Excitation of Surface Waves by an Electromagnetic Wave Packet

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It is demonstrated that, independent of its polarization, an electromagnetic wave packet propagating along a plasma-vacuum boundary excites surface waves associated with the boundary. The corresponding rate of loss is evaluated and it is shown that the rate is a maximum when the wavelength of surface wave is comparable with the width of the packet in the direction of propagation of the surface wave.

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It is well established, both theoretically (see Refs. 1-9) and experimentally (see, e.g., Ref. 10), that when a fast electron enters and passes through a plasma (metal) slab it experiences characteristic energy losses. Besides standard bremsstrahlung,[11] these losses have been attributed to Cherenkov radiation[2,5] (see also Ref. 8 for relativistic electrons) in the gas of conduction electrons, interband transitions,[2] and transition radiation[1,3,4,6,9] (see also Ref. 8 for relativistic electrons). Ritchie[2] suggested that part of these losses can also be due to the excitation of surface modes. Since the rate of loss corresponding to the volume effects is, obviously, proportional to the thickness of the slab, the surface effects become dominant for thin films. The idea that losses arise via the excitation of surface modes was further developed in Refs. 5 and 6. With the exception of the ultrarelativistic limit, fast electrons can only excite electrostatic surface modes with a frequency $\omega \cong \omega_p/\sqrt{2}$, where ω_p is the electron plasma frequency characterizing the plasma or the gas of conduction electrons. Therefore, the surface emission (see, e.g., Ref. 12) associated with these so-called low-lying ($\omega < \omega_p$) energy losses has a specific frequency making the fast electron-induced excitation of surface waves a useful diagnostic tool (see, e.g., Refs. 10 and 12).

Askar'yan[13,14] demonstrated that there is a direct analogy between fast electrons and an electromagnetic wave packet[13] or a modulated beam[14] in terms of Cherenkov and transition radiation. The presence of a wave packet in an electron gas gives rise to an average (with respect to the carrier frequency) potential force which polarizes the medium. This local polarization effect moves with the group velocity v_g of the wave packet and results in Cherenkov radiation of waves with phase velocity $v_p < v_g$. The effect was experimentally observed[15] when femtosecond laser pulses were launched into a nonlinear electro-optically active medium. The same polarization effect gives rise to transition radiation when a wave packet passes through a boundary between two media.

The same mechanism is responsible for the excitation of plasma waves by an electromagnetic wave packet propagating in a plasma.[16] Since, in this case, the phase velocity of the plasma waves v_p is simply $v_p = v_g$, this mechanism allows for the excitation of very fast plasma waves suitable for electron acceleration.[17]

In the present paper, we investigate the excitation of surface waves by an electromagnetic wave packet. It is well known that only the p component of a monochromatic plane wave can excite a surface wave propagating along a boundary between two media characterized by different values of the dielectric permittivity ϵ. Moreover, since such an electromagnetic wave cannot directly couple to a surface wave, one has to use various coupling techniques[18] (e.g., frustrated total reflection, corrugated surface, etc.) allowing for energy exchange between the incident and surface waves. We demonstrate that, independent of its polarization, an electromagnetic wave packet propagating along a plasma-vacuum boundary excites surface waves associated with the boundary. The coupling is direct, i.e., it does not require any refractive index matching technique[18] be applied. As in the case when plasma waves are excited,[16] the energy conversion is a maximum when the wavelength of the surface wave is of the same order of magnitude as the width of the packet in the direction of propagation of the wave. Since the group velocity of the packet can be very close to the speed of light, c, the wave-packet-induced surface waves can have a dominant electromagnetic component, i.e., their frequency ω can be $\omega \ll \omega_p$. Consequently, in contrast to the case of the excitation of plasma waves[16] where $k_y = \omega_p/v_g$, here k_y can be $k_y \ll \omega_p/v_g$ and, thus, the optimum conversion occurs for relatively long packets.

Let us assume that an electromagnetic wave packet propagates in the y direction along a boundary $x=0$ between a vacuum and a rare homogeneous plasma such that the packet carrier frequency $\omega_0 \gg \omega_p$. The presence of the packet induces a polarization[13] of the plasma $\mathbf{P}_0 = -(e/2m\omega_0^2)\beta\nabla\langle E_0^2\rangle$ where e, m are the electron charge and mass, β is the plasma polarizability, $E_0(x,y,t)$ is the electric field of the packet, and the angular brackets denote time averaging over the fast carrier period. The Fourier transforms (ω, k_y) of the response of the plasma to this externally imposed polarization are then governed by the following equations:

$$\frac{d^2E_y}{dx^2}+\frac{\omega^2}{c^2}(\epsilon-n^2)E_y=-\frac{\omega^2}{c^2}4\gamma\pi P_{0y}(x;\omega,k_y)\,, \tag{1}$$

