Electromagnetic Structure of the Pion and the Kaon

Véronique Bernard[(a)] and Ulf-G. Meissner

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 3 March 1988; revised manuscript received 23 May 1988)

We study the electromagnetic structure of the pseudoscalar Goldstone bosons (π,K) in the framework of a generalized $SU(3)_f$ Nambu-Jona-Lasinio model. Including vector-meson contributions beyond the Hartree-Fock approximation the q^2 dependence of the electromagnetic form factors in the spacelike region is well reproduced. We also find that $\langle r_{K^+}^2\rangle < \langle r_{\pi^+}^2\rangle$, in agreement with the data.

PACS numbers: 13.40.Fn, 11.30.Rd, 12.40.Au, 14.40.Aq

The pion (π) and the kaon (K) play a special role in the understanding of low-energy strong-interaction physics. They are the Goldstone bosons of broken SU(3) flavor symmetry (together with the η), and their couplings to other hadrons are determined by chiral-symmetry constraints. A realistic model to describe the spontaneously broken chiral symmetry of the strong-interaction vacuum is the Nambu-Jona-Lasinio (NJL) model.[1] Its generalization[2] to $SU(3)_f$ accounts for the properties of the pseudoscalar Goldstone bosons (π,K,η) as well as the η' and gives insight into the effects of flavor mixing. It can furthermore be extended to describe the properties of mesons at finite temperature and density, the quark condensates, and the chiral-restoration phase transition.[3]

Recently, the electromagnetic form factors of the π and the K have been measured very accurately[4] by direct scattering of pions and kaons from electrons at the CERN SPS in the spacelike q^2 range 0.015-0.20 GeV/c^2. The values for the radius of the charged pion and kaon extracted from a constrained best fit to the form factor data are $\langle r_p^2\rangle^{1/2}=0.663\pm0.006$ fm and $\langle r_K^2\rangle^{1/2}=0.58\pm0.04$ fm. The systematic error is minimized for the ratio $|F_K|^2/|F_\pi|^2$ and gives $\langle r_\pi^2\rangle-\langle r_K^2\rangle=0.10\pm0.045\ \mathrm{fm}^2$. It is our aim in this Letter to calculate these charge radii and form factors at low q^2 within the framework of a realistic $SU(3)_f$ Nambu-Jona-Lasinio model[2] and to test its limits of applicability.

The form factors $F_\pi(q^2)$ and $F_K(q^2)$ measure the modification from a pointlike interaction of the photon-pseudoscalar-meson (γP) vertex. To calculate them, consider the generalized $SU(3)_f$ NJL Lagrangian,[2]

$$\mathcal{L}=\bar\psi(i\partial\!\!\!/-\hat m)\psi+G\sum_{a=0}^{8}[(\bar\psi\lambda^a\psi)^2+(\bar\psi i\gamma_5\lambda^a\psi)^2]-K\{\det[\bar\psi(1+\gamma_5)\psi]+\det[\bar\psi(1-\gamma_5)\psi]\}\,, \tag{1}$$

with G and K coupling constants of dimensions $[\mathrm{mass}]^{-2}$ and $[\mathrm{mass}]^{-5}$, respectively. λ^a are the usual Gell-Mann $SU(3)_f$ generators, and $\hat m=\mathrm{diag}(m_u,m_d,\ m_s)$ is the current quark mass matrix. In what follows, we will only consider the isospin symmetric case $m_u=m_d$. The last term in Eq. (1) is a determinant in flavor space, i.e., a six-fermion interaction term. It breaks the global $U(1)_A$ symmetry, therefore giving the $\eta\eta'$ mass splitting. The form of this term is motivated by 't Hooft's instanton contribution which breaks $U(1)_A$ in QCD.[5] Since Eq. (1) constitutes an effective quark Lagrangian, loop integrals are divergent and have to be regularized. For that, we choose a Euclidean four-momentum cutoff $\theta(\Lambda^2-k_E^2)$. Fixing the quark masses m_u and m_s as given by Gasser and Leutwyler,[6] the remaining three parameters $G'(G,K,\Lambda)$ can be determined by an overall best fit to the pseudoscalar masses, the decay constants of the pseudoscalars, and the vacuum expectation values of the quark condensates, the latter being the order parameters of the spontaneously broken chiral symmetry.

