Quantum Beamstrahlung: Prospects for a Photon-Photon Collider

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In this paper the treatment of radiation in pulse-pulse collisions relevant to high-energy electron-positron colliders is extended to multiple-photon processes. The resultant multiple-photon spectrum is hard. We discuss the possibility of a photon-photon collider utilizing this radiation. The expected flux is calculated and turns out to be from 1 to 2 orders of magnitude larger than that of the conventional virtual-two-photon source, depending upon machine parameters. Photon-electron processes are also enhanced by this additional hard-photon source.

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We extend our previously published studies[1] of quantum beamstrahlung in the one-photon approximation (see also Refs. 2–4) by including the contributions of multiple-photon emission during the mutual traversals of pulses composed of electrons and positrons.[5] The new results substantially alter the spectral distribution of radiated photons and scattered $e^{\pm}$ but they have little effect on the average fractional energy loss from the particle beams (the beamstrahlung loss) calculated in the one-photon approximation (in fact, it *decreases*). Of particular interest, the equivalent luminosity for collisions of real, high-energy photons is considerably larger than the familiar result for two virtual photons. This is the case for linear colliders in the 500-GeV–1-TeV energy range now being studied and viewed as technically practical.[6–8]

This result introduces a new dimension into the design and application of $e^{\pm}$ linear colliders. For constant luminosity, the beam pulses can be shaped as desired in order to either maximize or minimize the fields and the resulting beamstrahlung (i.e., radiation due to beam-beam interaction). The beamstrahlung can be reduced by forming beams with thin ribbonlike cross sections, permitting study of e^+e^- interactions at maximum energy and with narrow energy spread. Oppositely, in order to study high-energy photon-photon processes, the collider can be tuned to maximize beamstrahlung by shaping the beam pulses to more nearly circular cross sections. After describing the calculation we will illustrate the potential for producing Higgs bosons in the 100–200-GeV mass range based on predictions of the standard model.

We work in the rest frame of the uniformly charged pulse of N positrons using the same approach as found in Ref. 1. The pulse length is $L=l_0\gamma$ where l_0 is its length in the colliding beam (laboratory) frame, and $E=\gamma m$ is the beam energy in the laboratory. The cross section of the pulse is elliptical with semimajor and semiminor axes a_x, a_y, and area $A=\pi a_x a_y$. It is convenient to introduce the aspect ratio $G\equiv(a_x+a_y)/2(a_x a_y)^{1/2}\geq 1$. As discussed in Ref. 1 our analysis is restricted to small values of the disruption parameter, which is the expansion parameter for our quantum treatment. Small disruption means that there is only a small fractional change in the value of the electron's impact parameter as it crosses the positron pulse. Furthermore, for all cases of interest (i.e., foreseeably realistic) the incident electrons are characteristically separated by transverse distances that are large relative to the dimension of their individual radiation patterns, i.e.,

$$\frac{A}{N}\sim\frac{1\,\mu\text{m}^2}{10^{10}}=10^{-18}\,\text{cm}^2\gg\frac{1}{m^2}\sim 10^{-21}\,\text{cm}^2,$$

and hence can be treated incoherently. The pulse effects, however, must be treated coherently. To that end, denote by $T(x,z)dl\,dx$ the probability that an electron with momentum fraction z will radiate a photon with momentum fraction $z-x$ and end up within dx of x after traversing a fraction dl of the pulse length $(0<l<1)$. In the approximation of single-photon emission

$$\delta(1\text{ photon})=\int_0^1 dx(1-x)T(x,1) \tag{1}$$

is the fractional energy loss—or beamstrahlung—and $(1-x)T(x,1)$ is the one-photon power spectrum ($d\delta/dx$ in Ref. 1). We also introduce

$$p(z)dl\equiv\int_0^z dx\,T(x,z)dl\,, \tag{2}$$

which is the probability that the electron with z radiates one photon while traversing dl.