$$E_x=\frac{ik_y}{\omega^2(n^2-\epsilon)/c^2}\frac{dE_y}{dx}+\frac{4\pi}{n^2-\epsilon}P_{0x}(x;\omega,k_y)\,, \tag{2}$$

where $n=ck_y/\omega$, $\gamma=(1-n^2/\epsilon-inc\kappa_0/\epsilon\omega)$, and $\epsilon=1-\omega_p^2/\omega^2$. For the sake of simplicity, but without losing any physics, we assume that only a "tail" of the packet propagates within the plasma $(x>0)$ and we assume that $P_0(x)\propto\exp(-\kappa_0x)$ for $x\ge 0$. Consequently, we assume that the energy losses of the packet are relatively small and, therefore, we neglect the effect of these losses on the packet itself. In plasma $(x>0)$, a general solution of Eq. (1), such that it vanishes for large x, is then

$$E_y=\frac{\omega^2}{c^2}\frac{4\gamma\pi P_{0y}(x;\omega,k_y)}{\kappa^2-\kappa_0^2}e^{-\kappa_0x}+Ce^{-\kappa x}\,, \tag{3}$$

$$E_x=\frac{-ik_y}{\kappa^2}\left[\kappa_0\frac{\omega^2}{c^2}\frac{4\gamma\pi P_{0y}(x;\omega,k_y)}{\kappa^2-\kappa_0^2}e^{-\kappa_0x}+\kappa Ce^{-\kappa x}\right]+\frac{\omega^2}{c^2}\frac{4\pi}{\kappa^2}P_{0x}(x;\omega,k_y)e^{-\kappa_0x}\,, \tag{4}$$

where $\kappa^2=\omega^2(n^2-\epsilon)/c^2$, while in vacuum $(x<0)$ $E_y=B\exp(\kappa_1x)$, $E_x=(ik_y/\kappa_1)B\exp(\kappa_1x)$, where $\kappa_1^2=\omega^2(n^2-1)/c^2$ and C, B are constants yet to be determined. Requiring continuity of E_y and ϵE_x at the boundary $x=0$, one obtains

$$C=-\frac{\omega^2}{c^2}\frac{4\pi}{\kappa k_yD}\left[k_y\gamma(\kappa-\kappa_0+\kappa_0D)\frac{P_{0y}(x;\omega,k_y)}{\kappa^2-\kappa_0^2}+i\alpha P_{0x}(x;\omega,k_y)\right]\,, \tag{5}$$

where $\alpha=\kappa_1\epsilon/\kappa$ and $D=1+\alpha$. The equation $D=0$ represents a dispersion relation for surface waves.[18] It can be satisfied only if $n>1$. Since $\langle E_0^2\rangle(y,t)=\langle E_0^2\rangle(y-v_gt)$, its Fourier transform is then $\langle E_0^2\rangle(\omega,k_y)\propto\delta(\omega-k_yv_g)$. Consequently, $n=c/v_g>1$, as required for a surface-wave-type solution. The general solutions (3) and (4) thus contain a contribution (all terms $\propto D^{-1}$) due to the excitation of the surface waves.

As usual, the rate of losses (per unit length in the z direction) of the packet is

$$-\frac{dW}{dt}=-v_g\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\rho E_y(x,y,t)\,dx\,dy\,, \tag{6}$$

where the induced charge density $\rho=-\nabla\cdot\mathbf{P}_0$ and we consider only those contributions to E_y which are $\propto D^{-1}$. There are two singularities [see (5)] at $D=0$ and $\kappa=\kappa_0$; however, only the first one represents a contribution to the losses due to the excitation of surface waves. Evaluation of the integral (6) is sensitive to the specific shape of the packet and, therefore, we will restrict ourselves to, for example, a Gaussian wave packet, i.e.,

$$\langle E_0^2\rangle(y,t)=A\exp\left[-\left(\frac{y-v_gt}{2L}\right)^2\right].$$

Now, one can calculate ρ, P_{0x}, P_{0y}, E_y, and their Fourier transforms in explicit forms and, after lengthy but straightforward calculations, one obtains the following expression for the rate of losses of a wave packet due to excitation of surface waves:

$$-\frac{dW}{dt}=Ar_{E_0}^2\beta^2\frac{\pi}{4}\left\{\frac{\kappa_0[2(1+\kappa_0^2L^2)-k_y^2L^2]}{\kappa+\kappa_0}\left(\frac{dD}{d\omega}\right)^{-1}\left[\frac{4\pi\omega^2\gamma'k_y}{\kappa\kappa_0(\kappa+\kappa_0)c^2}-1\right]e^{-2k_y^2L^2}\right\}_{k_y=\omega/v_g,D=0}, \tag{7}$$

where $r_{E_0}=eE_0/m\omega_0$ and $\gamma'=1-n^2/\epsilon$. The condition $D=0$ specifies the frequency $\omega=\omega_p[(1-n^2)/(1-2n^2)]^{1/2}$ of the excited surface waves. If, formally, $n=c/v_g\gg 1$, the frequency ω becomes $0<\omega_p/\sqrt{2}-\omega\ll\omega_p/\sqrt{2}$ and (as in the case of nonultrarelativistic electrons) only surface waves with dominant electrostatic component are excited. In contrast, for $n-1\ll 1$ (which corresponds to our case when $\omega_0\gg\omega_p$), one has $\omega\ll\omega_p$ and the excited surface wave has a dominant electromagnetic component.

