Memory Effects in Propagation of Optical Waves through Disordered Media

Isaac Freund and Michael Rosenbluh
Department of Physics, Bar-Ilan University, Ramat-Gan, Israel

Shechao Feng
Department of Physics and Solid State Science Center, University of California, Los Angeles, California 90024
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We verify experimentally for optical waves the striking memory effect predicted very recently by Feng, Kane, Lee, and Stone. We present data for both transmission and reflection, and find general agreement with the theoretical predictions for the linear scale dependence and asymptotic exponential falloff of the memory effect. The theoretical and experimental results suggest that significant information about the spatial variation of the incident waveform is preserved during passage through a highly disordered, strongly multiply scattering medium.

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Recently, Feng, Kane, Lee, and Stone (FKLS) predicted a striking memory effect in coherent wave propagation through disordered media.[1] At issue is the following: To what extent is information about the spatial variation in phase and amplitude of an incident wave preserved during its transmission through a highly random, multiply scattering medium? Implicit in the results of FKLS is the answer that transverse spatial variations on a scale finer than the sample thickness L are forgotten, but that information on larger scales is preserved. Thus, the emergent wave "remembers" much, but not all, about the incident wave from which it derives. In this Letter we present experimental data for both transmission and reflection which demonstrate (i) the existence of the predicted memory effect,[1] (ii) the predicted linear dependence on sample thickness of the scale over which memory loss occurs in transmission,[1] and (iii) the predicted exponential decay for the asymptotic falloff[1] of the angular correlation function $C(x)$ that serves as a quantitative measure of the extent of information preservation. Here x measures changes in the incident waveform. Although our experimental results generally verify the theoretical predictions of FKLS, we do find one significant difference between experiment and theory—namely, that the exponential decay of $C(x)$ is observed to set in almost immediately with nonzero x, while the theory predicts that its onset should be delayed.

Because of the randomness of the scattering medium, the intensity of the emergent wave always has an intrinsic, complex spatial variation which gives rise to the rapid, apparently random, intensity fluctuations known as speckle. When excited with a normally incident plane wave, this speckle pattern represents a sample-specific fingerprint that contains a great deal of (difficult to decipher) information about the particular realization which produces it. Well defined changes in this speckle pattern as the incident wave front is varied reflect some of the information remembered during transmission through the random medium, and this is accessible to measurement.

One of the simplest possible modulations of the incident wave front is a linear variation in phase across the sample face. To the extent that this linear phase variation is remembered during transmission, the resulting speckle pattern will be identical to the pattern excited by a normally incident plane wave (the reference pattern), except that it will be shifted in angle to one side. By changing the degree of phase variation across the sample face, the speckle pattern can be swept in a direction. When the extent of the linear phase variation becomes so large that a significant phase difference occurs over a transverse dimension of order the sample thickness L, the speckle pattern begins to change. By measuring the degree of correlation of this new pattern with the original reference pattern, one obtains a quantitative measure of the extent of the memory effect.[1]

The requisite linear phase variation can be obtained most simply by changing the angle of incidence. If this is done by rotating the incident laser beam direction around the sample, then the emergent speckle pattern tracks the laser direction. Thus, one has the striking result that although the sample appears totally diffuse and nearly opaque, and apparently nothing can be discerned upon attempting to look through it, nonetheless, the direction of the (invisible) incident laser beam can be immediately determined. Significant information about the incident wave front has thus been remembered and transmitted through the random medium, and one has to some extent succeeded in "looking" through a visually impenetrable obstacle.

The experimental confirmation of this predicted[1] behavior is shown in Fig. 1. In obtaining these data, the diameter of a polarized 5-mW He-Ne laser beam ($\lambda = 0.633$ μm) was expanded to 15 mm, and the central to 6 mm, representing a nearly flat wave front, was used to excite the speckle pattern. Placing this 6-mm mask

