Quasiparticle Statistics in Time-Reversal Invariant States

Kalmeyer and Laughlin[1] (KL) have shown that a frustrated spin-$\frac{1}{2}$ Heisenberg model (e.g., the triangular antiferromagnet) is equivalent to a Bose lattice gas with short-range repulsive interactions in an external magnetic field. KL suggest that the frustrated spin system and the continuum Bose plasma may be related by adiabatic continuation. This would imply that the ground state of the spin system is closely related to the lattice version of the Laughlin wave function,[2] which describes the fractional quantum Hall effect. They note that the properties of this state are very similar to those of the short-ranged resonating-valence-bond (RVB) state originally proposed by Anderson[3] and further characterized by Kivelson, Rokhsar, and Sethna[4] (KRS). In particular, KL identify the fractionally charged quasiparticles of the plasma with spin-$\frac{1}{2}$ quasiparticles of the spin system, the "spinons"[4,5] of RVB theory. Their assignment of fractional statistics to the quasiparticles, however, contradicts KRS's conclusion that spinons are fermions.

Despite their similarities, the spin and plasma problems are fundamentally different[6]; the spin system is time-reversal invariant while the plasma in a magnetic field is not. As long as time-reversal invariance is not spontaneously broken, the eigenstates of the spin model can always be chosen to be real, while the eigenstates of the plasma are intrinsically complex. KL report that the Laughlin wave function seems to be nearly real when evaluated on a many-site lattice, a remarkable discovery which supports their claim that it is a good approximate ground state. The reality of the exact eigenstates, however, has important consequences for the quasiparticle statistics. We prove below that in a time-reversal–invariant system the Berry phase[7] must be an integer multiple of π, and we argue that if the excitations of such a system have a quasiparticle interpretation, they are naturally described by either Bose or Fermi statistics.

Consider a many-body system whose Hamiltonian depends on a set of parameters $\{\zeta_i\}$. For any closed loop Γ in parameter space on which the ground state of the system is nondegenerate, Berry's phase can be computed[7] by $\oint_\Gamma \Psi_\lambda^* (d\Psi_\lambda/d\lambda)d\lambda$, where λ parametrizes the loop (running from 0 to 1) and Ψ_λ is any ground-state wave function that is continuous on Γ. If the system is time-reversal invariant, then Ψ_λ can be written as the product of a complex phase factor $e^{i\chi(\lambda)}$ and a normalized, purely real wave function Φ_λ that are continuous functions of λ from 0 to 1. (They may, however, be discontinuous on Γ between $\lambda=1^-$ and $\lambda=0^+$.) Berry's phase is then simply $\chi(1)-\chi(0)$. This difference can only be 0 or π, since $\Psi_1=\Psi_0$ (by continuity) and the real wave functions Φ_1 and Φ_0 can differ by at most a sign (they are real ground states of the same Hamiltonian). Thus in a nondegenerate, time-reversal invariant system, Berry's phase is always an integer multiple of π.

To relate Berry's phase to quasiparticle statistics, imagine that each ζ_i specifies the position of a potential which localizes a single quasiparticle.[8] If a quasiparticle interpretation is valid, Berry's phase for any closed circuit is the sum of three terms: (a) a statistical phase[9] for each exchange (e.g., 0 for bosons and π for fermions), (b) a phase due to gauge forces between quasiparticles, and (c) Aharonov-Bohm phases accompanying motion in an external gauge field. In two dimensions,[10] phases (a) and (b) are both proportional to the number of particle exchanges (a topological invariant). The statistics of two-dimensional particles are therefore not fundamental,[9] since the statistical phase can be continuously varied at the expense of introducing flux tubes bound to the particles. The "natural" statistics are those that eliminate the gauge forces between particles, i.e., set phase (b) equal to zero. The previous paragraph then establishes that in a time-reversal–invariant system, Bose and Fermi statistics are the only natural statistics. It follows that "anyon" quasiparticles as predicted by Kalmeyer and Laughlin can appear only if the two-quasiparticle ground state breaks time-reversal invariance.

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