In the broken phase, the quarks acquire a constituent mass (M_i) which is different from the current mass (m_i). The former is generated nonperturbatively in the Hartree-Fock approximation, which leads to a momentum-independent self-energy. All the dynamics are determined by the propagator of the constituent quarks, it reads

$$\begin{aligned}S(k)&=S_0(k)\lambda_0+S_8(k)\lambda_8\\&=\mathrm{diag}[S_u(k),S_u(k),S_s(k)]\,,\end{aligned} \tag{2}$$

with $[S_{u,s}(k)]^{-1}=k\!\!\!/-M_{u,s}$. In Fig. 1(a), we show the leading contribution to the photon-pseudoscalar $(\pi\gamma,K\gamma)$ vertex in the Hartree-Fock approximation. Because of the Ward identity,[1]

$$(p'-p)_\mu\Gamma^\mu_{\gamma P}(p',p)=i[QS^{-1}(p')-S^{-1}(p)Q]\,, \tag{3}$$

with $Q=\frac{1}{2}e(\lambda_3+\lambda_8/\sqrt{3})$ the SU(3) charge operator, no other diagram than the one shown in Fig. 1(a) contributes to the meson form factor in the mean-field approximation. Note that the Lagrangian (1) does not allow for anomalous magnetic couplings since the Fierz transforms of the two four-fermion terms give the same contribution to $\sigma_{\mu\nu}\sigma^{\mu\nu}$ up to a sign.[7] The form factor $F_P(q^2)$

$(P=\pi,K)$ can be easily read off from Fig. 1(a):

$$e(p+p')_\mu F_P(q^2)=g^2_{P_{qq}}\{\mathrm{Tr}[\gamma_\mu Q S(k-\tfrac{1}{2}p')\gamma_5 I_P S(k+\tfrac{1}{2}p')\gamma_5 I_P^\dagger S(k-\tfrac{1}{2}p'-q)] + \mathrm{Tr}[\gamma_\mu Q S(k+\tfrac{1}{2}p'+q)\gamma_5 I_P^\dagger S(k-\tfrac{1}{2}p')\gamma_5 I_P S(k+\tfrac{1}{2}p')]\}. \quad (4)$$

I_P are the SU(3) isospin matrices for the pseudoscalar mesons $[I(\pi^+)=(\lambda_1+i\lambda_2)/\sqrt{2};\ I(K^+)=(\lambda_4+i\lambda_5)/\sqrt{2}]$ and $g_{P_{qq}}$ are the pseudoscalar-quark coupling constants, subject to the Goldberger-Treiman relation $g_{P_{qq}}=M/f_P$ (in the chiral limit). The "Tr" denotes traces over color, spin, and isospin, as well as the four-momentum integration $\int d^4k/(2\pi)^4$. Obviously, the kaon form factor can be split into an isoscalar and an isovector part. The form factor $F_P(q^2)$ is of course divergent if one does not regularize the three propagators in (4). Charge conservation then fixes the cutoff, which does not necessarily agree with the cutoff in the two-point functions (cf. the discussion in Ref. 2). The charge radius of the pseudoscalar P becomes

$$\langle r_P^2\rangle = -6\frac{dF_P(q^2)}{dq^2}\bigg|_{q^2=0}. \quad (5)$$

$\langle r_P^2\rangle$ is, however, finite even in the limit $\Lambda\to\infty$. Along these lines, our result should agree with the one-loop calculation of Tarrach[8] for the same values of f_π, f_K, and the ratio of the constituent quark masses.

In the Hartree-Fock approximation discussed so far, one leaves out some important physics, namely, the coupling of the photon through intermediate vector-meson states. To incorporate them consistently, one would have to introduce a momentum-dependent self-energy $\Sigma(p)=M(p)+\gamma p B(p)$. Alternatively, one can get guidance from the old vector-meson-dominance picture,[9] in which the pion and the isovector kaon form factors are dominated by ρ-meson intermediate states, and the isoscalar kaon form factor by intermediate ω and ϕ states.[10,11] These contributions are shown in Fig. 1(b). The contribution to the form factor $F_P(q^2)$ from the diagram [1(b)] is given by

$$e(p+p')_\mu \tilde{F}_P(q^2)=2g^2_{P_{qq}}\mathrm{Tr}[\gamma_\mu Q S(p+\tfrac{1}{2}q)\gamma_\rho I_V S(p-\tfrac{1}{2}q)]P_V(q^2) \times\{\mathrm{Tr}[\gamma_\rho I_V S(k-\tfrac{1}{2}p')\gamma_5 I_P S(k+\tfrac{1}{2}p')\gamma_5 I_P^\dagger S(k-\tfrac{1}{2}p'-q)] + \mathrm{Tr}[\gamma_\rho I_V S(k+\tfrac{1}{2}p'+q)\gamma_5 I_P^\dagger S(k-\tfrac{1}{2}p')\gamma_5 I_P S(k+\tfrac{1}{2}p')]\}, \quad (6)$$