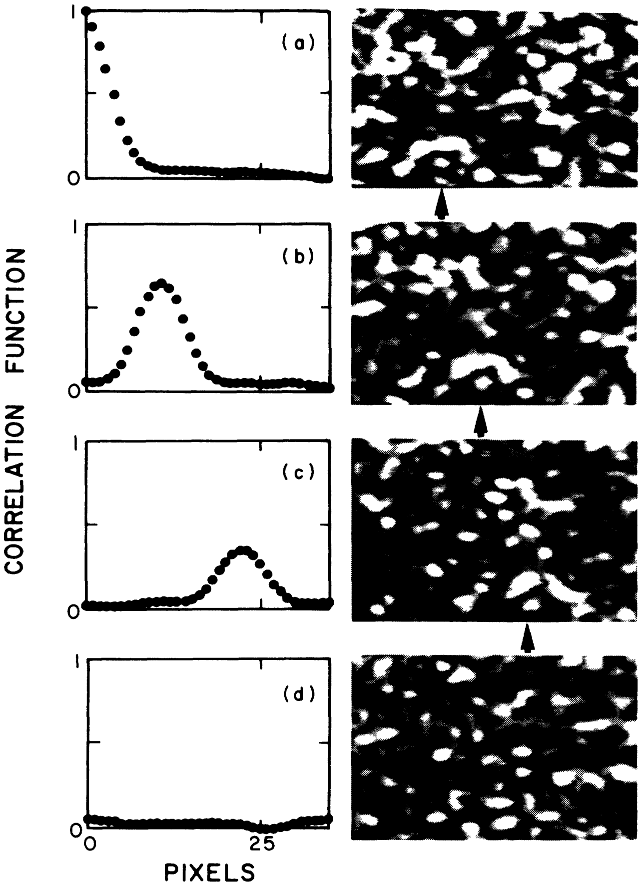

FIG. 1. Memory effect. The right-hand side shows images of a small portion of speckle patterns in transmission as the incident laser beam direction is varied. The arrow at the bottom of each image calls attention to the semicircular arc enclosing a bright spot (the "bulls eye") which serves as a convenient visual reference for tracking the motion of the patterns. The initial reference pattern produced by a normally incident laser beam is shown in (a), while in (b) the laser is rotated by 10 mdeg, and in (c) by 20 mdeg. The correlation function shown to the left of each image corresponds to the cross-correlation coefficients of the reference pattern with the corresponding image. This correlation is plotted as a function of pattern shift in pixels, and the peak represents the maximum degree of overlap of the two patterns. Note that the correlation function tracks the speckle patterns, which, in turn, "remember," and therefore track, the incident laser beam direction. The pattern in (d) is one which is unrelated to the reference pattern in (a), and the correlation function shows the expected small statistical fluctuations about zero.

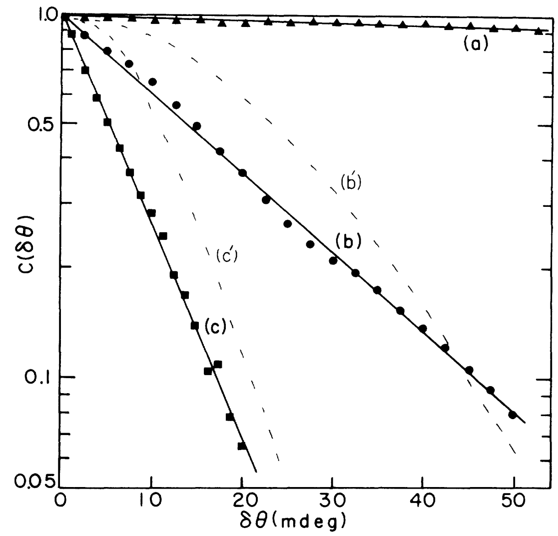

FIG. 2. Correlation function $C(\delta\theta)$ in transmission. $\delta\theta$ is the angle of rotation of either the laser beam direction or the sample. The solid lines are least-squares fits with an exponential function. Curve a, ground glass; curve b, 370-μm-thick opal glass; curve b', theory; curve c, 810-μm-thick opal glass; curve c', theory.

directly on the sample, together with careful system alignment, ensured that the laser beam did not walk across the sample face during rotation. The free-space speckle patterns were recorded with a sensitive charged-coupled-device video camera, and digitized by an on-line microcomputer. The sample was a thin sheet (370-μm thickness) of opal glass supported on a clear glass substrate. In Fig. 1(a) the laser beam was normally incident, and the correlation function shown is that of the initial (reference) speckle pattern with itself. This function is computed in terms of the rightward shift in pixels of a replica of the pattern against the original. For zero shift, the pattern and its replica are, of course, totally correlated, but as the shift increases, correlation is rapidly lost because of the apparently random arrangement of the speckle spots. The 0.12-mrad full width at half maximum of this autocorrelation function measures the mean angular diameter of a speckle spot, and is in close agreement with the expected width, which is that of the diffraction pattern of the 6-mm circular aperture used to define the incident laser beam. Other interesting correlation effects[2,3] can be studied by extending these methods.

In Figs. 1(b) and 1(c) the laser beam is successively rotated about the sample by a small amount $\delta\theta$. The transmitted speckle pattern may be observed to follow the incident beam direction while slowly changing. The cross correlation function (between the initial reference pattern and the new pattern) accurately tracks this motion, while its peak height decreases as correlation is lost. In Fig. 1(d), the cross-correlation function of two unrelated patterns is shown, and illustrates the expected small statistical fluctuations about zero.