One can formally introduce a surface-wave-induced lifetime τ_{eff} of a wave packet by setting $|dW/dt|=AL/8\pi\kappa_0\tau_{\text{eff}}$. Since the effect considered here is fundamentally nonlinear then τ_{eff} is intensity dependent and, in particular, $\tau_{\text{eff}}\propto 1/A$, i.e., wave packets with higher intensity are dissipated at a higher rate. Note, however, that our assumptions restrict the validity of (7) to $t\ll\max(\tau_{\text{eff}},T_L)$. Here $T_L\cong\omega_0^3L^2/c^2\omega_p^2$ is the characteristic time for the packet spread caused by linear dispersion. In Fig. 1 we present a plot of $-dW/dt$ (normalized to $Ar_{E_0}^2\beta^2\omega_0$) versus the width of the packet k_yL for $\epsilon(\omega_0)=0.99$ and $\kappa_0=0.5$. As can be seen, the rate of losses is a maximum when the wavelength λ of the sur-

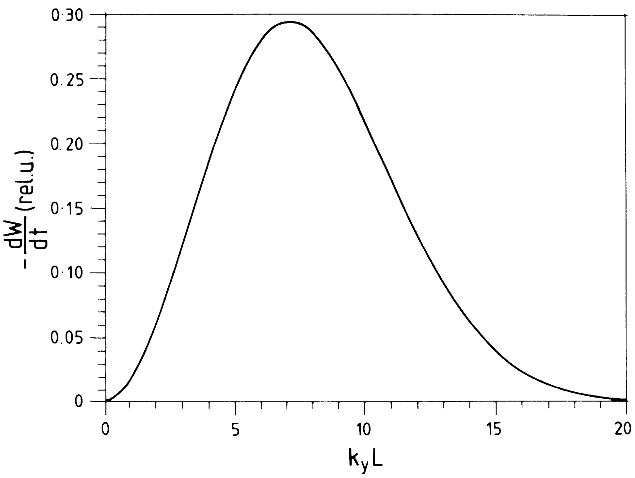

FIG. 1. Normalized rate of losses vs the packet width for k_yL for $\epsilon(\omega_0)=0.99$ and $\kappa_0=0.5$.

face wave becomes $\lambda \cong L$. For large L the effect of the wave-packet-induced polarization $P_0 \propto 1/L$ is diminished while the probability for a localized charge perturbation to generate a wave with a nonzero electrostatic component and with $\lambda > L$ is small. For the optimum duration of the packet τ_0 the characteristic time T_L becomes

$$T_L/\tau_0 \cong n^{-1}(1-\epsilon_0)^{3/2}[(1-2n^2)/(1-n^2)]^{1/2}$$

($\gg 1$ for our choice of the parameters). It is worth noting that the electron energy in the field of surface waves (normalized to the high-frequency potential $mv_{E_0}^2/2$, where $v_{E_0}=eE_0/m\omega_0$)

$$\left[\frac{16\pi^{7/2}\omega^2|\gamma| e^{-k_y^2L^2}}{c^2\kappa(\kappa+\kappa_0)} \frac{k_yL^2}{v_g(dD/d\omega)}\right]_{k_y=\omega/v_g, D=0}$$

also has a maximum (as a function of L) which is localized close to the peak of the rate of losses.

To conclude, we have demonstrated that, independent of its polarization, an electromagnetic wave packet propagating along a plasma-vacuum boundary with the group velocity $v_g < c$ excites surface waves with frequency $\omega=\omega_p[(1-n^2)/(1-2n^2)]^{1/2}$ and wave number $k_y = n\omega/c$ where $n=c/v_g$. In contrast to the linear case, this type of coupling is direct. In the limit $n-1 \ll 1$, the excited surface wave has a dominant electromagnetic component. Since in this case $k_y \ll \omega_p/v_g$ the optimum for excitation of surface waves by a wave packet occurs for a relatively wide packet, much longer than those corresponding to the optimum for excitation of plasma waves[16] at the same plasma density. Finally, the problem can, obviously, be extended in many ways, e.g., by the consideration of other angles of incidence ($<90°$), by the assumption that $\omega_0 < \omega_p$, by the consideration of other shapes of a wave packet, or by the replacement of a wave packet by a train of pulses produced, for example, by the beating of two electromagnetic beams with similar frequency; however, besides some extra mathematical difficulties, none of these modifications introduces any new physics into the problem.

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