with $V=\{\rho,\omega,\phi\}$ and I_V is the isospin matrix for the vector mesons, $I_\rho=\lambda_3$, $I_\omega=(\sqrt{2}\lambda_0+\lambda_8)/\sqrt{3}$, and $I_\phi=(\lambda_0-\sqrt{2}\lambda_8)/\sqrt{3}$, i.e., we consider ideal mixing for the $\omega\phi$ system. m_V is the vector-meson mass ($m_\rho=m_\omega=770$ MeV, $m_\phi=1020$ MeV) and $g_{V_{qq}}$ its coupling to the quarks. The momentum-dependent vector-meson propagation function $P_V(q^2)$ follows from a straightforward summation of ladder diagrams in the channels with the quantum numbers of ρ, ω, and ϕ mesons, respectively. Close to the vector-meson pole $q^2=m_V^2$, $P_V(q^2)$ reads

$$P_V(q^2\approx m_V^2)=g_{V_{qq}}(q^2)(m_V^2-q^2)^{-1}, \quad (7)$$

i.e., it can be parametrized as a propagating vector meson. For $q^2\not\approx m_V^2$, the momentum dependence of $P_V(q^2)$ is, however, more complicated. This accounts for the fact that one does not deal with mesons as fundamental (pointlike) but rather composite objects. At the vector-meson poles, we have $g_{\rho qq}=g_{\omega qq}=6$ and $g_{\phi qq}=2g_{\rho qq}$,[12] close to the predictions of broken flavor $\mathrm{SU}(3)_f$.[13] Evaluating the trace Tr in Eq. (6), one has to perform a subtraction to ensure gauge invariance leading to $\tilde{F}_P(0)=0$. This is the well-known problem connected to the self-energy of the photon in QED, which has already been stressed by Nambu and Jona-Lasinio.[1] Stated otherwise, $\tilde{F}_P(q^2)$ does not renormalize the charge. The total form factor is given by $F_P^{\mathrm{tot}}(q^2)=F_P(q^2)+\tilde{F}_P(q^2)$. The charge radius follows from Eq. (5) with $F_P(q^2)$ substituted by $F_P^{\mathrm{tot}}(q^2)$.

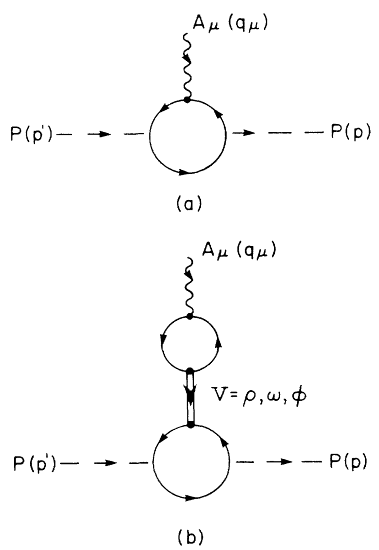

FIG. 1. The photon–pseudoscalar-meson vertex. (a) The contribution in the Hartree-Fock approximation, with $P(p)$ denoting pseudoscalar mesons (π,K) of momentum p. The photon field A_μ carries momentum q_μ. (b) The leading contribution beyond the Hartree-Fock approximation via coupling through intermediate vector-meson (V) states.

Let us now discuss the results. For the parameters of Ref. 2, which gives an overall good fit to m_π, m_K, m_η,

