

Two-Dimensional Nonlocal Electron Transport in Laser-Produced Plasmas

E. M. Epperlein

*Laboratory for Laser Energetics, University of Rochester, 250 East River Road,
Rochester, New York 14623-1299*

G. J. Rickard and A. R. Bell

*Blackett Laboratory, Imperial College of Science and Technology, Prince Consort Road,
London SW7 2BZ, United Kingdom*

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A two-dimensional Fokker-Planck code SPARK has been constructed to study the process of nonlocal thermal smoothing in planar laser-produced plasmas with no ion motion. The results from SPARK are compared with those from a standard fluid code with flux-limited Spitzer-Härm conductivity. It is found that even when the one-dimensional transport is well modeled by fluid theory the thermal smoothing arising from nonuniform laser energy deposition may be severely overestimated for scale-length modulations shorter than about 80 electron mean free paths.

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A great deal of effort has been devoted to the study of nonlocal electron transport in laser-produced plasmas.^{1,2} Most of the work has involved the numerical solution of the electron Fokker-Planck (FP) equation in one dimension by finite difference techniques. Comparisons with standard fluid transport calculations with use of Spitzer-Härm (SH)³ heat flow $\mathbf{q}_s = -\kappa \nabla T$ has revealed the occurrence of hot-electron "preheat," as well as reduced penetration of the bulk heat front, a phenomenon known as "flux inhibition."¹ In order to simulate the latter effect fluid codes are normally equipped with an artificial flux limiter which maintains the heat flux \mathbf{q} below some fraction f of its free-streaming limit $q_f = nm(T/m)^{3/2}$, i.e., $\mathbf{q} = \mathbf{q}_s / (1 + |q_s / q_f|)$.^{4,5}

This technique has been shown to be adequate for long pulse (~ 1 ns), short-wavelength ($< 1 \mu\text{m}$) lasers at moderate irradiances ($< 10^{15}$ W/cm²), where usually $q_s \lesssim f q_f$ (for $f = 0.1-0.2$).⁶ However, if the transport is two dimensional (2D), as would be the case for nonuniformly illuminated laser plasmas, the validity of fluid theory has not yet been tested. In particular, the appli-

cation of flux limiters requires special care, since the direction of the heat flow may not always be parallel to $-\nabla T$.^{5,7} Nevertheless, 2D fluid codes are widely used for thermal transport studies, with each direction individually flux limited.⁸

In this Letter we investigate the validity of the fluid modeling by constructing a code (SPARK) designed to numerically solve the electron FP equation in 2D planar geometry under conditions relevant to laser-produced plasmas. By imposing a spatial inhomogeneity of scale l in the incident laser beam, we show that the nonlocal electron transport is less effective at smoothing temperature gradients than the corresponding diffusive transport based on SH heat flow, irrespective of the size of the flux limiter. This effect becomes significant when $l \lesssim 80 \lambda_{\text{mfip}}$, where $\lambda_{\text{mfip}} [= 3T^2/4(2\pi)^{1/2} Z n e^4 \ln \Lambda]$ is the electron-ion mean free path at the critical surface.

The numerical algorithm of SPARK is briefly described as follows (for further details see Ref. 7). Using the diffusive approximation and taking the high- Z limit of the FP equation, we obtain (using the notation of Shkarofsky, Johnston, and Bachynsky)⁹

$$\frac{\partial f_0}{\partial t} + \frac{v}{3} \nabla \cdot \mathbf{f}_1 = \frac{1}{v^2} \frac{\partial}{\partial v} \left[\frac{v^2}{3} \mathbf{a} \cdot \mathbf{f}_1 + Y \left(C f_0 + D \frac{\partial}{\partial v} f_0 \right) + \frac{Y n Z}{6v} v_0^2 \frac{\partial}{\partial v} f_0 \right], \quad (1a)$$

$$\mathbf{f}_1 = -\tau (v \nabla f_0 - \mathbf{a} \partial f_0 / \partial v), \quad (1b)$$

