Possible Existence of Weyl's Vector Meson

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A quantum field theory for electroweak interaction and gravitational interaction with local scale invariance and local SU(2)⊗U(1) gauge invariance is proposed. The requirement of local scale invariance leads to the existence of Weyl's vector meson which absorbs the Higgs particle remaining in the Weinberg-Salam model.

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In these days when gauge invariance is employed in almost every branch of particle physics, is it not strange that the original idea of Weyl,[1] on which the concept of gauge-invariance sprang, is largely ignored? (Some of the works done on Weyl's theory are listed in Ref. 2–8.) As we recall, Weyl argued that the existence of a vector field is necessitated by local scale invariance, which is the invariance of the action under the change of the *magnitude*, not the *phase*, of the fields. There are, today, reasons which make it desirable to resurrect Weyl's vector meson.

(i) The Weinberg-Salam model[9] implies the existence of a Higgs meson so far unobserved experimentally. That this meson is not absorbed by the weak bosons is due to the fact that a gauge transformation alters only the phase, not the magnitude, of the Higgs field. Therefore, Weyl's vector meson would take care of precisely this unsatisfactory feature of the standard model: It absorbs the remaining degree of freedom—the magnitude of the Higgs field.

(ii) The Lagrangian for gravitation involves the gravitational constant $G \sim (10^{19} m_N)^{-2}$, which has a dimension. This is quite different from the coupling constants in strong, weak, and electromagnetic interactions, which are now known to be dimensionless. Related to this is the fact that R, the curvature scalar, is not scale invariant. As we shall see, this can be remedied by the introduction of Weyl's vector meson together with the use of the Higgs field in the Weinberg-Salam model.

Let us modify Einstein's Lagrangian $R/16\pi G$ to[10]

$$\tfrac{1}{2}\beta\phi^\dagger\phi\tilde{R}\,, \tag{1}$$

where ϕ is the same (isodoublet) Higgs field in the Weinberg-Salam model and $\tilde{R}$ is equal to R with the affine connection $\{{}^{\rho}_{\mu\nu}\}$ replaced by $({}^{\rho}_{\mu\nu})$, where

$$({}^{\rho}_{\mu\nu}) \equiv \tfrac{1}{2} g^{\rho\sigma}[(\partial_\mu + 2fS_\mu)g_{\nu\sigma} + (\partial_\nu + 2fS_\nu)g_{\mu\sigma} - (\partial_\sigma + 2fS_\sigma)g_{\mu\nu}] = \{{}^{\rho}_{\mu\nu}\} + f(S_\mu\delta^\rho_\nu + S_\nu\delta^\rho_\mu - S^\rho g_{\mu\nu})\,, \tag{2}$$

with S_μ denoting Weyl's vector meson. Note that the action of (1) satisfies local scale invariance. Thus the coupling constant f and β are both dimensionless. The Lagrangian term for the Higgs field ϕ is

$$g^{\mu\nu}(D_\mu\phi)^\dagger(D_\nu\phi)\,, \tag{3}$$

where

$$D_\mu\phi \equiv (\partial_\mu + \tfrac{1}{2} igW^i_\mu\sigma^i + \tfrac{1}{2} ig'B_\mu - fS_\mu)\phi\,, \tag{4}$$

which merely modifies the counterpart in the Weinberg-Salam model by the addition of the term $-fS_\mu\phi$. The Lagrangian for the Weyl's field as well as the gauge fields for the electroweak interaction is, as usual,

$$-\tfrac{1}{4} g^{\mu\rho}g^{\nu\sigma}(F^i_{\mu\nu}F^i_{\rho\sigma} + G_{\mu\nu}G_{\rho\sigma} + H_{\mu\nu}H_{\rho\sigma})\,, \tag{5}$$

where $F^i_{\mu\nu}$ and $G_{\mu\nu}$ are the usual field strengths for W^i_μ and B_μ, respectively, and

$$H_{\mu\nu} \equiv \partial_\mu S_\nu - \partial_\nu S_\mu\,.$$

The total Lagrangian L is equal to the sum of (1), (3), (5), and the terms for leptons and quarks, which we shall discuss later. The action for this L, equal to $\int d^4x\,|-g|^{1/2}L$, satisfies both local gauge invariance of SU(2)⊗U(1) and local scale invariance, the latter being the invariance under the transformation of

$$g^{\mu\nu} \to \Lambda^{-2}g^{\mu\nu}\,, \quad g_{\mu\nu} \to \Lambda^2 g_{\mu\nu}\,,$$
$$\phi \to \Lambda^{-1}\phi\,, \quad S_\mu \to S_\mu - f^{-1}\partial_\mu \ln\Lambda\,, \tag{6}$$

with W_μ and B_μ unchanged. In (6), Λ is a c function of $\mathbf{x}$ and t.

