Vortex Behavior in High-T_c Superconductors with Disorder

In a recent Letter,[1] Nelson studied the effect of thermally excited fluctuations on the vortices or flux lines (FL's) in high-T_c superconductors. As a result, he found that the FL density, n, vanishes as $n \sim (H - H_{c1})^{\bar{\beta}}$, with $\bar{\beta} = 1$ as the applied magnetic field H approaches the lower critical field H_{c1}. He also argued that FL fluctuations can lead to the appearance of two liquidlike states which can be distinguished by the degree of FL entanglement.

In this Comment, we consider the pinning and roughening of the FL's as a result of quenched impurities or defects. Strong pinning of the FL's by disorder is necessary in order to reach high critical currents and, thus, desirable from a technological point of view.[2] Here, we show that the disorder increases the distance between the lines, and that the FL density $n \sim (H - H_{c1})^{\bar{\beta}}$ vanishes with changed exponent $\bar{\beta} \simeq \frac{3}{2}$. In addition, we argue that the array of FL's may still exhibit long-range translational order even in the presence of impurities or defects.

Consider an array of N FL's of length L with coordinates $(z, \mathbf{r}_i(z))$ where $i = 1, 2, \ldots, N$, and with an energy as given by Eq. (1) in Ref. 1. The presence of disorder leads to a random potential of the form $\sum_{i=1}^{N} \int_0^L dz\, V(z, \mathbf{r}_i(z))$. For simplicity, we take the correlations of this potential to be short-ranged with

$$\langle V(z,\mathbf{r}) V(z',\mathbf{r}') \rangle = \Delta \delta(z - z') \delta_a^{(2)}(\mathbf{r} - \mathbf{r}'),$$

where $\delta_a(x)$ is a smeared-out δ function of width a. As mentioned by Nelson, an isolated FL of length L will then make transverse excursions of order $\xi_\perp \approx A_\perp L^{\zeta}$, with $\zeta > \frac{1}{2}$.[3] If $\delta_a(x)$ is taken to scale as $(1/x)(x/a)^p$, a Flory-type argument leads to $\zeta = 3/(6 - 2p)$ and $A_\perp = (\Delta/\Sigma^2 a^{2p})^{\zeta/3}$ for $T \lesssim T^* \simeq (\Sigma\Delta)^{1/3}$, where Σ is the line tension of a single FL.[4] The value $\zeta \simeq 0.6$ as found numerically[5] corresponds to $p \simeq \frac{1}{2}$.

Assuming that pinning arises from normal-conducting precipitates of diameter D and concentration C, we find that $\Delta \simeq \Sigma^2 a^6 C$, where $a = \max(D, 4(\ln\kappa)^{1/2}\xi)$, ξ denotes the correlation length of the superconductor, and κ the Landau-Ginzburg parameter. Using $C \simeq 10^{12}$ cm^{-3}, $D \simeq 5 \times 10^{-6}$ cm, and $L = 1$ cm, we obtain $\xi_\perp \simeq 10^{-3}$ cm which is larger than the thermally excited roughness even at $T \simeq 80$ K.

Within the FL array or lattice, the largest humps of a single FL are characterized by $\xi_\perp \simeq 1/\sqrt{n}$ and by a longitudinal correlation length, $\xi_\parallel \simeq (1/A_\perp \sqrt{n})^{1/\zeta}$. Then, the excess free energy per unit volume arising from the disorder-induced fluctuations scales as $n\Sigma(\xi_\perp/\xi_\parallel)^2 \sim \Sigma A_\perp^{2/\zeta} n^{1/\zeta}$, and the Gibbs free energy per unit volume, $g(n)$, has the asymptotic form

$$g(n) \approx (\Sigma - H/H_{c1})n + c_0 \Sigma A_\perp^{2/\zeta} n^{1/\zeta}, \tag{1}$$

for small n. Minimizing this free energy yields $n \sim (H - H_{c1})^{\bar{\beta}}$ with $\bar{\beta} = \zeta/(1 - \zeta)$. For thermal fluctuations, $\zeta = \frac{1}{2}$ and $\bar{\beta} = 1$ (Ref. 1); for fluctuations induced by disorder with short-ranged correlations, $\zeta \simeq 0.6$ and $\bar{\beta} \simeq \frac{3}{2}$.

On large scales, the state of the FL's can be described by a displacement field $\mathbf{u}(\mathbf{R}) = \mathbf{u}(z, \mathbf{r}) \simeq \mathbf{r}_i(z) - \mathbf{r}_i^0$, where $\mathbf{r}_i^0$ corresponds to a reference lattice with lattice parameter $1/\sqrt{n}$. Long-range translational order (LRTO) leads to

$$\Delta C_u(R) \equiv \langle [\mathbf{u}(\mathbf{R}) - \mathbf{u}(\mathbf{O})]^2 \rangle \approx \text{const}$$

for large R, while $\Delta C_u(R) \sim R^{2\zeta_c}$ with $\zeta_c > 0$ implies the destruction of this order. In previous theories, the random potential $V(z, \mathbf{r}_i(z))$ has been replaced by $\mathbf{h}_i(\mathbf{r})[\mathbf{r}_i(z) - \mathbf{r}_i^0] \simeq \mathbf{h}(\mathbf{R})\mathbf{u}(\mathbf{R})$, where $\mathbf{h}(\mathbf{R})$ is an uncorrelated random field.[6] This leads to $\zeta_c = (4 - d)/2$ which would imply the destruction of LRTO by pinning in $d \leq 4$ dimensions. However, such a replacement of V clearly overestimates the disorder and, in addition, yields the wrong result for roughness, $\xi_\perp$, of a single FL. The correct behavior of $\xi_\perp$ is recovered from a random field with anisotropic correlations, $\langle \hat{\mathbf{h}}_i(\mathbf{q}) \hat{\mathbf{h}}_j(-\mathbf{q}) \rangle \sim \delta_{ij} q^{3-2\zeta}$. Such a field leads to $\Delta C_u(R) \sim R^{2\zeta_c}$ with $\zeta_c = \zeta - (d-1)/2$. For $d = 3$ and $\zeta \simeq 0.6$, ζ_c is negative indicating the persistence of LRTO even in the presence of *weak* disorder. On the other hand, *strong* disorder will melt the FL lattice and can lead to the entanglement of the FL's. Thus, all three FL states considered by Nelson[1] could well survive even for superconductors with quenched impurities or defects.

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[2] See, e.g., E. H. Brandt and U. Essmann, Phys. Status Solidi (b) **144**, 13 (1987).

[3] In Ref. 1, $\xi_\perp$, $\xi_\parallel$, and Σ are denoted by Λ_L, l, and ϵ_1, respectively.

[4] For a FL in d dimensions, $\zeta = 3/[3 + d - p(d-1)]$, $A_\perp = [\Delta/\Sigma^2 a^{(d-1)p}]^{\zeta/3}$, and $T^* = (\Sigma\Delta a^{3-d})^{1/3}$.

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