Nuclear Stopping at Intermediate Beam Energies

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(Received 6 July 1988)

We perform an analysis of exclusive heavy-ion reactions around 100 MeV/nucleon in terms of multipole moments of the momentum distribution of nucleons. Numerical calculations based on the Boltzmann-Uehling-Uhlenbeck theory are presented and compared to an approximate scaling function. We find that nuclei completely stop each other even at 100 MeV/nucleon, if $A_P+A_T>200$ in symmetrical central collisions. We propose that the mass dependence of the quadrupole momentum tensor can be used to determine the nuclear compressibility.

PACS numbers: 25.70.Np, 21.65.+f

At temperatures of ≈ 20 to 30 MeV/nucleon we expect nuclear matter to undergo a second-order phase transition at the critical point.[1-6] This is essentially due to the fact that the nuclear equation of state has a short-range repulsive and a longer-range attractive component similar to a van der Waals equation of state.

We hope to achieve these kinds of excitation energies by bombarding of heavy ions with multi-GeV protons,[3] where the spectator matter is heated up, and by colliding heavy ions at beam energies around 100 MeV/nucleon, where the participants can in principle reach the necessary excitation energy. In this paper we want to concentrate on the latter.

Most theories predicting the existence of the fragmentation phase transition assume completely equilibrated infinite nuclear matter. In a previous paper,[7] we have addressed the influence of the finiteness of the fragmenting system on observables which can indicate the presence of a phase transition. In this paper we present a study on equilibration in nuclear collisions. We will attempt to answer the following questions: Can nuclei completely stop each other at beam energies around 100 MeV/nucleon? Which heavy-ion systems should we choose for experiments looking for signals of the phase transition? What are the most sensitive observables to nuclear equilibration?

In the following we will concentrate on a multipole analysis of the final-state momentum distribution of all nucleons, bound in fragments and unbound. Let as assume that the momenta $\mathbf{p}_i$ of all nucleons after the nucleus-nucleus collision are known (either measured experimentally or calculated theoretically), and that we can approximate the final-state momentum distribution by

$$\rho(\mathbf{p})=\sum_{i=1}^{A_T+A_P}\delta(\mathbf{p}-\mathbf{p}_i)\,. \tag{1}$$

The monopole moment of this distribution is then simply

$$q=\int d\mathbf{p}\,\rho(\mathbf{p})=A_T+A_P\,, \tag{2}$$

and the dipole moment is given by

$$\mathbf{D}=\int d\mathbf{p}\,\mathbf{p}\,\rho(\mathbf{p})=\sum_{i=1}^{A_T+A_P}\mathbf{p}_i=0\,, \tag{3}$$

where we have assumed that all momenta are measured in the center-of-mass system. These two lowest moments deliver only trivial results, but can be used in experimental measurements to correct the momentum distributions for undetected fragments.

The lowest-order multipole moment of interest is the quadrupole momentum tensor, which was previously used to study thermalization times in nuclear collisions.[8] Its matrix elements are given by

$$\begin{aligned}Q_{\alpha\beta}&=\int d\mathbf{p}(3p_\alpha p_\beta-p^2\delta_{\alpha\beta})\rho(\mathbf{p})\\&=\sum_{i=1}^{A_T+A_P}[3(\mathbf{p}_i)_\alpha(\mathbf{p}_i)_\beta-p_i^2\delta_{\alpha\beta}]\,.\end{aligned} \tag{4}$$

We would like to focus on the diagonal component of $Q_{\alpha\beta}$ in beam direction (which is assumed to be along the z axis). First of all, we note that for a completely equilibrated system $Q_{zz}=0$, because in this case $\langle p_x^2\rangle=\langle p_y^2\rangle=\langle p_z^2\rangle$. Second, we can calculate the value of Q_{zz} for a system of two noninteracting Fermi gases. For symmetric systems ($A_T=A_P=A$) the momentum distribution is then given by

$$\rho_{\mathrm{FG}}(\mathbf{p})=\theta(p_F-|\mathbf{p}-p_b\hat{\mathbf{e}}_z|)A+\theta(p_F-|\mathbf{p}+p_b\hat{\mathbf{e}}_z|)A\,, \tag{5}$$