where $\mathbf{a} = |e| \mathbf{E} / m$, $\tau = v^3 / [(Z+1)nY]$, $Y = 4\pi(e^2/m)^2 \times \ln \Lambda$, $C = I_0^0 f_0$, and $D = \frac{1}{3} v (I_2^0 + J_{-1}^0)$. To simplify the analysis we have assumed the ions to be cold and motionless. The last term in (1a) is the inverse-bremsstrahlung operator and v_0 is the electron quiver velocity.¹⁰ This reduced form of the FP equation has been successfully used for 1D transport studies,² where one is not too interested in highly collisionless phenomena (such as Landau damping). Here we have further neglected the effect of magnetic fields which may be justified by the fact that we will be comparing our results with fluid calculations based on the same approximation. Furthermore, we will simulate the transport under conditions where the effect of magnetic fields on the transport is not expected to be significant, i.e., short laser wavelength at moderate irradiances.⁵

Substituting (1b) into (1a) and defining $\alpha = -[\partial_v \times \ln(f_0)]/v$, $\beta = -\nabla \ln(f_0)$, and $\chi = v^2 \tau/3$, we have

$$\frac{\partial f_0}{\partial t} = \nabla \cdot [\chi(\nabla f_0 + \mathbf{a} a f_0)] + \frac{1}{v^2} \frac{\partial}{\partial v} \left\{ \chi \left[\mathbf{a} \cdot \mathbf{a} \frac{\partial}{\partial v} f_0 + v \beta \cdot \mathbf{a} f_0 \right] + Y \left[C f_0 + \left(D + \frac{n Z v_0^2}{6v} \right) \frac{\partial}{\partial v} f_0 \right] \right\}. \tag{2}$$

Assuming that the nonlinear coefficients α and β are known explicitly, as is usually done with the terms \mathbf{a} , C , and D , we have transformed the troublesome cross derivatives into convective terms. The resulting diffusion-convection equation may now be solved by standard numerical techniques.

Equation (2) is differenced in conservative form by adopting Cartesian geometry in x , z , and v . A generalized Chang-Cooper¹¹ weighting is applied in all directions by use of the zero flux condition across the cell boundaries. The electric field is calculated via the "implicit moment"¹² method by assuming that $\nabla \times \mathbf{E} = 0$ and taking the $\int_0^\infty v^2 dv$ moment of (2). (An alternative scheme that assumes that the total current is zero and allows for a finite $\nabla \times \mathbf{E}$ has also been tried with little effect on the transport calculations, apart from a deterioration in the quasineutrality.¹³) The resulting system of equations is solved by an "alternating-direction-implicit"^{7,14} scheme in v , z , and x , respectively, in conjunction with a "predictor-corrector" step. In the predictor stage f_0 is linearly extrapolated to the half time level and used to calculate the nonlinear coefficients. In the corrector stage f_0 is averaged in time to recalculate the coefficients.

To illustrate the process of 2D transport we consider a planar fully ionized plasma of $Z=4$ with an initial temperature of 250 eV. The ion background is kept fixed throughout the simulation with an exponential ramp of

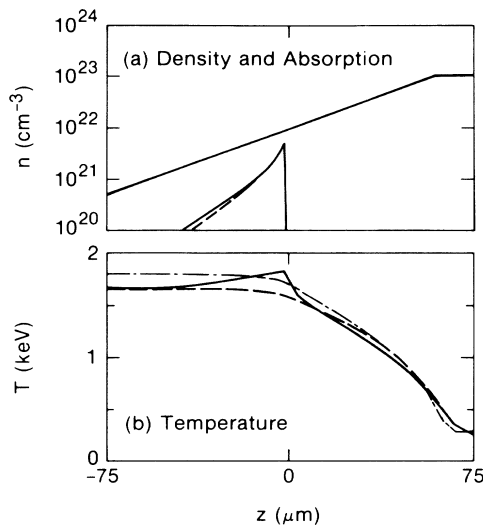


FIG. 1. (a) Density and energy deposition (in relative units), (b) temperature profiles for 0.35- μm irradiation at $5 \times 10^{14} \text{ W/cm}^2$ after 120 ps. Solid curves refer to FP results; dashed and dash-dotted curves refer to SH results with $f = \infty$ and $f = 0.2$, respectively.