We now demonstrate that it is possible to proceed classically in calculating multiple-photon emission. Theoretically a pulse can be chopped into thin slices, each of thickness $L/2y$, with $y\equiv(\alpha/m)(\pi\mathcal{L})^{1/2}$, where $\mathcal{L}$ is the luminosity per pulse. For future colliders of interest in the TeV energy range, $y\sim 100$–1000. The significance of $L/2y$ is that it is the length of the electron's trajectory over which it acquires a transverse momentum $\sim m$ from the electric field of the pulse (it defines the transverse coherence length, $l_\perp$).[1] Radiation from successive slices will therefore be incoherent since there will be no overlap. Furthermore, each slice is sufficiently thin that there is negligible probability for radiating more than one photon per slice.

Treating each slice of the pulse of thickness $l/2y$ as differentially small we simply construct the rate equation $P_e(x,l)$, defined as the probability of finding an electron with momentum fraction x at fractional depth l within the pulse:

$$P_e(x,l+dl)-P_e(x,l)$$
$$=dl\int_x^1 dz\,T(x,z)P_e(z,l)-dl\int_0^x dz\,T(z,x)P_e(x,l)\,,$$

or

$$\frac{dP_e(x,l)}{dl}=\int_x^1 dz\,T(x,z)P_e(z,l)-p(x)P_e(x,l)\,, \qquad (3)$$

where the two terms on the right-hand side are the standard "source" and "sink" contributions that represent falling into and out of the momentum fraction x as a result of radiating a photon.

Analogously, the rate equation for the probability of finding a photon with momentum fraction v at depth l is

$$\frac{dP_{\rm ph}(v,l)}{dl}=\int_z^1 dx\,T(x-v,x)P_e(x,l)\,. \qquad (4)$$

Equations (3) and (4) include only the first generation of shower development; neglected is subsequent pair conversion of the high-energy radiation formed in the pulses.

The fractional energy loss including the effects of all photon emission can be written in the equivalent forms

$$\delta({\rm total})=\int_0^1 dx(1-x)P_e(x,1)=\int_0^1 dv\,vP_{\rm ph}(v,1)\,. \qquad (5)$$

It is interesting to note that one expects that δ(total) will be somewhat smaller than δ(1 photon) for the same parameter values.[9]

Finally we define the photon flux through the pulse

$$F(v)\equiv\int_0^1 dl\,P_{\rm ph}(v,l)\,. \qquad (6)$$

The equivalent luminosity for beamstrahlung photon-photon collisions at total mass W is

$$\frac{d\mathcal{L}_{\gamma\gamma}}{dW^2}=\int_0^1 dv_1\int_0^1 dv_2 F(v_1)F(v_2)\delta(sv_1v_2-W^2)\,,$$

or

$$\frac{d\mathcal{L}_{\gamma\gamma}}{dz}=2z\int_{z^2}^1\frac{dv}{v}F(v)F\left[\frac{z^2}{v}\right]\,, \qquad (7)$$

where $z^2=W^2/s=W^2/4E^2$.

Equations (3) and (4) can be integrated numerically in terms of known single-photon radiation probabilities. Figure 1 shows the resulting spectra which depend only on momentum fraction x and the scaling varaibles y and $D\equiv CG$, where $C=ml_0/4\gamma y\propto l_\perp/l_z$, which also describe the one-photon approximation.[1]

$F(v)$ is to be compared with the equivalent virtual-photon flux, which takes the simple form for $v\ll 1$,

$$n_\gamma(v)\simeq\frac{\alpha}{\pi}\frac{1}{v}\left[\ln\frac{s}{m^2v^2}-1\right]\,. \qquad (8)$$

