposite direction of the magnetic field, the sum $R_{13,42}+R_{12,43}$ would be the appropriate quantity since the Hall voltage would have changed sign.) Until the barrier becomes very high, $V_g > -0.42$, the difference is approximately equal to $h/4e^2$ with fluctuations of $\simeq 10\%$ around that value. In the range $-0.32 < V_g < -0.24$, the difference is quantized exactly[12] to $h/4e^2$. Over the entire range in $V_g$, however, the difference between $R_{13,42}$ and $R_{12,43}$ tends to the quantized value, with deviations caused by the potential variations between the two probes used to execute the voltage measurements. These variations imply substantial phase coherence in the wire, which coherence results in the resistance fluctuations. Such magnetoresistance fluctuations are familiar in transport through samples in which the carriers retain phase coherence over distances of the order of the probe separation.[9]

The general character described above appears in the data for all values of magnetic field $H \gtrsim 3.5$ T. There are regions where the time-averaged measurement of the resistance is approximately $h/e^2 i$ and one, or ocassionally, two regions where they are true plateaus in $R_{13,42}$, $R_{12,43}$, and (hence) the difference between them. This exact quantization occurs irrespective of whether the system was started (at $V_g=0$) in a plateau or not. At some point the barrier pinches off the extended states at (or within $\sim k_B T$ of) the Fermi level and forces the system to conduct only via the localized states: The resistances are quantized. That the barrier and the concomitant backscattering induce plateaus in the longitudinal resistances is somewhat of a surprise. Büttiker's analysis which predicts such quantization in the presence of backscattering is based on very general considerations, and has as its foundation only two ingredients: Landauer's formula for the resistance and the existence of the Landau-level channels. It is by no means obvious that this relatively simple zero-temperature model should describe the physics in real devices. That it works so well is an indication of the degree to which the quantized Hall effect is robust against nonideal samples and thermal fluctuations. At intermediate barrier heights, the carriers are able to use either the backscattered or the nonbackscattered channels, and the time-averaged measurement of the resistance obtains a value which is not quantized, but which tends to be near the quantized resistance.

For magnetic fields $\simeq 1 < H < 3$ T, there are steplike structures in the resistances as functions of $V_g$, but there are no quantized plateaus. We infer that, in order for the plateaus to appear, the field must be strong enough to support the quantized Hall effect in the absence of the barrier. The "steps" that do appear gradually become rounder and shallower as the field decreases. They are not well correlated with the fractions obtained from Eqs. (1) and (2). We speculate that at lower temperatures, where the quantized Hall plateaus develop at these fields, the steps might approach the predicted values. The difference $R_{13,42}-R_{12,43}$ fluctuates around the "classical" Hall resistance at all fields.

In Fig. 3 we plot $R_{13,42}$ as a function of gate voltage for several values of the magnetic field. The 10.0-T curve starts in the $i=2$ plateau, and then the quantization is destroyed when the sample "searches" for the

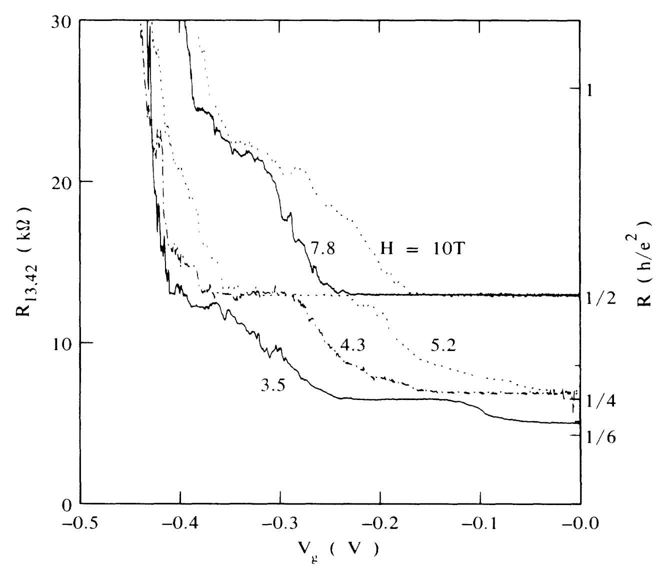


FIG. 3. $R_{13,42}$ vs $V_g$ at several values of magnetic field showing the shift of the plateau as the field changes and the paths move in the sample.

$i=1$ plateau, which has not completely developed. In sweeps at lower magnetic fields, $R_{13,42}$ inevitably finds a region of exact quantization at some intermediate gate voltage. The location (in $V_g$) of the plateau depends smoothly on the magnetic field—shifting to higher (more negative) voltage as the field is reduced. No matter the value of magnetic field, a particular quantized-Hall-effect plateau can be reached by increasing the barrier height until the necessary fraction of the Landau levels are being reflected.

To summarize, we remark that backscattering by the gate potential does not destroy the quantized Hall effect. Instead, there is always a region in which the various four-probe resistances are quantized to $h/e^2i$. For $R_{12,43}$ (or its equivalents among the possible lead permutations), $i$ is the number of channels which have been backscattered, and for $R_{13,42}$, $i$ is the number of channels which span the barrier.[6] The difference $R_{13,42}-R_{12,43}$ is quantized according to the number of occupied Landau levels in the absence of a barrier. As the particular measure resistance approaches $h/e^2i$, it exhibits at that value. When the resistances are not at plateaus, the difference between the two independent four-probe resistances fluctuates about the quantized value.

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