Twofold Symmetric Angular Distributions in Multiphoton Ionization with Elliptically Polarized Light

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The angular distributions of electrons in multiphoton multichannel ionization of hydrogen for the case of elliptically polarized laser light are calculated within a nonperturbative theoretical model taking into account the Coulomb interaction in the final state. It is found that the ellipticity of the radiation not only modifies the shape but also lowers the fourfold rotational symmetry occurring in linear polarization to a twofold one.

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This Letter is concerned with the calculation of angular distributions of electrons ejected from atoms as a result of multiphoton ionization by a strong laser field having an arbitrary elliptic polarization. In recently published experiments,[1] multiphoton ionization angular distributions for elliptic laser polarization have been studied in xenon and krypton. The experimental data points were taken in the polarization plane in a 90° angular range and at a retardation angle $\xi = 80°$, determining the ellipticity of the radiation via the polarization vector $\hat{\epsilon}$ given by

$$\hat{\epsilon} = \mathbf{x}\cos(\xi/2) + i\mathbf{y}\sin(\xi/2), \quad (1)$$

with $\xi = 0°$ and $\xi = 90°$ corresponding, respectively, to linear and circular polarizations.

In Ref. 1 it was assumed that the angular distributions had a fourfold symmetry, so that the data taken from 0° to 90° were symmetrically replicated in the remaining angular range and fitted accordingly. A theoretical basis to that fourfold symmetry assumption was provided by an extension to elliptic polarization of the Reiss model.[2]

Below we report on calculations showing that in the case of elliptically polarized laser light the angular distributions in the polarization plane exhibit only a twofold symmetry, a fourfold one holding only for linear polarization; in the general elliptic case, a fourfold symmetry is recovered only, neglecting Coulomb interactions in the final state and/or renouncing the gauge consistency. These last results provide an explanation of why the Reiss model predicts a fourfold symmetry; in fact, the latter model is known to be based on a hybrid procedure, as far as the gauge consistency is concerned[3,4] (see, in particular, Ref. 4 for a detailed analysis of the Reiss model and a comparison with the present treatment); further, Keldysh-type calculations, in which the Coulomb interaction is neglected in the final electron state, also provide a fourfold symmetry.

The predictions of our calculations can be verified by means of experiments like those of Ref. 1, by our taking data points in an angular range covering the entire polarization plane.

For our calculations, we use a theoretical model based on the S-matrix formalism and taking approximately into account the Coulomb interaction in the final state; the S-matrix element in the electric-field gauge is written as

$$S_{fi} = (i\hbar)^{-1}\int_{-\infty}^{\infty} dt \int d\mathbf{r}\, \Psi_f^*(\mathbf{r},t) e\mathbf{E}(t)\cdot\mathbf{r}\psi_i(\mathbf{r},t), \quad (2)$$

where the hydrogen ground state is taken as the initial state; the final one is approximated by a Coulomb-Volkov wave, i.e., a Volkov state whose spatial part is replaced by a Coulomb wave:

$$\Psi_f = U_G \exp\left\{-\frac{i}{2m\hbar}\int^t \left[\hbar\mathbf{k} + \frac{e\mathbf{A}(\tau)}{c}\right]^2 d\tau\right\}\Phi_k^-(\mathbf{r}), \quad (3)$$

$$U_G = \exp[ie\mathbf{A}(t)\cdot\mathbf{r}/\hbar c] = \exp[ik_G\mathbf{a}(\omega t)\cdot\mathbf{r}], \quad k_G = eE_0/\hbar\omega. \quad (3a)$$

In the above equations, $\Phi_k^-(\mathbf{r})$ is an incoming Coulomb wave and the unitary operator U_G accomplishes the transformation of the final-state wave function from the vector potential to the electric-field gauge; moreover, $\mathbf{A}(t)$ denotes the laser vector potential that, in dipole approximation, is given by

$$\mathbf{A}(t) = cE_0\mathbf{a}(\omega t)/\omega, \quad (4)$$

$$\mathbf{a}(\alpha) = [\hat{\epsilon}\exp(-i\alpha) + \text{c.c.}]/2, \quad (4a)$$

and the electric field has strength E_0 and angular frequency ω. The wave function (3) has been used recently in multiphoton ionization[5] and is used in other[6] contexts as well; it is expected to provide the main term of the interaction between a continuum Coulomb state and a strong radiation field for nonresonant wavelengths.