where p_F is the Fermi momentum, p_b is the beam momentum in the center-of-mass frame, and $\hat{\mathbf{e}}_z$ is a unit vector. Inserting this into Eq. (4), we obtain

$$Q_{zz}^{\mathrm{FG}}=4Ap_b^2\,. \tag{6}$$

To obtain the final-state momentum distribution in our calculations we use the nuclear transport theory based on the Boltzmann-Uehling-Uhlenbeck (BUU) equation[9] with an equation of state yielding a compressibility

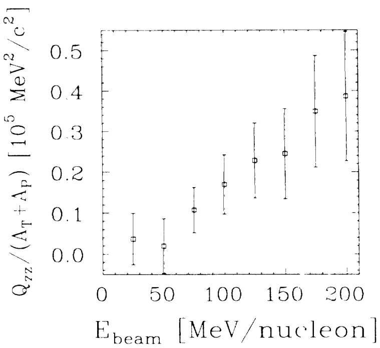

FIG. 1. $Q_{zz}/(A_T+A_P)$ as a function of beam energy for a central ($b=0$) collision of two heavy ions with mass $A_T=A_P=20$. The dotted line represents the zero value. For every data point 100 different events were calculated. The plot symbols stand for the average values, and the width of the distributions are indicated by the error bars.

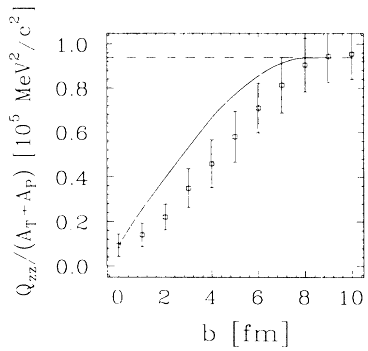

FIG. 2. Impact-parameter dependence of $Q_{zz}/(A_T+A_P)$ for a beam energy of 100 MeV/nucleon and $A_T=A_P=40$. The dotted line indicates the zero value, and the dashed line marks the value of $Q_{zz}/(A_T+A_P)$ for two noninteracting Fermi gases at a beam energy of 100 MeV/nucleon. The solid line is the result from the analytic scaling function, Eq. (8).

$\kappa=240$ MeV of nuclear matter. The use of the BUU transport theory has proven to be very useful to predict energy spectra of emitted nucleons, and so it can be expected that final-state momenta in heavy-ion reactions can be obtained from it with reasonable accuracy. Details of the numerical calculations can be found in Ref. 10. In terms of the Wigner function $f(\mathbf{r},\mathbf{p},t)$, which is calculated with use of the BUU equation, $\mathcal{Q}_{zz}$ is given as

$$\mathcal{Q}_{zz}=\lim_{t\to\infty}\int\frac{d\mathbf{x}\,d\mathbf{p}}{(2\pi)^3}(2p_z^2-p_x^2-p_y^2)f(\mathbf{r},\mathbf{p},t)\,. \qquad (7)$$

In Fig. 1, we present the results of our calculations for $Q_{zz}/(A_T+A_P)$ as a function of the beam energy. We chose a symmetric system with $A_T=A_P=20$ and central collisions. We calculated 100 events for each data point. The plot symbols represent the average value extracted from these calculations, and the error bars indicate the widths of the distributions. It can be seen from Fig. 1 that the value of $Q_{zz}/(A_T+A_P)$ is compatible with 0 up to a beam energy of about 50 MeV/nucleon. After that, $Q_{zz}/(A_T+A_P)$ rises roughly linearly with beam energy.

By looking at the ratio of transverse to longitudinal momenta, Kruse *et al.*[11] have reached quantitatively similar conclusions in a Vlasov-Uehling-Uhlenbeck study of the system Ar+Ca.