Since the action has gauge invariance and scale invariance, both the phase and the magnitude of ϕ can be chosen to be of any value. Therefore, the four degrees of freedom of the ϕ field can be transformed into the longitudinal degrees of freedom of the four vector mesons W^+, W^-, Z, and S. This is done as we replace ϕ by *precisely* $({}^0_v)/\sqrt{2}$, with no Higgs field remaining. Both gauge invariance and scale invariance are broken, and the mass scale for the physical world is set. More precisely, we set $v \simeq 250$ GeV, and $g_{\mu\nu} = \eta_{\mu\nu} + O(\sqrt{G})$. Then $W^\pm$ and Z acquire the same masses as in the

Weinberg-Salam model, the S meson acquires a mass as given by (9) below, the gravitational constant acquires a dimensional value[11]

$$G=(4\pi\beta v^2)^{-1}, \tag{7}$$

and the Higgs field disappears entirely. This may form a basis for the unification of the gravitational interaction with the electroweak interaction. Note that the present mechanism of generating a mass scale is not the same as the Higgs mechanism, although these two mechanisms share a resemblance. In particular, no Higgs potential is needed.

The terms in (1) which involve the Weyl field are

$$\tfrac{1}{2}\beta g^{\mu\nu}[6f^2\phi^\dagger\phi S_\mu S_\nu - 6fS_\mu(\phi^\dagger\partial_\mu\phi+\text{c.c.})]. \tag{8}$$

By setting $\phi=\binom{0}{v}/\sqrt{2}$ and $g^{\mu\nu}\simeq\eta^{\mu\nu}$ in (8), we find that Weyl's meson acquires a mass

$$M_S\simeq\left(\frac{3f^2}{4\pi G}\right)^{1/2}\simeq 0.5\times10^{19}f\ \text{GeV}. \tag{9}$$

[There is also a contribution to M_S^2 from the Lagrangian term (3). This contribution is about $(250f\ \text{GeV})^2$ and will be ignored.]

Let us next discuss the Lagrangian for a fermion field ψ. In general relativity, and with the SU(2)⊗U(1) gauge invariance taken into account, this Lagrangian is

$$\bar\psi i\gamma^c e_c^\mu[D_\mu - \tfrac{1}{2}\sigma_{ab}e^{b\nu}(\partial_\mu e_\nu^a - \{{}^{\rho}_{\mu\nu}\}e_\rho^a)]\psi, \tag{10}$$

where e_μ^a is the tetrad, and

$$D_\mu\psi=(\partial_\mu+igW_\mu^iT^i-\tfrac{1}{2}ig'YB_\mu)\psi, \tag{11}$$

with T^i the ith isospin matrix and Y the hypercharge for ψ. Under scale transformation, we have

$$\psi\to\Lambda^{-3/2}\psi,\quad e_\nu^a\to\Lambda e_\nu^a. \tag{12}$$

Therefore, to make the Lagrangian scale invariant, we should add a term $-\frac{3}{2}fS_\mu$ to D_μ in (11), and replace $\partial_\mu e_\nu^a$ and $\{{}^{\rho}_{\mu\nu}\}$ in (10) by $(\partial_\mu+fS_\mu)e_\nu^a$ and $({}^{\rho}_{\mu\nu})$, respectively. It turns out that all such additional terms of S_μ cancel.[5] Therefore, the Lagrangian for ψ is simply (10) and Weyl's vector meson does not interact with leptons or quarks. Neither does it interact with other vector mesons. The only interaction the Weyl's meson has is that with the graviton. It is interesting to ponder whether Weyl's meson may account for at least part of the dark matter of the universe.

We close with the following remarks.

(i) It is allowable to add a term $\lambda(\phi^\dagger\phi)^2$, which is scale invariant, to the Lagrangian. Upon setting $\phi=\binom{0}{v}/\sqrt{2}$, this term becomes $\lambda v^4/4$, which is identified as the cosmological constant. One may speculate whether the pure $\lambda\phi^4$ theory being a trivial field theory in four dimensions[12] has something to do with the cosmological constant being zero.