The differential ionization rate is found as a sum over the allowed (on the grounds of energy conservation) multiphoton channels

$$dw/d\Omega = \textstyle\sum_s (dw/d\Omega)_s, \quad s=0,\ldots,\infty, \tag{5}$$

where the differential ionization rate to the channel with absorption of s photons beyond the minimum n_0 is

$$(dw/d\Omega)_s = C(k_s)(I/I_a)\,|M_{n_0+s}(\mathbf{k}_s)|^2, \tag{6}$$

$$C(k) = (e^2/\hbar a_0)(2^5/\pi^2) \times [1-\exp(-2\pi\nu)]^{-1}, \quad \nu=(ka_0)^{-1}. \tag{7}$$

The electron kinetic energy (inside the laser beam) in the channel with s "above-threshold" photons is given by

$$\hbar^2 k_s^2/2m = (n_0+s)\hbar\omega - I_0 - \Delta, \tag{8}$$

$$\Delta = e^2 E_0^2/4m\omega^2, \tag{9}$$

where n_0 is related to the ionization energy I_0 by

$$n_0 = [I_0/\hbar\omega] + 1, \tag{10}$$

with the square brackets denoting the largest integer smaller than $I_0/\hbar\omega$; moreover, in the above expressions, a_0 is the Bohr radius, $I=cE_0^2/8\pi$ is the laser intensity, and $I_a \simeq 3.51\times 10^{16}$ W/cm^2 is its atomic unit; the quantity Δ has the classical interpretation of the average (over the laser period) of the oscillatory kinetic energy of an electron in the field of a plane wave.

The dependence of the differential ionization rates on the orientation of kinetic momentum $\hbar\mathbf{k}$, whose unit vector has spherical components $(\sin\theta_k\cos\phi_k, \sin\theta_k\sin\phi_k, \cos\theta_k)$, with respect to the polarization vector $\hat{\epsilon}$ is completely contained in M_n entering Eq. (6); it is found, with integral representations of the Bessel functions involved, as the following one-dimensional integral

$$M_n(\mathbf{k}) = -\int_{-\pi}^{\pi} B(\mathbf{k},\alpha) f_n(\alpha)\,d\alpha, \tag{11}$$

$$f_n(\alpha) = \exp\{i[n\alpha - (\Delta/\hbar\omega)\mathbf{e}(\alpha)\cdot\mathbf{a}(\alpha) - eE_0\mathbf{k}\cdot\mathbf{a}(\alpha)/m\omega^2]\}, \tag{12}$$

$$\mathbf{k}_G = eE_0\mathbf{e}(\alpha)/\hbar\omega, \quad \mathbf{e}(\alpha) = [i\hat{\epsilon}\exp(-i\alpha) + \text{c.c.}]/2, \tag{13}$$

$$B(\mathbf{k},\alpha) = (T/S)^{i\nu}\nu(1-i\nu)T^{-3}a_0\mathbf{a}(\alpha)\cdot\{\mathbf{k}_G[(T/S)(2-ika_0) + (T/S)^2(1+i\nu)(1-ika_0)/(1-i\nu) - (\nu+2i)/\nu] + \mathbf{k}[(T/S)(1-ika_0) - (\nu+2i)/\nu]\}, \tag{14}$$

$$T = 1 + a_0^2(\mathbf{k}+\mathbf{k}_G)^2, \quad S = a_0^2 k_G^2 + (1-ika_0)^2. \tag{14a}$$

The experimental data for angular distributions are taken in the polarization plane, i.e., variation of the ϕ_k, with θ_k fixed at the value 90°; the angular dependence in M_n appears in terms of the type $\mathbf{k}\cdot\mathbf{a}(\alpha)$ and $\mathbf{k}\cdot\mathbf{e}(\alpha)$, with the presence of the latter ones being linked with the presence of $\mathbf{k}_G$, needed on the ground of the gauge consistency, and responsible for the reduction of symmetries of the angular distributions in the polarization plane. It can be shown, by inspection of the above equations, that the an-

