Diagonal and Off-Diagonal Density Fluctuations in Hot Nuclear Matter, and Applications to Neutrino Transport in Supernovae

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We calculate the effects of interactions on the density fluctuations of an n,p,e plasma. The scattering of neutrinos arising from the Fermi part of the neutral current interactions with protons, under supernova conditions, is practically extinguished by Coulomb effects in the plasma. Charged-current reactions are discussed in terms of off-diagonal density fluctuations. Inclusion of the symmetry energy term in the nuclear interaction gives rise to a large reduction in the Fermi part of the cross section for $\nu+n\rightarrow e^-+p$.

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According to present theory the greater part of the lepton number of the presupernova core is trapped, ephemerally, in the collapse process.[1-3] Most of the lepton number then diffuses out, as electron neutrinos, within a few seconds after the collapse.[4-6] The excitation energy of the core is radiated over a similar (but not necessarily identical) time scale, through emission of all species of light neutrinos and antineutrinos. The rates of lepton-number loss and cooling are governed by the neutrino opacities in the interior of the core.[4] The spectrum depends on the fluxes, as determined by the solution to the interior problem, and also on the opacities of the outer layers, insofar as these opacities affect the radius of the neutrino "photosphere" and therefore the emitting area.

In current scenarios the outer layers of importance to the neutrino problem consist of nuclear matter in the density range (5×10^{12})–10^{14} g cm^{-3}, at a temperature (postshock) of 5 to 20 MeV. In this region of density and temperature, nuclei may (or may not) be totally dissociated.[7] In the present Letter we shall report new results for opacities for the case of dissociated nuclei, that is for the case of a gas of neutrinos, protons, electrons, and trapped neutrinos. The electrons remain degenerate throughout the region, but the neutrons, much of the time, and the protons, most of the time, are nondegenerate. The only interactions taken into account, beyond the weak interactions, will be the Coulomb interactions among the charged constituents, and a nuclear force term responsible for the "symmetry" energy in nuclear matter.

There are three observations which underly our treatment.

(a) The wavelengths of the neutrinos under consideration are a few times the interparticle spacing, in most of the regions of interest.

(b) The scattering of a neutrino of energy, say, $3\beta^{-1}=3k_BT$, is determined by the thermal fluctuations in the density, for the Fermi part, and the fluctuations in spin density, for the Gamow-Teller part of the neutral-current interaction. Consider first the neutral-current scattering of a ν in our medium (a medium containing one species of nucleon, for simplicity). Since the energy transferred to the nucleon is small, we can use closure to write an expression for the differential cross section per unit volume, from the Fermi interaction,[8]

$$V^{-1}\frac{d\sigma}{d\cos\theta}=(8\pi)^{-1}G_W^2E_\nu^2(1+\cos\theta)\rho_0S(\mathbf{q})\,, \quad (1)$$

where V denotes volume and where

$$S(\mathbf{q})=(\rho_0V)^{-1}\langle\rho(\mathbf{q})\rho(-\mathbf{q})\rangle\,. \quad (2)$$

Here $\mathbf{q}$ is the momentum transfer to the nucleons, $\langle\cdots\rangle$ denotes thermal average, and the operator, $\rho(\mathbf{q})$ is the Fourier transform of the nucleon density. The thermal average is to be taken over the states, $|i\rangle$, of the hot nuclear medium. ρ_0 is the average nucleon density and $S(\mathbf{q})$ is the static liquid structure factor. All effects of lepton degeneracy will be expressed as occupation factors multiplying the right-hand side of (1); it is the nucleon density-density correlation function which will be of concern to us here.

(c) The long-wavelength limit of the correlation function in (2) is given by the classical result,

$$\lim_{\mathbf{q}\rightarrow0}S(\mathbf{q})=\left(\beta\rho_0\frac{\partial^2F}{\partial\rho_0^2}\right)^{-1}=\beta^{-1}\rho_0K_T^{-1}\,, \quad (3)$$

where F is the free-energy density, and K_T is the isothermal bulk modulus of the medium.

To compare this approach to the conventional wisdom, of neutrino transport, viz. (mean free path)$^{-1}=\sigma_0\times\rho_0\times$(Fermi factors for degenerate species), where σ_0 is the cross section on an isolated nucleon, we note the following:

(a) For a noninteracting Boltzmann gas of nucleons the results of the two approaches are identical, when the ideal gas equation of state is used to determine K_T in (3).