25 μm , as shown in Fig. 1. A 0.35- μm laser is propagated along the positive z direction and is absorbed via a 1D ray-trace package, as used in the hydrocode LILAC,¹⁵ with a full reflection at the critical surface. The intensity of the beam is modulated in the x direction by $I = I_0 [1 + \epsilon \cos(2\pi x/\lambda)]$ where $I_0 = 5 \times 10^{14} \text{ W/cm}^2$ at all times.

Figure 1 shows the density, temperature, and absorption profiles of a 1D simulation at 120 ps with use of SPARK and a fluid code with $f = \infty$ (i.e., no flux limit) and $f = 0.2$. As expected, for uv light at moderate irradiances, the agreement between FP and SH calculations is very good, and the mild flux inhibition is fairly well modeled by $f = 0.2$, in agreement with Ref. 6.

The result of nonuniform illumination is shown in Fig. 2 for $\epsilon = 1$ and $\lambda = 150 \mu\text{m}$, where the comparison is made with fluid results under no flux limitation. Here we used (30×30) cells in the x - z plane and obtained a maximum charge separation of 5×10^{-6} and an energy conservation error of 6×10^{-3} for a time step of 0.1 ps. As observed from the isotherms, the smoothing predicted by the FP solution is less than predicted by the SH solution, though the average temperature $\langle T \rangle$ follows the 1D result of Fig. 1 very closely. This effect is highlighted in Fig. 3, where we plot

$$\sigma_{\text{rms}} = \left\{ \left[\int dx (T - \langle T \rangle)^2 \right] / \int dx \right\}^{1/2} \langle T \rangle^{-1}$$

as a function of z . The peaked structure of σ_{rms} at high density is essentially an artifact of the locally sharp tem-

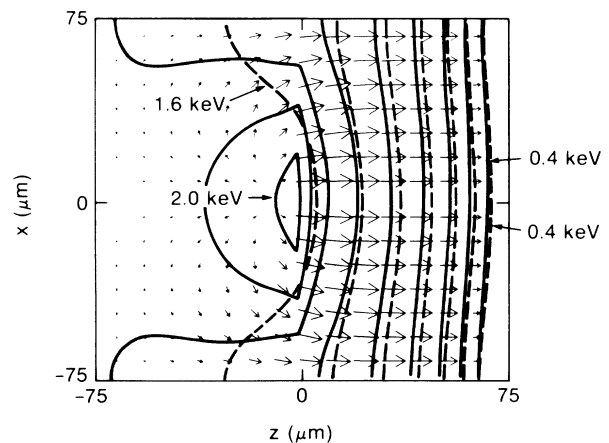


FIG. 2. Isotherms for nonuniform laser irradiation, in steps of 0.2 keV. Vector plot of FP heat flow \mathbf{q} normalized to the maximum value of $|\mathbf{q}|$. As in Fig. 1.

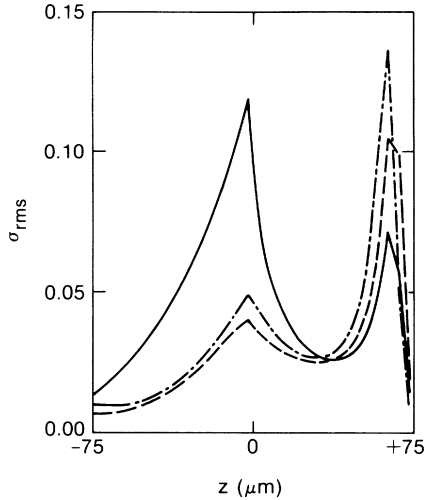


FIG. 3. Plot of σ_{rms} as a function of z . As in Fig. 1.

