On the Existence of Quantum Electrodynamics

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We explain from an intuitive renormalization-group perspective how "collapse of the wave function" in quenched quantum electrodynamics leads to coupling-constant renormalization and an interacting ultraviolet stable fixed point. A diagrammatic expansion in N_f, the number of fermion species, suggests that vacuum polarization leads the fixed point of the quenched model stable, and computer-simulation data support this possibility. The scaling region of the quenched lattice model is discovered numerically.

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What happens to quantum electrodynamics (QED) as the bare charge e_0 is taken large? For $e_0 \sim 1$, perturbation theory is an unreliable guide to the physical content of the theory. However, most physicists appear to believe that the coupling-constant renormalization problem in the theory is controlled entirely by the ultraviolet structure of the photon propagator and that vacuum polarization reduces the coupling as the cutoff is reduced. In fact, impressive arguments have been made based on high-order perturbative analysis that the resulting renormalized theory is a free field (the "Moscow-zero" or "Landau-ghost" problem).[1]

In this paper we consider a radical but hopefully compelling physical picture of strongly coupled QED. We suggest on the basis of the solution of the chiral dynamics of the quenched model, a diagrammatic analysis for small N_f (N_f=number of species of dynamical fermions), and computer simulations of the $N_f=0$, 2, and 4 theories, that QED possesses an ultraviolet-stable, infrared-unstable fixed point at strong coupling. The physical basis of this nonperturbative fixed point is "collapse of the wave function"[2]—for sufficiently large e_0 the attractive potential between a e^+e^- pair overwhelms their centrifugal barrier and their wave function develops a $1/r$ singularity which leads to chiral-symmetry breaking and a probability of order unity that the e^+e^- pair collide. The $1/r$ singularity is a nonperturbative source of coupling-constant renormalization which produces a *negative* Callan-Symanzik function. The stability of this physical picture to the introduction of light dynamical fermions is discussed within a N_f expansion. We speculate that the crucial physical phenomenon of collapse of the wave function survives vacuum polarization for small N_f and leads to an interacting field theory in which there is intense screening. Computer simulations of the $N_f=0$ theory expose the scaling law of the chiral condensate $\langle\bar\psi\psi\rangle$ predicted by this physical picture and computer data for $\langle\bar\psi\psi\rangle$ at $N_f=2$ and 4 suggest that this mechanism survives vacuum polarization. We comment on the relation of this physical picture to the fermion-monopole kinematics the underlying catalysis of proton decay.

We will illustrate our ideas in the context of the Dirac equation describing a light fermion bound by a static Coulomb potential.[2] The results we emphasize can also be obtained from a Schwinger-Dyson equation analysis of two light fermions in QED when vacuum polarization is ignored. It is easily seen from the square of the Dirac equation that for $\alpha \gtrsim \alpha_c = 1$, the Coulomb attraction overwhelms the centrifugal barrier and the system implodes. For small distances the wave function behaves as[2]

$$\psi(r) \sim \sin[(\alpha^2-\alpha_c^2)^{1/2}\ln(r|\epsilon|)]/r(\alpha^2-\alpha_c^2)^{1/2}, \quad (1)$$

where ϵ is the bound-state energy of the lowest-lying s wave. Letting $r \to 0$, $\Psi(r)$ oscillates at an ever increasing rate and the system fails to exist. To understand this problem better we cut off the $1/r$ potential at a distance r_0 and solve the equation again. The cutoff at r_0 provides a nonperturbative regulation of the theory. If we demand that ϵ is fixed, independent of r_0, then α must become a function of r_0. From Eq. (1) we see that if

$$\alpha^2(r_0) = \alpha_c^2 + \frac{\pi^2}{\ln^2(r_0|\epsilon|)}, \quad (2a)$$

then

$$\epsilon = -\frac{1}{r_0}\exp[-\pi/(\alpha^2-\alpha_c^2)^{1/2}] \quad (2b)$$

is finite and independent of the cutoff r_0 as $r_0 \to 0$. When $\alpha \to \alpha_c$, the bound-state wave function Eq. (1) becomes

$$\Psi(r) \underset{\substack{r\to 0\\ \alpha\to\alpha_c}}{\sim} \ln(1/r|\epsilon|)/r, \quad (3)$$

which is a sensible, renormalized wave function. Following Miransky we interpret α_c as an ultraviolet-stable fixed point.[3] One can define a beta function from Eq.

