Gravitomagnetic Interaction and Laser Ranging to Earth Satellites

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The contribution of the "gravitomagnetic" interaction to a well-measured perturbation term in the orbits of Earth satellites is calculated. Laser-ranging determination of the Moon's orbit and Lageos-satellite orbit requires the precise participation of the gravitomagnetic interaction; otherwise anomalous orbital perturbations exist. For the Lageos-satellite orbit, the gravitomagnetic interaction's contribution to the orbit is a 100-m altitude variation of frequency $\omega-\Omega$ (ω and Ω are satellite and Earth orbital angular frequencies, respectively), but which is nullified by effects from other well established post-Newtonian gravitational potentials.

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"Gravitomagnetism" is a post-Newtonian gravitational interaction between mass currents analogous to the magnetic interaction between electric-charge currents. Gravitomagnetism is present in general-relativity theory as well as most viable alternative theories of gravity, and although gravitomagnetism is ubiquitous in its contributions to relativisitc, post-Newtonian gravitational effects,[1] there is still a frequently expressed view that this interaction has not yet been measured or even "seen."[2] The purpose of this Letter is to calculate a particular perturbation of the orbits of Earth satellites (including the Moon) for which inclusion of the gravitomagnetic interaction is essential in order to fit the laser-ranging observations. The analysis below is an adaptation of part of a general examination of possible post-Newtonian perturbations of Earth-satellite orbits which was performed some time ago.[3]

Working within the framework of metric gravity, the linearized metric gravitational field is sufficient for calculation of the particular effects of interest here; the linear-order components of metric field being the diagonal ones,

$$g_{00}=1-2u\,, \tag{1a}$$

$$g_{ij}=-(1+2\gamma u)\delta_{ij} \tag{1b}$$

and the off-diagonal gravitomagnetic potentials,

$$g_{0i}\equiv\mathbf{h}=\Delta\frac{Gm\mathbf{w}}{c^3r}+\Delta'\nabla\left(\frac{Gm\mathbf{w}\cdot\mathbf{r}}{c^3r}\right), \tag{1c}$$

with

$$u=\frac{G}{c^2}\left(\frac{M}{|\mathbf{R}-\mathbf{r}|}+\frac{m}{r}\right).$$

i,j range over the three spatial dimensions and 0 indicates the time-dimension index; G is Newton's constant and c is the speed of light. The metric is given in the isotropic spatial coordinate gauge. M and m are the Sun's and Earth's masses, respectively, while $\mathbf{R}$ and $\mathbf{r}$ are coordinate positions of the Sun and satellite from Earth. γ is the well known parametrized post-Newtonian coefficient measured in photon-deflection and time-of-flight experiments as well as many other post-Newtonian gravitational effects;

$$\gamma_{\text{expt}}\simeq 1\pm 10^{-4}.$$

Δ and Δ' are coefficients parameterizing the gravitomagnetic potentials. $\mathbf{w}(t)$ is the velocity of the Earth in orbit around the solar-system barycentric inertial frame in which we perform the calculation ($d\mathbf{R}/dt=\mathbf{w}$).

The Lagrangian for Earth-orbiting test bodies is then

$$L=-\left(g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}\right)^{1/2}=-c^2[1-2u+2\mathbf{h}\cdot(\mathbf{w}+\mathbf{v})/c-(1+2\gamma u)(\mathbf{w}+\mathbf{v})^2/c^2]^{1/2}, \tag{2}$$

with $d\mathbf{r}/dt=\mathbf{v}$. Collecting the specific perturbations in the equation of motion which results from Eq. (2) and which are proportional to $Gm\mathbf{r}/r^3$ and linear in both $\mathbf{w}$ and $\mathbf{v}$, one has

$$\delta\mathbf{a}=\frac{Gm}{c^2r^3}(\mathbf{w}\cdot\mathbf{v}\,\mathbf{r}+\mathbf{r}\cdot\mathbf{w}\,\mathbf{v})-(2\gamma+1)\frac{Gm\mathbf{w}\cdot\mathbf{v}\,\mathbf{r}}{c^2r^3}+c\left(\frac{d\mathbf{h}}{dt}-\nabla(\mathbf{h}\cdot\mathbf{v})\right), \tag{3}$$

with, for circular orbits ($\mathbf{v}\cdot\mathbf{r}=0$),

$$\frac{d\mathbf{h}}{dt}=\Delta'\frac{Gm}{c^3r^3}\mathbf{w}\cdot\mathbf{v}\,\mathbf{r}\,, \tag{4a}$$

and

$$\nabla(\mathbf{h}\cdot\mathbf{v})=(\Delta'-\Delta)\frac{Gm\mathbf{w}\cdot\mathbf{v}\mathbf{r}}{c^3r^3}. \tag{4b}$$

The first perturbation term in Eq. (3) results from special-relativistic particle mechanics, the second term proportional to $2\gamma+1$ results from the well established linear gravitational potentials g_{00} and g_{ij}, while the net contribution of gravitomagnetism to Eq. (3) is the magneticlike term

$$c\mathbf{v}\times(\nabla\times\mathbf{h}).$$

Altogether the perturbation of interest is

$$\delta\mathbf{a}=(\Delta-2\gamma)\frac{Gm\mathbf{w}\cdot\mathbf{v}\mathbf{r}}{c^2r^3}+\frac{Gm\mathbf{r}\cdot\mathbf{w}\mathbf{v}}{c^2r^3}. \tag{5}$$

The application of Eq. (5) to a circular orbit $[r(t)=r_0+x(t)]$ yields approximately (I neglect perigee precession in this solution)

$$x(t)\cong\left(\frac{1+\cos i}{2}\right)\left(\Delta-2\gamma-2\frac{\omega}{\omega-\Omega}\right)\frac{Gm}{r^2}\frac{wv}{c^2}\frac{\cos(\omega-\Omega)t}{\omega^2-(\omega-\Omega)^2}, \tag{6}$$

in which i is the inclination angle of the satellite orbit relative to Earth orbit, ω is the satellite's orbital angular frequency ($\omega=v/r$), and Ω is the Earth's orbital angular frequency ($\Omega=w/R$).