(b) For the case of noninteracting degenerate nucleons we have the familiar result for the structure function, for the case of small q and small T,

$$S(\mathbf{q})=\tfrac{3}{4}qk_F^{-1}+3M_N(\beta k_F^2)^{-1}.$$

The second term on the right-hand side dominates the first, for a neutrino of momentum, e.g., $3\beta^{-1}$. This dominant term is exactly the classical result, (2), substituting the bulk modulus of a free Fermi gas. Thus, even in the degenerate limit, the classical term dominates. It is straightforward to include the effects of Coulomb interactions for the case of the Fermi part of the neutral-current scattering of a neutrino from the protons in the medium. To calculate the mean fluctuations of $\rho(q)$ we use a thermal fluctuation distribution

$$\mathrm{Prob}[\delta\rho(\mathbf{q})]=N\exp(-\beta\delta F), \tag{4}$$

where δF is the variation of the free-energy density of the protons in the system (the presence of neutrons being irrelevant at the moment), and is given by

$$\delta F_{\mathrm{small\,q}}=(2V)^{-1}\sum_{\mathbf{q}}\delta\rho(\mathbf{q})\delta\rho(-\mathbf{q})\left(\frac{\partial^2F_0}{\partial\rho_0^2}+\frac{4\pi e^2}{\mathbf{q}^2\epsilon(\mathbf{q},0)}\right). \tag{5}$$

The first term on the inside of the parenthesis gives the fluctuation energy of a noninteracting gas, and the second term takes into account the effects of the screened proton-proton interactions, following a Born-Oppenheimer treatment of the electrons. We take the long-wavelength limit, in which the relativistic Thomas-Fermi screening momentum determines the dielectric constant,

$$\mathbf{q}^2\epsilon(\mathbf{q},0)=\frac{12\pi}{(3\pi^2)^{1/3}}e^2\rho_0^{2/3}\equiv 2\pi e^2b^{-1}. \tag{6}$$

The structure factor is determined from (2), (4), (5), and (6):

$$\lim_{\mathbf{q}\to 0}S(\mathbf{q})=\beta^{-1}\left(\rho_0\frac{\partial^2F_0}{\partial\rho_0^2}+2b\rho_0\right)^{-1}, \tag{7a}$$

$$S(0)_{\mathrm{ND}}=(1+2b\beta\rho_0)^{-1}, \tag{7b}$$

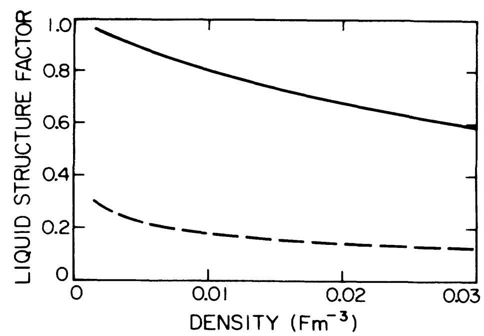

FIG. 1. The solid curve is the structure function, $S(0)$, for noninteracting protons, for the case $T=10$ MeV; it differs from unity only because of the Pauli-principle effects. The dashed curve is $S(0)$ from (7a), where the Coulomb effects are included.

where we have evaluated in the Boltzmann limit in (7b). Figure 1 gives a comparison of the structure factor of (7a) with that of the noninteracting Fermi gas, for $k_BT=10$ MeV, over a range of densities covering the transition from quite degenerate to almost nondegenerate.

There are some circumstances under which we can apply our nearly classical approach to the charged-current interactions $\nu+n\to e^-+p$, as well. In the domain of high density ($\rho>10^{14}$ g cm^{-3}) and (initially) fairly low temperature ($k_BT<10$ MeV), which characterizes the inner core of the star, the lepton kinematics are constrained by the four Fermi factors for the degenerate species in such a way that the lepton energy transfers are not small compared to the lepton energies themselves. But in matter at lower densities, 10^{12} to 10^{14} g cm^{-3}, at postshock temperatures in the region of 10 MeV, the nucleon degeneracy ranges from partial to none. In this case, the average lepton energy transfer (for a 10–20-MeV neutrino) becomes quite small, and the closure approximation again applicable. We obtain, for the vector current contribution to the reaction $\nu+n\to e^-+p$, an expression analogous to (1), with the liquid structure factor replaced by $S^{-+}(\mathbf{q})$, where

$$S^{-+}(\mathbf{q})=(\rho_nV)^{-1}[\mathrm{Tr}\rho_-(\mathbf{q})\rho_+(\mathbf{q})\exp(-\beta K)][\mathrm{Tr}\exp(-\beta K)]^{-1},$$

where $\rho_\pm=\psi^\dagger\tau_\pm\psi$. (We also define $\rho_3=\psi^\dagger\tau_3\psi$, and $\rho=\psi^\dagger\psi$.) For the operator, K, we take

$$K=\int(H_0+\lambda\boldsymbol{\rho}\cdot\boldsymbol{\rho})d^3x-\tfrac{1}{2}\mu\rho(0)+\tfrac{1}{2}\hat{\mu}\rho_3(0), \tag{8}$$

where $\hat{\mu}=\mu_n-\mu_p$, determined from the conditions for equilibrium with electrons and trapped neutrinos. The λ-dependent term in (8) is chosen to provide the symmetry energy of nuclear matter. The numerical value of λ is taken from Vautherin and Brink's[9,10] Skyrme-potential fit to nuclear data, $\lambda=390$ MeV fm^3. An addition of an isospin-independent term to the potential would make no difference in the results to follow. From the identity,