(2b), or, even better, from the Schwinger-Dyson analysis of the relativistic e^+e^- problem,[4]

$$\beta(\alpha)=\Lambda\frac{\partial}{\partial\Lambda}\alpha(\Lambda)=-\frac{2\alpha_c}{\theta}(\alpha/\alpha_c-1)^{3/2}, \quad (4)$$

$$\Lambda=1/r_0, \quad 0<\theta<\pi,$$

whose minus sign indicates that the effective coupling grows large at low momenta.

These equations can be interpreted in a renormalization-group picture. Our analysis below indicates that the $1/r$ behavior of $\Psi(r)$ in Eq. (3) is the source of a new ultraviolet divergence and coupling-constant renormalization in QED. Consider $\alpha<\alpha_c$. Then the theory's bound-state wave functions are less singular than Eq. (3) and energy levels computed in the quenched theory are insensitive to short distance modifications of the α/r potential. This is simply because the probability to find the bound state e^+e^- separated by a distance less than r falls to zero as $r\to 0$, i.e., $|\Psi(r)|^2r^2dr\,d\Omega$ approaches zero as $r\to 0$. Such relatively well-behaved wave functions underlie most physicist's intuitions into quantum mechanics. However, for the collapsed wave function Eq. (3) the corresponding probability does *not* fall to zero. This means that the low-energy spectrum of the potential model is sensitive to forces at vanishingly small length scales. To understand the minus sign in Eq. (4) we consider a renormalization-group calculation of ϵ. Regulate the theory in momentum space and integrate out the high-frequency ($|q|>q_0$) fluctuations in the potential α/r. The resulting theory has a lower cutoff in momentum space and a *larger* effective coupling. The effective coupling is amplified because it incorporates implicitly the high-frequency photon exchanges which cause the e^+ and e^- to *attract* one another at short distances. Equations (2a) and (2b) are the configuration-space realization of this effect. Equation (2a) states that if the Coulomb potential is cut off at a larger r_0 then to compensate for some otherwise lost e^+e^- attraction, $\alpha(r_0)$ must be *increased* to keep the binding energy unchanged.

The same physics can be discovered in the quenched planar field theory analyzed with the Schwinger-Dyson equations. Collapse of the wave function occurs and dynamical chiral-symmetry breaking occurs in the new, stable strong-coupling vacuum. One finds $\alpha_c=\pi/3$ and the scaling law Eq. (2b) governing the dynamical fermion mass m. The renormalized chiral condensate also scales,[2]

$$\langle\bar\psi\psi\rangle_{\text{ren}}=-8m^3/\pi^4 \quad (5a)$$

with

$$m=a^{-1}\exp[-\theta/(\alpha-\alpha_c)^{1/2}], \quad 0<\theta<\pi, \quad (5b)$$

where a is the space-time cutoff. Equation (5b) is the relativistic generalization of Eq. (2b) and the source of Eq. (4). The quenched theory has nontrivial chiral dynamics with a composite massless pion which is a Goldstone boson and $f_\pi\neq 0$.[2] Equation (5a) allows us to search for the crucial scaling law Eq. (5b) in computer simulations, as will be demonstrated below. The perplexing feature about Eq. (5b) is the following. Critical behavior in four space-time dimensions is usually well approximated by the Landau mean-field theory where an order parameter such as $\langle\bar\Psi\Psi\rangle$ should satisfy the scaling law $(\alpha-\alpha_c)^{1/2}$, $\alpha>\alpha_c$. Only if we analyze the Schwinger-Dyson equations at strong coupling $\alpha\gg\alpha_c$, where the dynamical fermion mass is of order $1/a$, do we find[5]

$$m\simeq a^{-1}(\alpha-\alpha_{\text{MF}})^{1/2}, \quad \alpha_{\text{MF}}=4\alpha_c \quad (5c)$$

instead of Eq. (5b). Thus, this theory displays a classic crossover phenomenon between mean-field behavior away from the critical region to a fluctuation dominated nontrivial scaling behavior at the critical point. Equation (5c) has also been confirmed for $\alpha\gg\alpha_c$ in computer simulations, as will be demonstrated below.

The central question is whether this physical picture and its scaling law survive the introduction of dynamical fermions. The quenched model is simply a toy and could be misleading. Collapse of the wave function is the crucial phenomenon because its space-time singularity generates a new source of coupling-constant renormalization which leads to an ultraviolet-stable fixed point. Does the collapse of the wave function survive vacuum polarization?