Figure 2 contains the results of our calculations as a function of impact parameter b. Here we use $A_T=A_P=40$. We fixed the beam energy at 100 MeV/ nucleon. As expected, $Q_{zz}/(A_T+A_P)$ rises with impact parameter from the central value up to the value of $2(p_b^{\text{c.m.}})^2$ which is the value for the noninteracting system, and which is represented by the dashed line in Fig. 2. From Fig. 2 we conclude that it is possible to distinguish between different impact parameters, if one is able to measure the complete momentum distribution of all fragments in an event by event technique using 4π detectors. We believe that $Q_{zz}/(A_T+A_P)$ can be complimentary centrality trigger to the presently used total charged particle multiplicity trigger.

The most important result of our calculation is summed up in Fig. 3. Here we fix the beam energy at 100 MeV/nucleon and calculate $Q_{zz}/(A_T+A_P)$ for central collisions as a function of A_T+A_P. We can see a decrease of $Q_{zz}/(A_T+A_P)$ as a function of the mass of the system. For a combined mass of roughly 200, the value of $Q_{zz}/(A_T+A_P)$ becomes compatible with 0, indicating a thermalized system. This means that we can obtain completely thermalized heavy-ion systems even at 100 MeV/nucleon, if we only choose target and projectile masses high enough. These kinds of systems should then be used to study the fragmentation phase transition in 4π experiments.

Aichelin and Stöcker mention similar findings[12] by studying the number of colliding nucleons in 85-MeV/nucleon ^{12}C collisions with various targets.

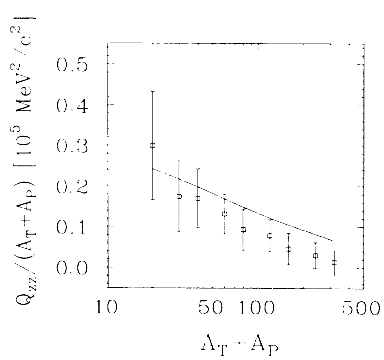

FIG. 3. $Q_{zz}/(A_T+A_P)$ as a function of the target and projectile mass for central collisions of symmetric ($A_T=A_P$) heavy-ion systems at a beam energy of 100 MeV/nucleon. The dotted line marks the zero value. The solid line is the result from the analytic scaling function, Eq. (8).

In order to understand the scaling behavior of $Q_{zz}/(A_T+A_P)$ we may attempt to formulate approximate scaling laws. To do this, we assume that all nucleons in the overlap region experience stopping due to nucleon-nucleon collisions and the mean field of the other nucleus. The other nucleons are assumed to be unstopped. Furthermore, we assume that the stopping due to the mean-field effects and nucleon-nucleon collisions may be separated into individual reduction factors:

$$Q_{zz}=Q_{zz}^{\mathrm{FG}}[(1-\alpha)+\alpha R_{\mathrm{MF}}R_{NN}]\,, \tag{8}$$

where α is the fraction of nucleons inside the overlap region for a given impact parameter.

To obtain the reduction factor R_{MF} for the mean field, we assume that the energy to compress the nuclear medium is taken out of the beam energy. The expansion is then assumed to proceed without a preferred direction. Since $Q_{zz}/(A_T+A_P)\propto E_{\mathrm{beam}}$ for the case of noninteracting nuclei, the reduction factor due to the mean field is then given by

$$R_{\mathrm{MF}}=\frac{E_{\mathrm{beam}}-E_{\mathrm{comp}}}{E_{\mathrm{beam}}}\,. \tag{9}$$

For a given equation of state, we can calculate the energy E_{comp} to compress nuclear matter from normal nuclear-matter density to some higher density. It can be assumed that in these intermediate-energy heavy-ion collisions nuclei are compressed to about twice nuclear-matter density. Using a functional form[10]

$$U(\rho)=A(\rho/\rho_0)+B(\rho/\rho_0)^\sigma \tag{10}$$

for the mean-field potential and $\sigma=\frac{7}{8}$, $\frac{4}{3}$, and 2 for nuclear compressibilities of 200, 240, and 380 MeV, respectively, we obtain $R_{\mathrm{MF}}(\rho_0\rightarrow 1.9\rho_0)=0.69$, 0.63, and 0.31, respectively.