(ii) The coupling of ϕ to fermions is scale invariant and is hence also allowed in a scale-invariant theory. Therefore, in the present theory, quarks and leptons may derive masses in the same way as in the Weinberg-Salam model. The present theory, however, offers no clues to the hierarchy of lepton masses and quark masses, nor does it throw any light on why the gravitational interaction is so weak ($\beta^{-1}\approx10^{-32}$).

(iii) Consider the model in which a number of massless real scalar fields ϕ^a, $a=1,2,\ldots,N$ couple to Weyl's field. The Lagrangian term involving these scalar fields is

$$\tfrac{1}{2}g^{\mu\nu}(\partial_\mu-fS_\mu)\phi^a(\partial_\nu-fS_\nu)\phi^a. \tag{13}$$

Since such a theory satisfies local scale invariance a degree of freedom is redundant. This redundancy can be eliminated by the choice of a scale. Let us choose the scale so that

$$\phi^a(x)\phi^a(x)=v^2. \tag{14}$$

Then the term in (13) which is linear in the coupling constant f vanishes. The Lagrangian in (13) becomes

$$\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^a+\tfrac{1}{2}g^{\mu\nu}f^2v^2S_\mu S_\nu. \tag{15}$$

Note that the scalar fields are decoupled from Weyl's field. If we approximate $g^{\mu\nu}$ by $\eta^{\mu\nu}$, then (15) together with (14) is the nonlinear σ model for the scalar fields.

(iv) A major difficulty with the theory of gravitation is that it is not renormalizable. We do not expect that the addition of Weyl's field changes that. Indeed, even if we ignore the gravitational interaction as we did in (iii), the resulting theory for the scalar fields ϕ^a is the nonlinear σ model, which is not renormalizable. Similarly, neither is the Weinberg-Salam model, modified by the inclusion of Weyl's vector meson, renormalizable. Indeed, this modified model, with gravitation ignored, is just the massive Yang-Mills theory with masses satisfying Weinberg's relation, and is known to be not normalizable at the two-loop level. If we require all physical theories be renormalizable, then we must reject Weyl's theory. On the other hand, if we adopt the view that symmetry, not renormalizability, is the ultimate criteria, then Weyl's theory remains worthy of exploration, particularly at a time when the physical Higgs meson is still undetected. We also mention that since its one-loop diagrams are renormalizable, experimental consequences can be extracted from this theory, as long as two-loop diagrams give too small contributions for experimental purposes.

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[1]H. Weyl, Z. Phys. **56**, 330 (1929).

[2]D. K. Sen and K. A. Dunn, J. Math. Phys. **12**, 578 (1971).

[3]P. A. M. Dirac, Proc. Roy. Soc. London A **333**, 403 (1973); R. Utiyama, Prog. Theor. Phys. **50**, 2080 (1973), and **53**, 565 (1975).

[4]P. G. O. Freund, Ann. Phys. (N.Y.) **84**, 440 (1974).

[5]K. Hayashi, M. Kasuya, and T. Shirafuji, Prog. Theor. Phys. **57**, 431 (1977), and **59**, 681 (1978).

[6]J. Hayashi and T. Kugo, Prog. Theor. Phys. **61**, 334 (1979).

[7]M. Nishioka, Fortsch. Phys. **33**, 241 (1985).

[8]D. Ranganathan, J. Math. Phys. **28**, 2437 (1987).

[9]S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity,* edited by N. Svartholm, Nobel Symposium No. 8 (Wiley, New York, 1968), p. 369.

[10]The Lagrangian of (1) differs from that proposed by Dirac in Ref. 3 in only one aspect: The ϕ field is the isodoublet Higgs field of the Weinberg-Salam model, not a new scalar field as in Dirac's model.

[11]For another way to give the gravitational constant a dimension, see A. Zee, Phys. Rev. Lett. **42**, 417 (1979), where the Higgs mechanism is employed, see also A. Zee, Phys. Lett. **143B**, 368 (1984).

[12]K. G. Wilson, Phys. Rev. **84**, 3184 (1971); K. G. Wilson and J. Kogut, Phys. Rep. **12C**, 78 (1974); G. Parisi, Nucl. Phys. **B100**, 368 (1975); G. A. Baker, Jr., and J. Kincaid, Phys. Lett. **42**, 1431 (1979), and J. Stat. Phys. **24**, 4691 (1981).