The luminosity spectrum $d\mathcal{L}_{\gamma\gamma}/dz$ is shown in Figs. 2 and 3 and compared with that computed from the virtual-photon spectrum for a specific but interesting choice of machine parameters for a ~1-TeV collider[6]:

$$C=1.5 \text{ and } y=400,$$
$$\text{or } \mathcal{L}\sim 0.7\times 10^{30}\ {\rm cm}^{-2} \text{ and } l_0\sim 0.9\ {\rm mm}\,,$$
$$C=0.5 \text{ and } y=800, \qquad (9)$$
$$\text{or } \mathcal{L}\sim 2.8\times 10^{30}\ {\rm cm}^{-2} \text{ and } l_0\sim 0.3\ {\rm mm}\,.$$

The sensitivity to the beam geometry is illustrated by the three curves plotted in Fig. 2 corresponding to different values of the pulse aspect ratio, $G=1$, 2, and 5, respectively. The latter value corresponds to a flat beam with aspect ratio $a_y/a_x=100$ and its beamstrahlung

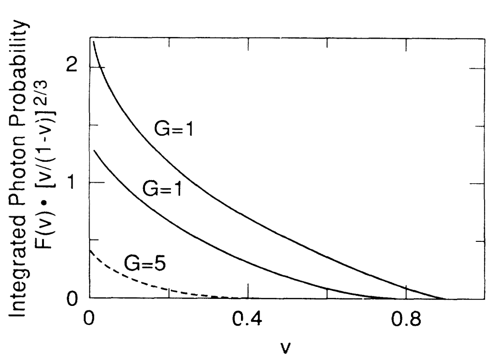

FIG. 1. The photon flux integrated through the pulse as defined in Eq. (6) in the text. The upper and lower solid curves are for $C=0.5$, $y=800$ and $C=1.5$, $y=400$, respectively, for $G=1$. The dashed curve is for the latter case with a ribbon beam, $G=5$.

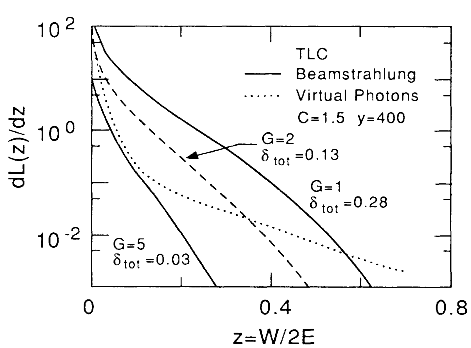

FIG. 2. The photon-photon luminosity relative to the incident electron-positron flux plotted for the case $C=1.5$ and $y=400$. The effect of pulse shaping is illustrated by the three values of G. The virtual-two-photon flux also plotted as the dotted line for a collider energy of 1 TeV.

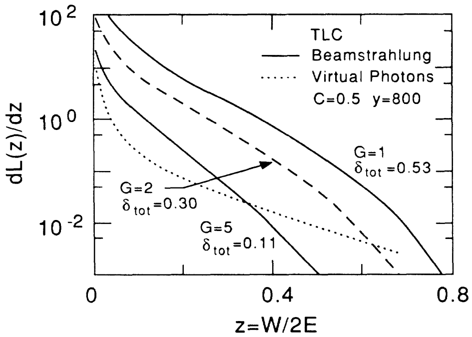

FIG. 3. The same as Fig. 2 but for machine parameters of $C=0.5$ and $y=800$. The increase in luminosity is evident.

luminosity is smaller than that due to the well-known virtual photons for all values of the photon-photon mass for the parameters shown. However, for higher values of the luminosity per pulse, the situation changes. In Fig. 3, the beamstrahlung photon-photon flux for $G=5$ is larger by a factor of 5-10 for $W\sim(0.1-0.25)\sqrt{s}$. The curve for $G=2$ corresponds to $a_y/a_x=15$. The virtual-photon curves are drawn for a 1-TeV collider[10]; the beamstrahlung curves are independent of s.

Thus depending on specific design parameters, $\mathcal{L}_{\gamma\gamma}$ can be made negligible or can be made to dominate the two-photon flux in future high-energy colliders. The large ratio to the virtual-photon luminosity shown in Figs. 2 and 3 suggests naturally the potential for linear colliders operated in a mode to maximize beamstrahlung to explore relatively rare reactions initiated by the two photons as well as the more copious hadronic states that they produce. One example is the search for a neutral Higgs boson via the coupling $\gamma\gamma H_0$.