The ensemble-averaged correlation function, $C(\delta\theta)$, is shown for transmission in Fig. 2, and for reflection in Fig. 3. These data represent the recording, digitizing, and analysis of some 250 separate patterns, each containing several hundred speckle spots. As may be seen, in every instance the data are well approximated by a

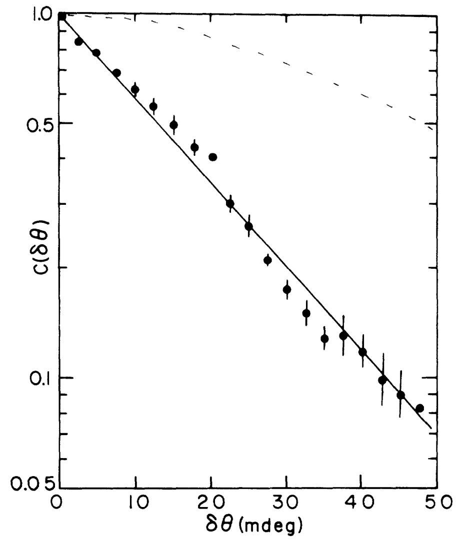

FIG. 3. Correlation function $C(\delta\theta)$ in reflection. $\delta\theta$ is the angle of rotation of the 370-μm opal-glass sample. The dashed line is theory, Eq. (1), corrected for the asymmetric experimental geometry.

simple decaying exponential.

Curve a in Fig. 2 shows the results in transmission for a coarsely ground glass surface, which corresponds to a sample with a near zero value for L. These data were obtained by rotating the laser direction around the sample, as in Fig. 1. For this sample, $C(\delta\theta)$ is expected to be nearly independent of $\delta\theta$, in agreement with experiment. Since these data were obtained and processed in exactly the same fashion as for thick samples (the laser was attenuated to keep the transmitted intensity the same), this agreement also verifies that system alignment, the recording apparatus, the digitizing process, and the data analysis were all well behaved. We attribute the small residual decay of $C(\delta\theta)$ with increasing $\delta\theta$ to the finite surface roughness, which was estimated from microscopic examination to be about 15 μm.

Curves b and c of Fig. 2 show the results in transmission for 370-μm- and 810-μm-thick opal-glass multiple scatterers. These data were obtained in two ways. The first was to rotate the direction of the incident laser around the sample, as in Fig. 1, and to measure the peak height of the correlation function. In accordance with expectation, we found that the position of the peak of $C(\delta\theta)$ accurately tracked the laser beam to within the half-pixel experimental error (0.4 mdeg). Since in the forward direction only the angle between the laser and sample normal is relevant, equivalent results should be obtained by rotating the sample in a stationary laser beam. In this case the speckle pattern, which remembers the laser direction, also remains stationary, but slowly changes in form as the sample rotates. Both methods did, in fact, give entirely equivalent results, and the data shown are the average of the two.

For a finite transport mean free path l, the effective thickness of the sample is expected to be approximately $L-l$, since at least one transport mean free path is required before the injected light becomes diffusive. From transmission measurements we estimate $l=100$ μm.[4] Fitting the data by $C(qL)=\exp[-bq(L-l)]$, where the momentum difference is $q=2\pi\delta\theta/\lambda$, we find that for both the 380-μm sample and the 810-μm sample, $b=1.07$. Considering the likely magnitude of the various experimental errors, we estimate that the uncertainty in b is of order 10%, and that a value of unity would be consistent with the data.

The theory of FKLS predicts[1] that $C(qL)=[qL/\sinh(qL)]^2$, where it is assumed that $l\gg\lambda$ (weak localization regime) and the calculation is performed for a waveguide geometry with dimension W. Though the experiment has an open geometry we expect that as long as the beam diameter is much greater than the sample thickness the use of the calculation for the waveguide geometry (with W set to the beam diameter) should provide approximately correct results. Our data indeed support the major predictions of this theory as regards to the existence of the memory effect, the linear scaling of the half-width of $C(qL)$ with L, the scale of its half-width ($qL\sim 1$), and the exponential form for its asymptotic decay. There is one significant difference, however. The predicted form for $C(qL)$ always starts flat for small q, then rolls over, and finally becomes asymptotic to an exponential decay. The experimental data, on the other hand, do not exhibit a flat region, but begin an immediate exponential falloff. At present it is not clear if the differences between theory and experiment are simply due to the fact that the theory is written for a waveguide geometry, rather than for unconstrained optical propagation, or whether there is some additional mechanism for memory loss that needs to be added.

In Fig. 3 we plot the results for reflection. These data were obtained by rotating the sample. In order to avoid specular reflections, we used an asymmetric geometry in which the speckle pattern was measured at an angle of "reflection" $\theta_r=0$ (i.e., along the surface normal), while the angle of incidence of the laser beam was $\theta_i=30°$. If the penetration depth of the light is assumed to be negligibly small compared to the beam diameter, the angle of rotation of the speckle pattern, $\delta\theta_r$, is related to the angle of rotation of the sample, $\delta\theta$, by

$$\delta\theta_r=[1+\cos(\theta_i)/\cos(\theta_r)]\delta\theta\,.$$

We found experimentally that the peak of the correlation function followed this form to within the half-pixel experimental uncertainty. Fitting an expression of the form $C(ql)=\exp(-aql)$ to the data, and again taking $l=100$ μm, we find that $a=3.0$.