Consider QED with N_f species of fermions with N_f infinitesimal. In the quenched ($N_f=0$) case the e^+e^- bound state is generated by the graphs of Fig. 1(a). Actually the Schwinger-Dyson equation includes only the planar graphs, but we will include all photon exchanges since they occur in the computer simulations. Now consider the first correction to this figure in a N_f expansion. It contains one fermion loop whose momentum is labeled k in Fig. 1(b) and a sum over all photon exchanges is

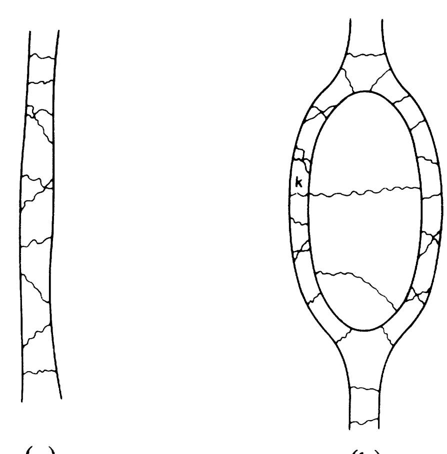

FIG. 1. (a) The quenched graphs binding a e^+ and e^- together into a collapsed wave function. (b) First loop correction to (a).

implied. The high-frequency photons of Fig. 1(a) produce the coupling-constant renormalization Eq. (4) of the quenched model and the collapsed e^+e^- wave function. In Fig. 1(b) there are two regions of the k integration which produce the coupling-constant renormalization. The first region is *small* k^2 where, as suggested by the figure, two collapsed bound states form and propagate. Each bound state is electrically neutral so they interact via dipole forces in this set of graphs. The second region is large k^2 where there is a familiar fermion vacuum-polarization correction to the photon propagator attached to the two through-going fermion lines. Since the coupling is large we must consider all internal photon corrections to the loop. We cannot calculate such effects, but let us assume that they lead to screening of the photon propagator as occurs at low orders of perturbation theory. This effect may not lead to a free field theory because the low k^2 piece of Fig. 1(b) persists and it leads to two collapsed wave functions (mesons) through the virtual dissociation of the incoming meson. In a N_f expansion there will be an $O(N_f)$ correction to the Callan-Symanzik function of Eq. (4) coming from Fig. 1(b). Vacuum-polarization corrections to single photon exchange will presumably contribute positively while collapse of the wave function effects will contribute negatively to this term. If this expansion makes sense, then the β function of Eq. (4) will continue to have a nontrivial zero order by order in N_f. It is certainly not clear, however, if there will be a nontrivial zero when the N_f expansion is summed.

To make further progress consider the computer simulation study of this theory. We simulated the $N_f=0$, 2, and 4 theories using the hybrid algorithm of Ref. 5. The gauge fields were noncompact and the pseudofermion fields were represented by stochastic noise so that any number of fermions could be simulated. The data to be shown here were taken on 8^4 and $8^3\times 16$ lattices with dynamical fermion masses of $m=0.02$ and 0.04 to allow $m\rightarrow 0$ extrapolations of the physical quantities of interest. We will emphasize the chiral condensate $\langle\bar{\Psi}\Psi\rangle$ here since it should expose the scaling laws of the continuum theory. In the quenched theory, we measured $\langle\bar{\Psi}\Psi\rangle$ from $\beta=1/e^2=0.32$ to 0.10 in steps of $\Delta\beta=0.01$ as shown in Fig. 2. 125000 sweeps of the algorithm with a time step of 0.02 were done and $\langle\bar{\Psi}\Psi\rangle$ was measured at five distant lattice sites after every 1000 sweeps. In Fig. 2 we show a fit $\langle\bar{\Psi}\Psi\rangle\sim(\beta-2.4)^{1/2}$ at strong coupling and a fit $\langle\bar{\Psi}\Psi\rangle\sim\exp[-\theta/(\beta-0.36)^{1/2}]$ at weak coupling. The scaling region extends from $\beta=0.29$ to 0.32. Finally, in Fig. 3 we show $\langle\bar{\Psi}\Psi\rangle$ vs β for $N_f=0$, 2, and 4. The $N_f=2$ and 4 simulations required an order of magnitude with more computing power per iteration than the $N_f=0$ curve so that only 10^4–5×10^4 sweeps of the algorithm could be done at each β and m. Nonetheless, the curves again suggest scaling regions with essential singularities, although the scaling windows are narrower than in the quenched case. Plausible fits similar to Fig. 2 can be made[6] and simulations on larger lattices are in progress to remove finite-size effects. The steepness of the $N_f=4$ curve suggests that for N_f slightly higher the mean-field scaling law will become exact at all β and such theories will be free in the continuum limit.

