Quantized Multichannel Magnetotransport through a Barrier in Two Dimensions

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Magnetoresistance measurements in a $Al_xGa_{1-x}As/GaAs$ heterojunction with a gate covering a small region of the sample reveal quantized values for certain ranges of gate voltage; these are explainable in terms of a Landauer resistance formula. The quantization occurs when the voltage probes are located in a region of dissipationless current flow, where the quantum Hall effect provides a physical realization of ideal leads. The measurements distinguish dramatically between different multichannel generalizations of the Landauer formula.

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In studies of quantum transport through small devices at low temperatures the elastic-scattering approach due to Landauer[1] has been extremely useful. He argued that the resistance of electrons in a one-dimensional disordered medium is given by

$$R=\frac{h}{e^2}\frac{r}{t}, \tag{1}$$

where r and t are reflection and transmission coefficients for electrons incident from and transmitted to ideal leads on the left- and right-hand sides of the sample. Recently, it has been realized[2-4] that closely related arguments provide a useful picture of transport in two-dimensional (2D) systems in a strong magnetic field. A novel feature of the strong-field case is the spatial separation of left-going and right-going states which, in the case of ideal leads and the Fermi energy not equal to a Landau-level energy, are localized on opposite edges of the 2D sample. The experiments described below were motivated in part by the fact that the occurrence of the quantum Hall effect[5] allows, for the first time, a physical realization of Landauer's ideal leads.

Our samples are $Al_xGa_{1-x}As/GaAs$ heterojunctions with two undisturbed regions separated by a small region covered by a gate. (Figure 1 gives a schematic picture of the sample.) In the quantum Hall regime the Fermi energy corresponds to an integer number of filled Landau levels, and in the absence of a gate voltage a nondissipative current is carried by the edge states.[6-8] When a gate voltage is applied a barrier appears between source and drain which reduces the carrier concentration in the gate region and causes a potential difference to develop along either edge across the gate section. We see below that this resistance across the barrier exhibits a novel quantization, which is explainable in terms of a Landauer formula and which, in the case where several Landau levels are occupied in the ideal leads, distinguishes dramatically between different multichannel generalizations of the Landauer formula.[9-11] Our experimental results are consistent with those obtained earlier by von Klitzing and Ebert[12] and by Syphers and Stiles[13] who studied samples with two regions of different carrier density. The three-region geometry studied here is intented to address issues raised by recent developments in the study of quantum transport phenomena.

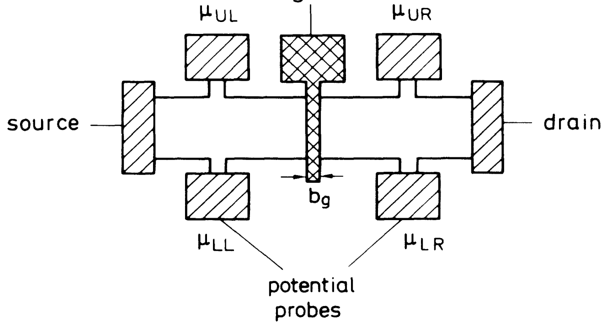

FIG. 1. Schematical view of the device.

The conventional $Al_xGa_{1-x}As/GaAs$ heterojunction used has a cap layer of 8-nm GaAs, a 40-nm-thick Si-doped $Al_xGa_{1-x}As$ layer, and a spacer of 21.3-nm thickness. The carrier concentration at a temperature of $T=4$ K has a value of $n_s=3.21\times10^{11}$ cm^{-2} and the mobility is 6.2×10^5 cm^2/V s. The samples have been etched in a Hall-bar-like structure with a channel width of 100 μm. The gate is covering the whole width for a distance b_g. The gates used ($b_g=10$ and 20 μm) are of macroscopic dimensions in comparison to the magnetic length l_c. For the measurements the samples were immersed in ^{3}He ($T\approx0.5$ K) and magnetic fields up to 13 T were applied perpendicular to the two-dimensional electron gas in order to achieve the ideal-land condition. The voltage across the potential probes was measured while a constant current was applied to the sample. In order to avoid heating of the electron gas in the narrow gate region an ac lock-in technique and a current of $I=10^{-8}$ A were used. As the gate voltage was varied, the magnetic field was kept fixed at values of B corresponding to integer filling factors $\nu=n_sh/eB$ in the undisturbed parts of the sample. Typical experimental curves obtained by varying the gate voltage are shown in Fig. 2.