$$\rho_+(0)\exp(-\beta K)=\exp(-\beta K)\rho_+(0)\exp(\beta\hat{\mu}),$$

and the commutation relation,

$$[\rho_+(0),\rho_-(0)]=\rho_3(0),$$

we obtain an expression for the structure function $S^{-+}(0)$,

$$S^{-+}(0)=\rho_n^{-1}(\rho_n-\rho_p)(1-e^{-\hat{\mu}\beta})^{-1}, \tag{9}$$

where we have used $\rho_n - \rho_p = -\langle \rho_3(0)\rangle V^{-1}$. The effect of the symmetry-energy term on the structure function is to change the relation between neutron and proton densities and the quantities, $\mu, \hat{\mu}$,

$$\rho_n = F[(\mu+\hat{\mu})/2 - \lambda(\rho_n - \rho_p)],$$
$$\rho_p = F[(\mu-\hat{\mu})/2 + \lambda(\rho_n - \rho_p)], \quad (10)$$

where $F(\mu)$ is the density of a free nucleon species of chemical potential, μ, i.e., the standard Fermi integral. Solving for $\hat{\mu}$ in terms of ρ_n, ρ_p, and substituting in (9) gives an expression for the structure function in terms of densities. We specialize to the case of Boltzmann statistics, for the purposes of illustration,

$$S_{\mathrm{ND}}^{-+} = \{1 - \rho_p(\rho_n - \rho_p)^{-1}\exp[-2\beta\lambda(\rho_n - \rho_p)]\}^{-1}.$$

In order to calculate the transport of neutrinos in the medium we need the collision term in the transport equation, which involves both the reaction $\nu + n \rightarrow e^- + p$ and the reverse process, $e^- + p \rightarrow \nu + n$. The effective transport differential cross section for a neutrino of energy E is proportional to the quantity W,[11,12]

$$W = V^{-1}\{\langle \rho_-(0)\rho_+(0)\rangle[1+\exp\beta(\mu_e - E)]^{-1} + \langle \rho_+(0)\rho_-(0)\rangle[1+\exp\beta(E-\mu_e)]^{-1}\},$$

where we have substituted the neutrino energy into the electron Fermi factor, following our approximation of quasielasticity for the leptons. The effects of the second term are sometimes referred to as induced absorption. In analogy with (9) we have

$$\langle \rho_+(0)\rho_-(0)\rangle = V(\rho_n - \rho_p)(e^{\hat{\mu}\beta} - 1)^{-1},$$

which leads directly to

$$W = (\rho_n - \rho_p)\{[1+\exp\beta(\mu_e - E)]^{-1} + [\exp(\beta\hat{\mu}) - 1]^{-1}\}. \quad (11)$$

Since the electron chemical potential is substantially greater than the neutrino chemical potential (and greater than the energies of most of the neutrinos, at any density of trapped leptons), the second term on the right-hand side of (11) provides most of the transport cross section. We calculate the ratio of this term to its value at $\hat{\mu} = 0$,

$$R = [\exp(\hat{\mu}_0\beta) - 1]/[\exp(\hat{\mu}\beta) - 1] \quad (12)$$

for fixed neutron and proton densities. Here $\hat{\mu}_0$ is the $n-p$ chemical potential difference for $\lambda = 0$; $\hat{\mu}$ and $\hat{\mu}_0$ are determined from the solutions to (10). The results are plotted in Fig. 2, for a proton fraction $Y = 0.3$, $k_B T = 10$ MeV. The reduction in cross section is dramatic over a significant region of temperature.

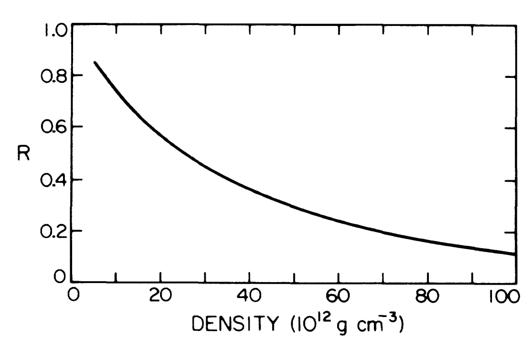

FIG. 2. The quantity R of Eq. (12), for a range of density, for the case $T = 10$ MeV, $Y_p = 0.3$.

In this Letter we have incorporated those effects of interactions which we think are the dominant ones for the case of the Fermi cross sections. The Gamow-Teller cross sections, numerically more important, can be treated by similar, but somewhat more elaborate means. This treatment also requires more extrapolation of the nuclear force terms (now the spin-spin, and the spin-isospin-spin-isospin interactions) from the domains in which they have been studied[12]; they are not included in the Skyrme-potential phenomenology of Ref. 9. This work is in progress. The indications are that the effects will be larger than those, as a result of the symmetry energy, calculated in the present paper for the charged-current interactions.

The most important result of the present work is the conclusion that interactions can have a large effect on neutron opacities, even in regions of density in which it has commonly been assumed that they could be neglected.[11,12] It is not within the scope of this Letter to attempt a quantitative estimate of the effects of the modifications in opacity on the neutrino pulse from the supernova core. They will be in the direction of a shorter deleptonization time and a reduction of the radius of the photosphere, for electron neutrinos. Both of these effects shift the neutrino spectrum upward in energy. The new opacities should be incorporated into future calculations of the neutrino output of a supernova.

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