A more realistic estimate for R_{MF} can be obtained from solving the collisionless Vlasov equation for the systems in question. Doing this for the three equations of state above, we obtain values of $R_{\mathrm{MF}}=0.60$, 0.55, and 0.48 for the three nuclear compressibilities of 200, 240, and 380 MeV, respectively. Therefore we obtain differences up to 20% in the values of $Q_{zz}/(A_T+A_P)$, and a careful measurement of $Q_{zz}/(A_T+A_P)$ might give us additional insight into the nuclear equation of state.

To calculate R_{NN} we assume that in two-body collisions the final-state momenta of the two scattered nucleons are isotropically distributed on the surface of a sphere, the radius of which is given by the relative momentum of the initial state. This approximation is valid for beam energies which are not too high. In this case, only the nucleons which have not experienced two-body collisions still carry the original nonzero value of Q_{zz}. Therefore R_{NN} is in this approximation just equal to the fraction of nucleons which have not experienced two-body collisions during the course of the nucleus-nucleus reaction. With use of Poisson statistics this fraction is given by

$$R_{NN}=\exp(-\bar{N})\,, \tag{11}$$

where $\bar{N}$ is the average number of nucleon-nucleon collisions per nucleon[13]

$$\bar{N}=\lambda_P\sigma_{NN}\int_{\mathcal{O}}dx\,dy\int_{-\infty}^{\infty}dz_1\,dz_2\,\rho_T(x^2+y^2+z_1^2)^{1/2}\rho_P(x^2+y^2+z_2^2)^{1/2}\,, \tag{12}$$

where $\sigma_{NN}=40$ mb is used, and the integration over x and y is performed over the geometrical overlap area $\mathcal{O}$. λ_P is a correction factor resulting from the fact that the final-state phase space for the scattering nucleons is partially Pauli forbidden. With use of geometrical considerations, it can be approximated by

$$\lambda_P=\left[1-\frac{2p_F^3-\frac{1}{2}h^2(3p_f-h)}{(p_F+p_b)^3}\right]^2, \tag{13}$$

with $h=(p_f-p_b)\theta(p_F-p_b)$.

Even though we expect the above scaling behavior to be only an approximation, we can still test it by comparing it to the numerical calculations based on the solution of the BUU equation. The results obtained from Eq. (8) are displayed in Figs. 2 and 3 and are represented by the solid lines. As can be seen, the qualitative features of the numerical calculations are reproduced. Even though the right magnitude of $Q_{zz}/(A_T+A_P)$ as well as the rise with increasing b and the falloff with increasing A_T+A_P is obtained in the approximate analytic calculations, some quantitative differences remain. However, it can be stated that the gross features of $Q_{zz}/(A_T+A_P)$ and therefore nuclear stopping are understood in intermediate heavy-ion reactions.

It is important to point out that $Q_{zz}/(A_T+A_P)$ as a function of mass appears to be a tool to determine the nuclear compressibility κ from heavy-ion collisions. Attempts to determine this quantity via a flow analysis were inconclusive, because the interplay between nucleon-nucleon collisions and compression of the mean field could not be properly separated. If the scaling behavior of Eq. (8) holds, then such a separation will be possible, because R_{MF} is independent of the mass of projectile and target, whereas R_{NN} is not. The use of different nuclear compressibilities in numerical calculations will therefore result in an overall shift along the vertical axis in Fig. 3, whereas different numbers of nucleon-nucleon collisions will change in the steepness of the curve. Of course, Eq. (8) is an oversimplification, and careful numerical calculations with different assumptions about the nuclear equation of state are necessary.

With the use of 4π detectors the predictions made above can be tested. Since our calculations show that complete stopping can be achieved in central symmetrical collisions of heavy systems, we expect that the fragmentation phase transition can be studied in heavy-ion collisions at intermediate beam energies. In addition, a comparison of the predictions made above to the data should yield a better understanding of the nuclear equation of state.

The author thanks Professor S. E. Koonin for a critical reading of the manuscript. This work was supported by NSF Grants No. PHY85-05682 and No. PHY86-04197.

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