FKLS did not give an expression for the correlation function in reflection. Extending their treatment to the reflected case yields

$$C(q,L,l) = \{L \sinh[ql] \sinh[q(L-l)]/ql(L-l)\sinh[qL]\}^2, \quad (1)$$

where, as before, $q = 2\pi\delta\theta/\lambda$. Although this function describes the major features of the data, the same differences between theory and experiment that we refound for transmission exist also for reflection.[5]

Why is L the smallest scale over which information is preserved upon transmission? We believe that a simple physical answer is that photon diffusion scrambles the incident waveform over this scale. If the photon trajectories were ballistic, then the incident waveform, superimposed upon the intrinsic random fluctuations, would be exactly replicated at the output face of the sample. This is essentially what happens for ground glass, and accounts for the near perfect degree of correlation found experimentally. But in thick samples the trajectories are diffusive, with approximately equal probabilities for diffusing in any direction. Accordingly, light injected at a given point on the input face ends up being spread out over an area of diameter $2L$ on the output face. Thus, photon diffusion scrambles the phase of the incident wave on this scale, leading to the observed loss of correlation. Could an arbitrary incident waveform be reconstructed (with limiting resolution L) if a suitable set of reference speckle patterns were available? Both theory and experiment suggest that the information needed to do so is actually preserved during transmission through the random medium, thereby holding out the possibility, in principle at least, of imaging through visually impenetrable objects.[6]

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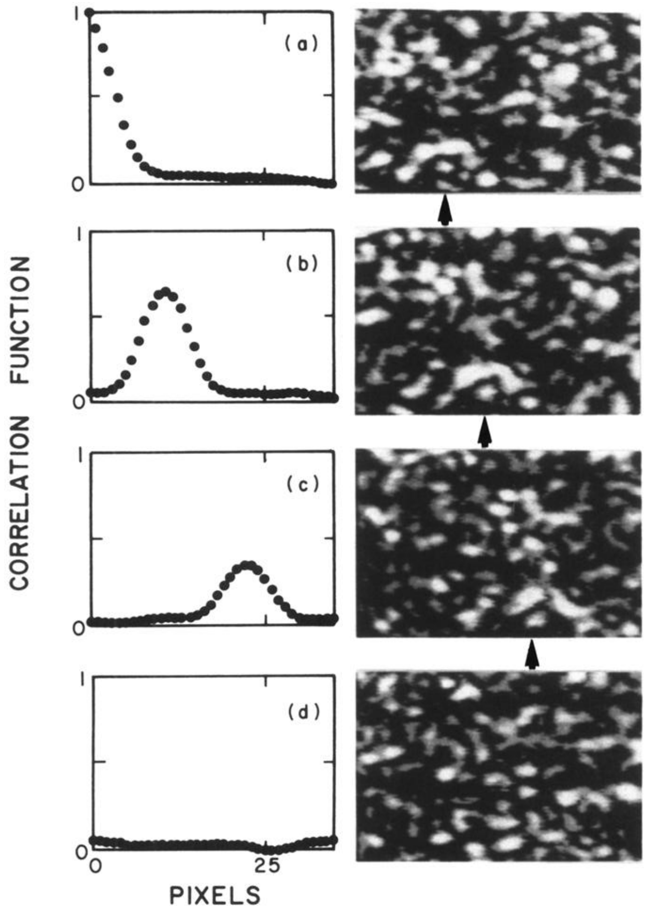
[2]M. J. Stephen and G. Cwillich, Phys. Rev. Lett. **59**, 285 (1987).

[3]B. Shapiro, Phys. Rev. Lett. **56**, 1809 (1986).

[4]The 19-μm transport mean free path of 0.46-μm-diam polystyrene spheres in a 10% (volume ratio) concentration was used as a reference. [E. Akkermans, P. E. Wolf, and R. Maynard, Phys. Rev. Lett. **56**, 1471 (1986)].

[5]Intensity correlation functions for transmission and reflection are considered in P. A. Mello, E. Akkermans, and B. Shapiro, Phys. Rev. Lett. **61**, 459 (1988), in the limit of $L \rightarrow \infty$. In this limit the memory effect does not appear in their results.

[6]I. Freund and M. Rosenbluh, to be published. Suitable imaging methods are related to those employed in stellar speckle interferometry [J. C. Dainty, *Laser Speckle and Related Phenomena* (Springer-Verlag, Berlin, 1984)].



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