The single neutral Higgs boson of the standard model should be produced copiously in hadron-hadron collisions[11] but will be very difficult to pull out of the large backgrounds if its mass lies below the W^+W^- or Z^0Z^0 threshold.[12] In the mass region 100–200 GeV it will be necessary to rely on e^+e^- colliders. The process of W fusion,

$$e^+e^-\to(W^+W^-)\nu\bar{\nu}\to H_0\nu\bar{\nu}, \tag{10}$$

leads to an observable signal for high-luminosity colliders[13]:

$$\begin{aligned}\sigma_{W\,\text{fusion}}&\sim1.5\sigma_{\text{point}}\text{ for a 1-TeV collider with }m_{H_0}=200\text{ GeV},\\ \sigma_{W\,\text{fusion}}&\sim0.3\sigma_{\text{point}}\text{ for a 600-GeV collider with }m_{H_0}=120\text{ GeV},\end{aligned}\tag{11}$$

where

$$\sigma_{\text{point}}=4\pi\alpha^3/3s=(86.8\times10^{-39}\text{ cm}^2)/[s\text{ (TeV}^2)]\,.$$

These predictions correspond to ten and six H_0's per day, respectively, for a luminosity of 10^{33} cm^{-2} sec^{-1}.

Beamstrahlung photons also lead to H_0 production in the standard model that can be estimated theoretically.[14] The predictions depend on collider parameters through the $\mathcal{L}_{\gamma\gamma}$ flux and the coupling $H_0\to2\gamma$ which is sensitive to theoretical details, including the mass ratio of the heavy lepton and the W to the Higgs boson.

Using the definition of $\mathcal{L}_{\gamma\gamma}$,

$$\frac{d}{dz}\sigma_{\text{beamstrahlung}}(e^+e^-\to e^+e^-X)=\left[\frac{d}{dz}\mathcal{L}_{\gamma\gamma}\right]\sigma_{\gamma\gamma\to X}(W)\,, \tag{12}$$

we find in the narrow-resonance approximation for the H_0,

$$\sigma_{\text{beamstrahlung}}(H_0)=\left[z\frac{d}{dz}\mathcal{L}_{\gamma\gamma}\right]_{z=m_{H_0}/\sqrt{s}}\left[\frac{4\pi^2\Gamma(m_{H_0}\to2\gamma)}{m_{H_0}^3}\right]. \tag{13}$$

This gives approximately $\sigma_{\text{beamstrahlung}}(H_0)\sim10^{-37}$ cm^2 with use of $\Gamma(m_{H_0}\to2\gamma)\sim5$ keV for Higgs-boson masses in the 100–200-GeV mass range and $d\mathcal{L}_{\gamma\gamma}/dz\sim10$ for $z=0.2$ from Fig. 3. This corresponds to roughly 100 H_0's per day for a collider luminosity of 10^{34} cm^{-2} sec^{-1}.

This example is used only to suggest that two-photon reactions via beamstrahlung can be significantly larger than previously appreciated on the basis of virtual-photon processes.[15] Note also that the beamstrahlung source works with electron-electron as well as positron-electron colliders.[16] These hard photons may lead to troublesome backgrounds or, conversely, raise the attractive possibility of doing interesting new physics with linear colliders in the 500-GeV–1-TeV energy range that is now being studied.[13] The colliders can be tuned by beam shaping to minimize or to maximize the beamstrahlung contribution as demanded by experiment.

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[9]This arises from the energy loss during the traversal of the pulse. For the two cases plotted in Figs. 2 and 3, the ratio $\delta(\text{total})/\delta(1\text{ photon})$ for $G=1$ is 0.8 and 0.7, respectively. A fuller discussion of this effect, as well as more details of our calculation, is now being prepared for publication.

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