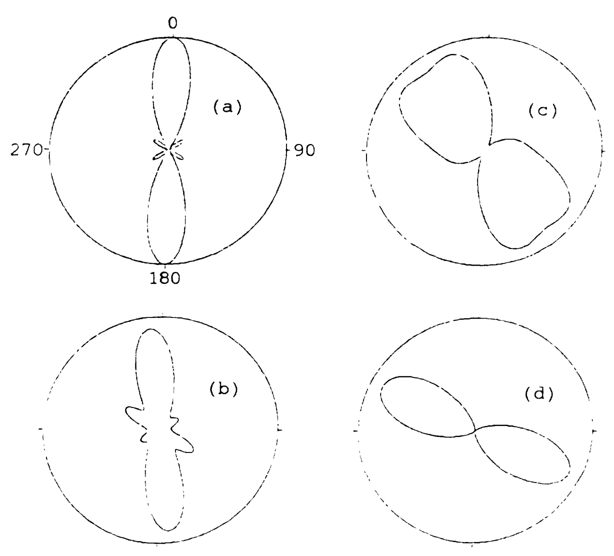

FIG. 1. Angular distributions in the polarization plane of photoelectrons emitted in multiphoton ionization of hydrogen by an elliptically polarized laser field; laser intensity and wavelength are 10^{12} W/cm^2 and 1064 nm, respectively; the number s of photons absorbed over the minimum required is 0. Distributions are shown for several values of the retardation angle: (a) $\xi=0°$; (b) $\xi=30°$; (c) $\xi=50°$; (d) $\xi=85°$.

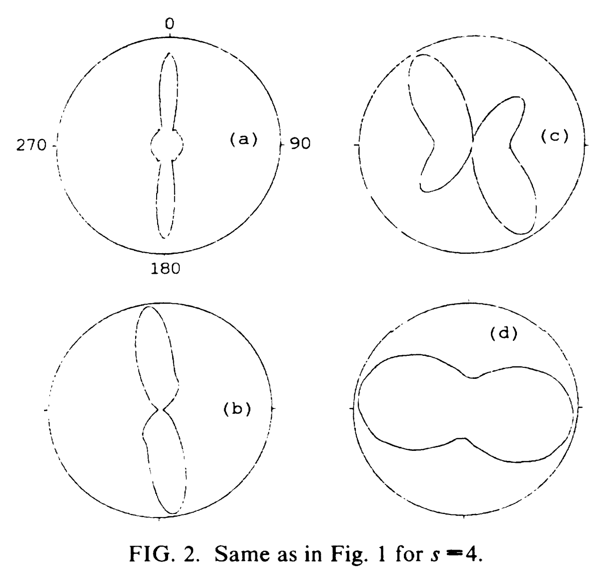

FIG. 2. Same as in Fig. 1 for $s=4$.

gular distributions in the polarization plane are independent of ϕ_k for circular polarization while, in general, they have the following symmetric property:

$$D_s(\xi,\phi_k)=D_s(\xi,\pi+\phi_k)\,, \qquad (15)$$

with $D_s(\xi,\phi_k)$ denoting the differential ionization rate to a fixed multiphoton channel, computed at a value ξ of the retardation angle, and for θ_k equal to 90°. According to Eq. (15), only a twofold rotational symmetry holds. In the case of linear polarization, besides (15), the following holds:

$$D_s(\phi_k)=D_s(-\phi_k)\,, \qquad (16)$$

leading to a fourfold rotational symmetry in the polarization plane. This more extended symmetry is also found for any polarization if one neglects completely the Coulomb interaction in the electron final state or alternatively if one sets $\mathbf{k}_G=0$, i.e., adopts a hybrid gauge procedure. Thus these results provide some insight into physical mechanisms responsible for the reduction of the fourfold symmetry.

In Figs. 1 and 2 we show angular distributions of photoelectrons in the polarization plane for several values of the retardation angle and for two representative values of s, i.e., 0 and 4, at the intensity of 10^{12} W/cm^2 and laser wavelength 1064 nm ($n_0=12$); the value of the differential rates for $\phi_k=0°$ is taken as the normalization value. These figures show that the fourfold rotational symmetry, as said above, holds only for $\xi=0°$ and that the direction in which electrons are mostly emitted varies with the retardation angle.

In conclusion, we have considered the angular distributions of photoelectrons in multiphoton multichannel ionization of atomic hydrogen by a strong, elliptically polarized laser field. The theoretical treatment, as well as the numerical results, have in particular shown the crucial role of the Coulomb interaction in the final state and of the gauge consistency in the determination of symmetry properties; besides they have strongly suggested the need for more detailed and extended experimental informations in order to compare the predictions of different approaches.

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