perature gradients in z . Paradoxically, the simulation predicts $|q_x| < |q_z|$ and $|q_x| \ll fq_f$ (apart from the low-density corona, as seen in Figs. 2 and 4), which is normally taken to imply that the transverse heat transfer is close to classical. However, as has been previously shown by Bell,¹⁶ in the context of 1D thermal transport in ion waves, the relevant criterion for nonlocal effects to be significant is that the mean free path of the heat-carrying electrons ($\approx 80\lambda_{mfp}$) is greater than the relevant spatial scale length l ($=\lambda/2\pi$), irrespective of the magnitude of the heat flux. This is demonstrated quantitatively in Fig. 5, where we plot the ratio of FP to SH σ_{rms} as a function of l/λ_{mfp} , calculated at the critical surface. The observed reduction in smoothing by the nonlocal transport is a direct consequence of heat flux in-

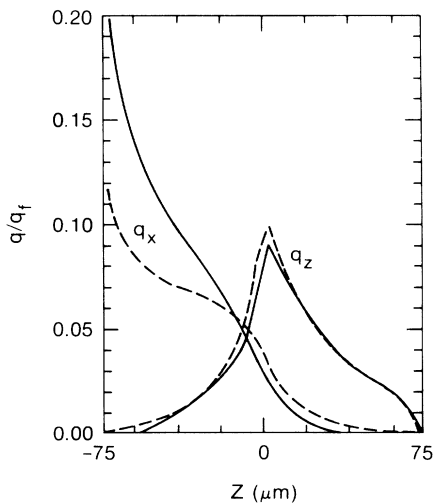


FIG. 4. Plots of q_z/q_f and q_x/q_f as functions of z , calculated at $x = 32.5 \mu\text{m}$. As in Fig. 1.

hibition, and is independent of ϵ (for $\epsilon \leq 1$). Similar results have also been obtained for different laser wavelengths and irradiances.¹³ It must be noted, however, that although $(\sigma_{rms})_{FP} \gg (\sigma_{rms})_{SH}$ for $l \ll \lambda_{mfp}$, both models predict that $\sigma_{rms} \rightarrow 0$ in the same limit, as seen in Fig. 5.

The effect of flux limiting the SH heat flow in 2D is shown in Fig. 3 for $f=0.2$. The relatively small improvement in the results is not surprising since the flux limitation is only weakly dependent on the lateral heat flow, which is well below fq_f . Moreover, the increased coronal temperature, arising from the axial flux inhibition (see Fig. 1), has a tendency to increase the thermal-conductivity coefficient, and hence the amount of smoothing.

In summary, we have shown that even when 1D transport in laser plasmas is well described by fluid equations, the same is not necessarily true in 2D. Particularly, when there are small scale ($\lesssim 80\lambda_{mfp}$) temperature modulations in the plasma, the thermal smoothing becomes less effective as a result of the nonlocal nature of the electron transport. This reduced smoothing, which is not adequately treated by a simple flux-limited diffusion theory, may have important implications to the process of thermal self-focusing and ultimately to the estimation of ablation pressure uniformity. In order to adequately assess these effects, further study is required through the inclusion of the hydrodynamic response of the plasma.

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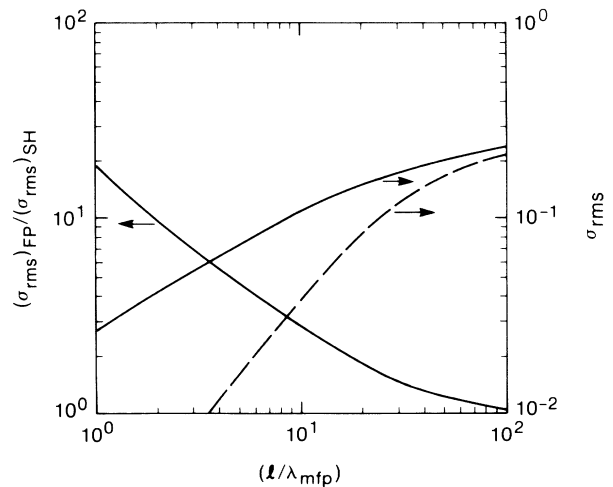


FIG. 5. Plots of $(\sigma_{rms})_{FP}$, $(\sigma_{rms})_{SH}$, and $(\sigma_{rms})_{FP}/(\sigma_{rms})_{SH}$ as functions of l/λ_{mfp} , calculated at the critical surface. As in Fig. 1.

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