$m_{\eta'}$, f_π, f_K, $\langle\bar{u}u\rangle_0$, and $\langle\bar{s}s\rangle_0$, i.e., $G\Lambda^2=3.43$, $K\Lambda^5=46.3$, $\Lambda=1$ GeV, $m_u=6$ MeV, and $M_s=200$ MeV, the square of the pion factor $|F_\pi(q^2)|^2$ for $0\leq q^2\leq 0.2$ $(\mathrm{GeV}/c)^2$ and the square of the kaon form factor $|F_K(q^2)|^2$ for $0\leq q^2\geq 0.1$ $(\mathrm{GeV}/c)^2$ are shown in Figs. 2 and 3, respectively. For comparison, we also show the data from Ref. 4 and their best constrained fit $[F_{\pi,K}(0)=1]$ (dashed line in Fig. 2). For the kaon form factor, the prediction of the NJL model and the best fit of Ref. 4 agree (on the scale of this figure). The pion radius, $\langle r_{\pi^+}^2\rangle^{1/2}=0.66$ fm, comes out of the experimental value, whereas the kaon radius, $\langle r_{K^+}^2\rangle^{1/2}=0.56$ fm, is underestimated by 4%. These values amount to $\langle r_\pi^2\rangle-\langle r_K^2\rangle=0.12$ fm^2, independent of m_s and close to the experimental value quoted in Ref. 4. The direct photon-quark contribution [Eq. (4)] for the pion amounts to an "intrinsic" charge radius $\langle r_\pi^2\rangle_{\mathrm{int}}=0.139$ fm^2 (for $m_s=200$ MeV), and the coupling via the ρ meson adds another 0.299 fm^2. In the case of the kaon, the situation is rather different. The intrinsic charge radius is $\langle r_K^2\rangle_{\mathrm{int}}=0.178$ fm^2, i.e., larger than the pion one. However, because of a destruction interference of isoscalar (ω,ϕ) and isovector (ρ) contributions, it is only increased by 0.136 fm^2 from the vector mesons [Eq. (6)]. The kaon radius is of course sensitive to the ratio of the vector-meson-quark coupling constants. Concerning the sensitivity of our results to parameter changes, we observe the following. The intrinsic pion radius can be written as $\langle r_\pi^2\rangle=(3/4\pi_\pi^2 f_\pi^2)f(\Lambda/m)$ with $f(\Lambda/m)$ a slowly varying function. As $\Lambda/m\rightarrow\infty$, we have $f(\infty)=1$ and thus we recover the values $\langle r_\pi^2\rangle^{1/2}=\sqrt{3}/2\pi f_\pi=0.58$ fm and similarly $\langle r_{K^+}^2\rangle=0.34$ fm^2 quoted by Tarrach[8] from the direct photon-quark coupling in the soft-pion limit. Note that in this case the vector-meson contribution vanishes. We should mention here that a purely mesonic theory with (vector) mesons as fundamental particles also describes well the pion and the kaon radii.[10] This is, however, not in contradiction to the model presented here since it uses different degrees of freedom in the underlying theory. In our model, constituent quarks and their interactions determine the properties of mesons, where as in pure meson theories quarks are integrated out. Also, the vector-meson-dominance principle used in the meson-theory calculations has never been derived from QCD.

Let us now compare our results with a recent phenomenological description of the pion form factor in the spacelike and timelike regions.[14] Brown, Rho, and Weise describe the pion form factor in a similar way as it is done here, namely, in terms of a pion core and a ρ-meson induced cloud contribution. The contribution from the pion core is phenomenologically parametrized with an intrinsic radius of $\langle r_\pi^2\rangle_{\mathrm{int}}^{1/2}=0.35$ fm, and should be the analog of the direct $\gamma\pi$ coupling to the model presented here. Our intrinsic radius of $\langle r_\pi^2\rangle_{\mathrm{int}}^{1/2}=0.37$ fm agrees well with theirs and the one obtained in a model which also includes confinement,[15] where the direct $\gamma\pi$ coupling leads to $\langle r_\pi^2\rangle_{\mathrm{int}}^{1/2}=0.42$ fm. The authors of Ref. 15 did not, however, consider the vector-meson contribution to the pion radius. Loosely spoken, we can say that our model calculation justifies the phenomenological parametrization of Brown, Rho, and Weise in Ref. 14.

In summary, we have calculated the charge form factors of the pion and the kaon at low q^2 within the framework of a generalized $\mathrm{SU}(3)_f$ Nambu-Jona-Lasinio model, with all parameters fixed and taken from a previous study of the low-lying pseudoscalar mesons.[2] The form factors receive important contributions from the direct quark-photon as well as vector-meson-photon coupling. They are in good agreement with experimental data.[4] We furthermore show that for a reasonable