This orbital perturbation has the same frequency as terms which have been accurately measured for the Moon's orbit and satellite orbits by use of laser ranging,[4,5] since an identical frequency perturbation occurs if the Earth's gravitational to inertial mass ratio differs from 1.[6] For the Moon, the amplitude of such a range-to-Earth oscillation has been measured to better than 10-cm accuracy; evaluation of Eq. (6) for the lunar case gives [I approximate Eq. (6) by neglecting $\Omega/\omega\sim\frac{1}{13}$ which contributes to other effects[3]]

$$x(t)_{\text{E-M}}\simeq 80(\Delta-2\gamma-2)\cos(\omega-\Omega)t\ \text{cm}. \tag{7a}$$

From the lunar laser-ranging observations alone, then, we have

$$\Delta-2\gamma-2\simeq 0\pm 0.1.$$

For a low-Earth-orbit satellite the perturbation is larger, e.g., for the Lageos satellite ($r_0=12\,300$ km):

$$x(t)_{\text{Lageos}}\cong\left(\frac{1+\cos i}{2}\right)2700(\Delta-2\gamma-2)\cos(\omega-\Omega)t\ \text{cm}. \tag{7b}$$

A similar 10-cm accuracy fitting of that satellite's orbit by laser ranging then gives a very accurate measurement of the presence of gravitomagnetism:

$$\Delta-2\gamma-2\cong 0\pm 0.004.$$

An angular-position perturbation of the satellite orbits accompanies these radial perturbations. Δ also parametrizes the spin-spin interaction between Earth and a gyroscope which would be measured in orbiting gyroscope experiments, such a free-falling gyroscope being expected to precess at the rate

$$\boldsymbol{\Omega}=\frac{\Delta}{4}\frac{G}{c^2}\nabla\times\left(\frac{\mathbf{r}\times\mathbf{J}}{r^3}\right), \tag{8}$$

in which $\mathbf{J}$ is the spin angular momentum of the Earth. The Δ' potential plays no role in any of the above effects. Elsewhere the wide spread role of the gravitomagnetic interaction in producing a variety of other post-Newtonian gravitational effects which have been measured in the solar system and in the binary system PSR 1913+16 has been discussed.[1]

This calculation could be performed in other inertial frames in which the solar-system barycenter moves at constant velocity $\mathbf{u}$. Additional unseen $\mathbf{u}$-dependent perturbations would then also result in the orbits of Earth satellites unless the gravitomagnetic interaction existed in just the amount established here. This makes even stronger the necessity for the gravitomagnetic interaction.

A few comments are relevant to the above analysis. Certainly the measurement of gravitomagnetism described here depends on the metric gravitational field hypothesis and the independent measurement of the gravitational potentials g_{00} and g_{ij} through other effects. Indeed, most gravitational effects are the results of contributions from more than one of the various gravitational potentials. Post-Newtonian gravity must be considered as a whole, in which a complete theoretical framework which permits calculation of all gravitational effects from the point of view of all inertial observers is tested by the whole collection of interlocking empirical constraints.

It should be noted that most tests of post-Newtonian gravity are null tests in which the observable perturbation is found to have zero amplitude. This is the case with the present effect given by Eqs. 7(a) and 7(b); there is observed no anomalous post-Newtonian orbital polarization of Earth-satellite orbits proportional to $\cos(\omega-\Omega)t$.[4,5] I feel that many workers have underestimated the degree to which post-Newtonian gravity has been mapped out, by neglecting to include the many null observations in the set of empirical constraints on the gravitational interaction.

If the various nongravitomagnetic post-Newtonian terms in the metric-based gravitational equation of motion are accepted to exist as a consequence of various observations, then the gravitomagnetic interaction is empirically established by such effects as calculated in this Letter. Gravitomagnetism has been seen and has been well measured! If direct observation of gravitomagnetism by measurement of the precession rate of an orbiting gyroscope or of an orbit, itself, in the environment of the spinning Earth does not confirm the precise form and strength of the gravitomagnetic interaction as already measured, then a fundamental breakdown of our very basic framework would result, forcing a radical modification of theory whose form is not obvious today.

The conclusions of this Letter rest on the assumption of metric-field-based gravity. Elsewhere we will publish a framework for interpreting post-Newtonian gravitational phenomena which does not presume the metric-field hypothesis, but nevertheless permits arrival at the same conclusions reached in this Letter

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[1]K. Nordtvedt, Int. J. Theor. Phys. (to be published).

[2]"At present there is no experimental evidence arguing for or against the existence of the gravitomagnetic effects predicted by general relativity. This fundamental part of the theory remains untested." In *Physics Through the 1990's* (National Academy Press, Washington, DC, 1986).

[3]K. Nordtvedt, Phys. Rev. D **7**, 2347 (1973).

[4]J. Williams *et al.*, Phys. Rev. Lett. **36**, 551 (1976).

[5]I. I. Shapiro *et al.*, Phys. Rev. Lett. **36**, 555 (1976).

[6]K. Nordtvedt, Phys. Rev. **170**, 1186 (1968).