Other computer studies of the theories have been made. We have good evidence from simulations on 2×8^3, 4×8^3, 6×12^3, and 8×16^3 lattices that the $N_f=4$ theory has nontrivial temperature dependence characteristic of an infrared-unstable fixed point.[6] We plan to improve the quenched data by simulating a $16^3\times 32$ lat-

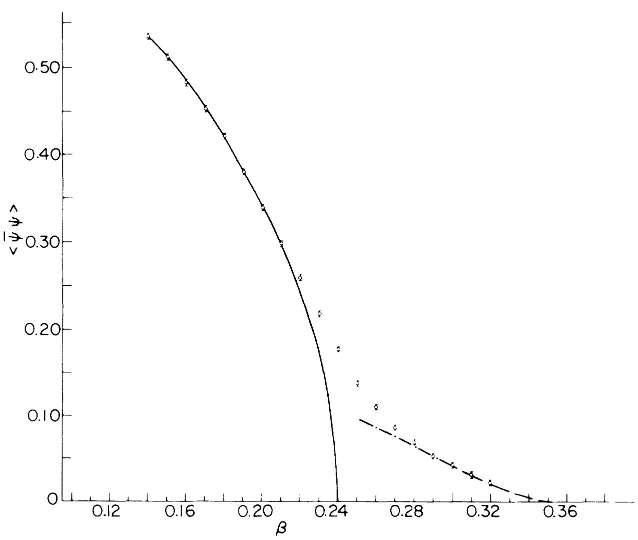

FIG. 2. Quenched data of $\langle\bar{\Psi}\Psi\rangle$ extrapolated to zero fermion mass vs $\beta=1/e^2$. The fits are described in the text.

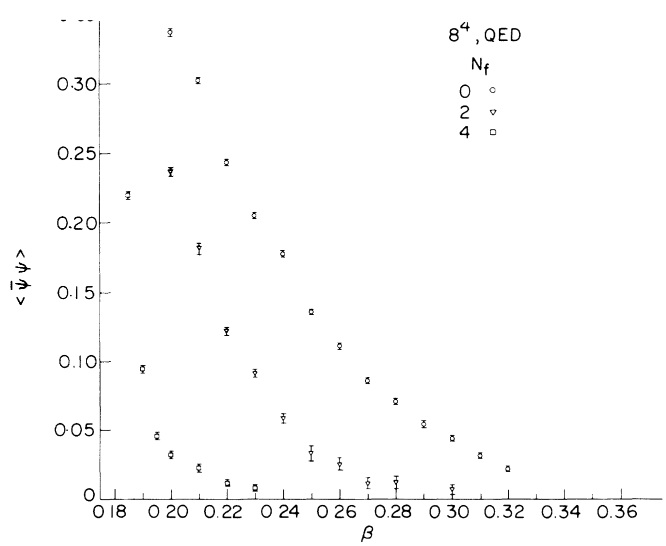

FIG. 3. $N_f=0$, 2, and 4 data of $\langle\bar{\Psi}\Psi\rangle$ vs β.

tice using the accelerated hybrid algorithm and measuring $\langle\bar{\Psi}\Psi\rangle$ using the Lanczos algorithm so that $m \rightarrow 0$ extrapolations can be avoided.

In summary, this Letter suggests a simple physical picture of strongly coupled QED based on a nonperturbative source of coupling-constant renormalization associated with the collapse of the wave function. This new source of renormalization may generate an interacting theory of mesons in which free fermions do not exist.[5] Finally, the similarity of the collapsed wave function Eq. (3) and the fermion wave function in the presence of a monopole suggests other common features of these two phenomena. The collapsed fermion wave function around a monopole leads to catalysis of proton decay with decay amplitudes of unit strength.[7] Similarly, we believe that strongly coupled QED acts as an amplifier of symmetry breaking interactions at short distances. This and related effects will be the subject of a forthcoming Letter. And, of course, the sketchy ideas of this Letter will be amplified themselves in a series of full-length articles containing Schwinger-Dyson analysis and extensive supercomputer simulations.

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