For zero gate voltage the carrier concentration under the gate is almost the same as in the undisturbed parts and the measured resistance is minimal as a result of the integer filling factor in all parts of the sample. Application of a negative voltage U_g lowers the electron concentration and the filling factor ν_g in the gated region decreases. For high enough voltages ($U_g \approx -0.3$ V) the gate region can be totally depleted and the resistance grows rapidly. In the curve for a magnetic field of $B=3.01$ T, corresponding to the filling factor of $\nu=4$ in the undisturbed parts of the sample, three local maxima are observed. We ascribe these maxima to the filling factors ν_g approaching integer values $\nu_g=3$, 2, and 1 in the gated region since the observed periodicity on the gate-voltage scale is consistent with the change of the carrier concentration expected from the capacitance of the sample. For the filling factor $\nu_g \approx 2$ ($U_g \approx -0.18$ V) the local maximum is spread into a real plateau. It is flat to an accuracy of 10^{-3} in a gate-voltage range of $\Delta U_g \approx 50$ mV and the measured resistance value of $R=6.453$ kΩ is just equal to $h/4e^2$ with the same accuracy. This plateau occurs because of the splitting between the lowest Landau levels and it is thus more developed than the maxima at filling factors $\nu_g=3$ and 1, which reflect the spin splittings within the two lowest Landau levels. A similar dependence of the resistance on the gate voltage is obtained for a magnetic field value of $B=6.01$ T. On account of the low filling factor of $\nu=2$ the only integer value obtainable for ν_g by decreasing the carrier concentration in the gate region is $\nu_g=1$, which corresponds to the spin splitting of the lowest Landau level. Because of the higher magnetic field the spin splitting is better resolved and the resulting plateau has a value of $R=h/2e^2$ (with an accuracy of 10^{-3}). The minima at the left side of the plateaus in both curves are more pronounced for narrower gate structures. These minima have a striking similarity with the unexplained minima observed sometimes in Hall resistance measurements.[14] The total device resistance is just equal to the sum of the quantum Hall resistance of the ideal leads and the resistance of the gate region.

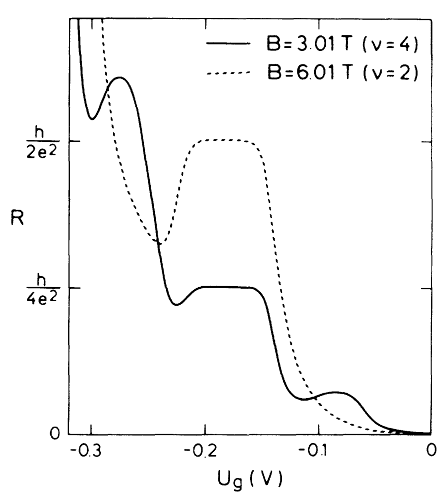

FIG. 2. The resistance measured across a gate region with $b_g=10$ μm vs the gate voltage U_g at a temperature of $T=0.55$ K for magnetic fields corresponding to the filling factors $\nu=4$ and $\nu=2$.