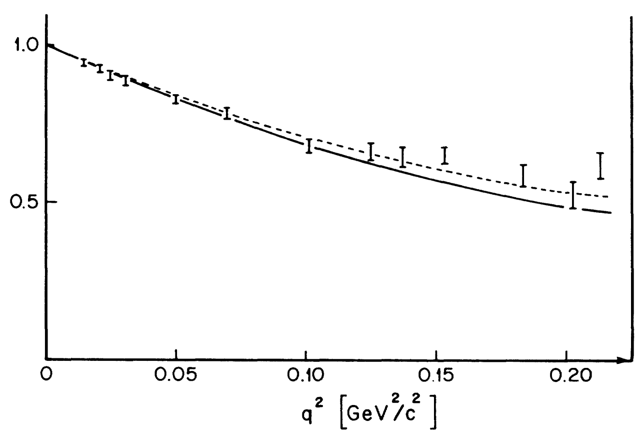

FIG. 2. The pion charge form factor $|F_\pi(q^2)|^2$ in the spacelike region for $q^2\leq 0.2$ $(\mathrm{GeV}/c)^2$. The solid line gives the prediction of our model, the data are taken from Ref. 4. The dashed line gives the best constrained fit of Ref. 4 with $\langle r_\pi^2\rangle=0.431$ fm^2.

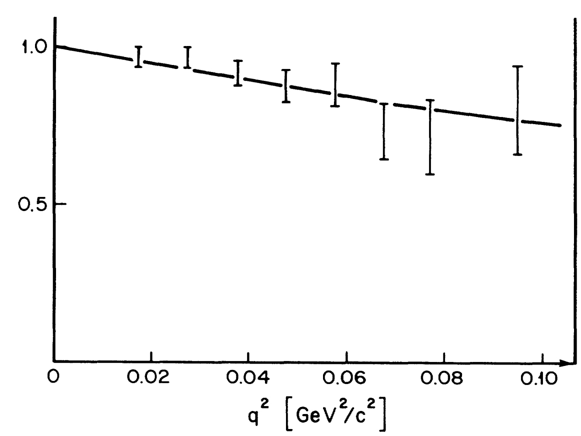

FIG. 3. The kaon charge form factor $|F_K(q^2)|^2$ in the spacelike region for $q^2\leq 0.1$ $(\mathrm{GeV}/c)^2$; notations otherwise as in Fig. 2. In this case, the best constrained fit of Ref. 4 is identical with the model prediction within the accuracy of the figure.

choice of the input parameters $\langle r_{K^+}^2 \rangle < \langle r_{\pi^+}^2 \rangle$, in agreement with experiment. These results indicate to us that a systematic evaluation of meson properties (such as, e.g., η decays, vector mesons, ...) in the generalized $SU(3)_f$ Nambu-Jona-Lasinio model should be pursued.

We thank W. Weise for a stimulating discussion. This work is supported in part by the U.S. Department of Energy under Contract No. DE-AC02-76ER03069.

[a]Permanent address: Groupe de Physique Nucléaire Théorique, Centre de Recherches Nucléaires, F-67037 Strasbourg CEDEX, France.

[1]Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961), and **124**, 246 (1961).

[2]V. Bernard, R. L. Jaffe, and U.-G. Meissner, Phys. Lett. B **198**, 92 (1987), and Nucl. Phys. **B308**, 753 (1988).

[3]V. Bernard, U.-G. Meissner, and I. Zahed, Phys. Rev. D **36**, 819 (1987), and Phys. Rev. Lett. **59**, 966 (1987); T. Hatsuda and T. Kunihiro, Phys. Lett. B **185**, 304 (1987), and **198**, 126 (1987).

[4]S. R. Amendolia *et al.*, Phys. Lett. **146B**, 116 (1984), and **178B**, 435 (1986), and Nucl. Phys. **B277**, 168 (1986).

[5]G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).

[6]J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[7]The inclusion of such terms is recently under investigation by A. Blin, B. Hiller, and M. Schaden, Regensburg Report No. TPR-87-25, 1987 (to be published).

[8]R. Tarrach, Z. Phys. C **2**, 221 (1979).

[9]J. J. Sakurai, *Currents and Mesons* (Univ. Chicago Press, Chicago, 1969).

[10]N. Zovko, Phys. Lett. **51B**, 54 (1974).

[11]J. Schechter, Phys. Rev. D **34**, 868 (1986).

[12]For a similar calculation in the framework of the bosonized NJL model compare D. Ebert and H. Reinhardt, Nucl. Phys. **B271**, 188 (1986), and Phys. Lett. B **173**, 453 (1986).

[13]U.-G. Meissner, Phys. Rep. **161**, 213 (1988).

[14]G. E. Brown, M. Rho, and W. Weise, Nucl. Phys. **A454**, 669 (1986).

[15]V. Bernard, R. Brockmann, and W. Weise, Nucl. Phys. **A440**, 605 (1985).