We interpret our experimental results using the multichannel, multiprobe Landauer resistance formula proposed by Büttiker,[10] which asserts that

$$I_i=\frac{e}{h}\left[(M_i-R_{i,i})\mu_i-\sum_{j\neq i}T_{i,j}\mu_j\right]. \tag{2}$$

In Eq. (2), μ_i is the chemical potential at contact i, M_i is the number of channels in contact i, I_i is the current from the reservoir at contact i, $T_{i,j}$ is the total probability, after summing over all channels, for carriers incident from contact j to be transmitted to contact i, and $R_{i,i}=M_i-\sum_{j\neq i}T_{i,j}$ is the total probability for reflection in the ith contact. We shall apply this equation for each of the six contacts (the current source and drain and the four voltage probes) labeled as in Fig. 1. We assume that the condition necessary for the quantum Hall effect holds outside the gate region, i.e., that the right-going states are localized along the upper edge, that left-going states are localized along the lower edge, and that the electrons incident from an edge state are certain to enter a contact.

Applying Eq. (2) at the source, at the drain, and at the contacts on the upper left and lower right it follows that

$$I=\nu_l\frac{e}{h}(\mu_{\rm UL}-\mu_{\rm LL})=\nu_r\frac{e}{h}(\mu_{\rm UR}-\mu_{\rm LR}), \tag{3}$$

where ν_l and ν_r are the integer filling factors specifying the number of channels in the areas to the left and to the right of the gate region. (In the experimental situation $\nu_l=\nu_r=\nu$.) Equation (3) expresses the quantized Hall conductance achieved in the ungated regions. To calculate the magnetoresistance we must consider the lower-left and upper-right contacts. Electrons transmitted to the upper-right contact can originate from the upper-left contact and be transmitted over the barrier, with total probability T, or originate from the lower-right contact and be reflected by the barrier with total probability R'. It follows that

$$\mu_{\rm UR}=\frac{T}{T+R'}\mu_{\rm UL}+\frac{R'}{T+R'}\mu_{\rm LR}. \tag{4}$$

Similarly, electrons transmitted to the lower-left contact can originate from the lower-right contact with total probability T' and from the upper-left contact with total probability R so that

$$\mu_{\rm LL}=\frac{T'}{T'+R}\mu_{\rm LR}+\frac{R}{T'+R}\mu_{\rm UL}. \tag{5}$$

Note that $T+R=\nu_l$ and $T'+R'=\nu_r$ by conservation of probability and that $T+R'=\nu_r$ and $T'+R=\nu_l$ by conservation of probability in a reversed field $[T_{i,j}(H)=T_{j,i}(-H)]$, so that we must have $T=T'$ and all potential differences can be expressed in terms of ν_r, ν_l, and a single total transmission probability. In particular the magnetoresistances measured along the upper and lower edges are given by

$$R_{\rm U}=\frac{\mu_{\rm UL}-\mu_{\rm UR}}{eI}=\frac{h}{e^2}(T^{-1}-\nu_r^{-1}) \tag{6}$$

and

$$R_{\rm L}=\frac{\mu_{\rm LL}-\mu_{\rm LR}}{eI}=\frac{h}{e^2}(T^{-1}-\nu_l^{-1}), \tag{7}$$

and the tunneling probability can be directly measured.

In our experiments the negative gate voltage, U_g, depletes the gated part of the sample and lowers the filling factor of this region, ν_g. In other words, a potential barrier is formed in the middle of the sample which, in the simplest approximation, enlarges the potential energy by a constant amount, V_g, which is a monotonic function of U_g. It is intuitively clear that for high-mobility samples with narrow Landau levels and $\nu_l=\nu_r=\nu$ equal to an integer, ν_g will equal an integer over wide ranges of V_g, close to Landau-level separation energies. In this case the total transmission probability must equal $T=N_t=\nu_g$ and electrons in the $N_r=\nu-\nu_g$ channels corresponding to Landau levels below the barrier will be totally reflected as illustrated schematically in Fig. 3. We emphasize that for high-mobility samples with narrow well-separated Landau levels, this conclusion is independent of complexities associated with a realistic treatment[15] of the potential distribution at the edges of the gate region. This is the mechanism responsible for the magnetoresistance plateaus versus gate voltage in Fig. 2, and the quantized magnetoresistance values are predicted by Eqs. (6) and (7) to equal

$$R=\frac{h}{e^2}(\nu_g^{-1}-\nu^{-1}). \tag{8}$$

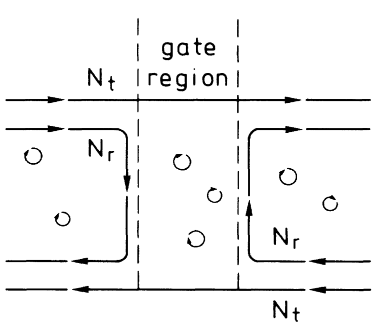

FIG. 3. The trajectory network model for integer values of both filling factors, ν and ν_g.

Similar equations have recently been independently derived by van Houten *et al.*[16] and by Büttiker.[4] For $\nu=4$, $\nu_g=2$ and $\nu=2$, $\nu_g=1$ the measured plateau values agree with Eq. (8) to 1 part in 10^3. For $\nu=4$ and $\nu_g=3$ and 1, the maxima in R are, respectively, 82% and 89% of the plateau values predicted by Eq. (8) for a fully developed spin splitting.

An interesting feature of our results is the surprising decrease in R with increasing gate voltage as ν_g is decreased below an integer value. This decrease is responsible for the minima on the left-hand side of the plateaus in Fig. 2. As explained in more detail elsewhere[17] these minima are due to the occurrence of bulk states in the gate region which allows the originally fully reflected channels to be partly transmitted after diffusing through the bulk region of the gate. In agreement with experiment, this mechanism produces a stronger effect as the width of the gate b_g becomes small compared to its length, i.e., to the width of the Hall bar. Eventually, the decrease in transmission for the channels which were originally fully transmitted begins to dominate and the magnetoresistance rises again.

It should be realized that many different multichannel generalizations have been proposed for the Landauer formula[11] based on different assumptions about the nonequilibrium distributions of carriers in the leads and different models for the voltage measurement process. In the present context it has been unclear whether or not in averaging chemical potentials to determine $\mu_{\rm UR}$ and $\mu_{\rm LL}$ [Eqs. (4) and (5)] the sources of the electrons incident upon these contacts should be weighted according to their densities of states (inverse channel velocities). When there are few channels and large velocity differences between channels, the quantitative difference between these options can be large. The discussion here is based on Büttiker's resistance formula[10] in which these weighting factors do not appear, while in the original discussion of the strong-field case,[2] which is based on the multichannel resistance formula of Ref. 9, they do appear. If the latter formula were used, the magnetoresistance on the plateau for $\nu=4$ and $\nu=2$ would equal

$$R=\frac{h}{4e^2}\left[\frac{2}{1+v_0/v_1}\right], \tag{9}$$

where v_1 and v_0 are the velocities in the ungated region of the sample for the edge-state channels corresponding to the $n=0$ and $n=1$ orbital Landau levels.[18] For parabolic confinement and a Fermi level midway between the bulk $n=1$ and $n=2$ Landau levels $v_0/v_1=\sqrt{3}$. It is easy to verify that more realistic confining potentials give similar values for v_0/v_1 so that Eq. (9) is in stark disagreement with the precise quantization found experimentally. Our results therefore favor, at least for the strong-field case, the resistance formula used here, which is based on a model where the chemical potential in the voltage probe is chosen so that the equilibrium current ejected from the contact cancels the nonequilibrium current transmitted to the contact.

In closing we remark that the novel device described here provides a new way of studying quantum magnetotransport properties. For the first time, because of the occurrence of quantum Hall effect away from the gate region, the energy dependence of transmission through a barrier can be measured directly. This, combined with the spatial separation of left-going and right-going states in a strong magnetic field, also makes possible unique tests of the theoretical approach to resistance formulated by Landauer.

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