Argyrakis Replies: The information dimension D_I for a random walk on a lattice was defined[1] as the following:

$$D_I = \left[-\sum_{k=1}^{S_N} P_k \ln P_k \right] \Big/ \ln N \,, \qquad (1)$$

where N is the number of steps, P_k is the probability of visiting the kth site, and S_N is the number of distinct sites visited at least once in a walk [Eq. (3) of Ref. 1]. This should be considered as a general definition provided that $N \to \infty$. From this equation and also from Eq. (5) of Ref. 1 it is rather obvious that $D_I = D_s/2$ only when all sites have exactly the same probability of visitation (i.e., $P_k = 1/S_N$). While this is intuitively true for a perfect, symmetric, and homogeneous lattice, it is not obvious if this should also hold true for an impurity doped, random, and inhomogeneous lattice, all properties of a fractal structure.

To test this point, numerical calculations were performed[1] for the 2D percolating clusters, and it was initially found that, indeed, $D_I \neq D_s/2$, for the time range examined (10000 steps), as $D_I = 0.62$ vs $D_s/2 = 0.65$. Since the accompanying Comment[2] suggests otherwise, more numerical calculations were performed on other fractal systems, and their behavior as a function of time was studied in detail.[3] The Sierpinski gasket (the fractal prototype) was first tested and then the 3D perfect and fractal lattices for walks up to 10^5 and 10^6 steps. Our preliminary results for all systems checked showing that for the range up to 10^4 steps our original calculations were correct, i.e., the difference between D_I and $D_s/2$ persists (about 5%–10%, depending on the system). However, the extended time calculations show that this difference starts to diminish, and one can clearly see that as $t \to \infty$ indeed D_I and $D_s/2$ coincide. For example, for the Sierpinski gasket, where $D_s/2 = 0.685$, we find that $D_I(N = 1000) = 0.659$, but $D_I(N = 30000) = 0.667$, and $D_I(N = 100000) = 0.677$. Thus what we calculate here is an *effective exponent* $D_I(N)$ which depends on time and not its theoretically limiting value.

In the present Comment, de Arcangelis, Coniglio, and Paladin argue that the P_k function is always asymptotically homogeneous, even in the fractal lattice, and also that the P_k function *is not* an example of a multifractal property that could possibly be amenable to independent exponents. The authors are basing this claim on the obvious identity[4]

$$-D_I \ln N = \textstyle\sum_k P_k \ln P_k = \lim_{q \to 1} \frac{\ln \sum_k P_k}{q-1} \,, \qquad (2)$$

which has meaning only at the limit of infinite time. With this restriction the above data are thus in agreement with the main idea that this Comment offers. The implications of my work are that for most other transport properties examined up to now, such as S_N, R_N^2, etc., finite, short times suffice to bring out the fractal behavior, while for the P_k function tested here this is not true. For example, in past work the R_N^2 exponent was derived[5] from calculations of $N = 400$ steps, and the S_N exponent from $N = 1000$ steps.[6] This is important for experimental measurements of fractal and critical exponents based on *excited-state* kinetics.[7] These experiments describe the low-temperature transport range on a molecular alloy or a chemical reaction mechanism on a surface, and certainly the excited state of the measurement has a well-known *limited* lifetime (restricted number of steps). Thus it is of the utmost importance to clearly define the regions of validity of these exponents. Precisely for this reason I introduced the $\sum_k P_k \ln P_k$ function, since it is well known[8,9] that this function constitutes a probability measured in the most general sense.

Concluding, our earlier,[1] and most recent work,[3] do indeed show that $D_I = D_s/2$ for fractals *only* in the limit of very long times, unlike other well-known related exponents.[6–8] This point has practical implications in the interpretation of related experimental results.[7]

I would like to thank Dr. G. L. Bleris for helpful comments on the character of these exponents.

Panos Argyrakis
Department of Physics 313-1
University of Thessaloniki
54006 Thessaloniki, Greece

Received 25 February 1988
PACS numbers: 72.90.+y, 02.50.+s, 05.40.+j

[1] P. Argyrakis, Phys. Rev. Lett. **59**, 1729 (1987).
[2] L. de Arcangelis, A. Coniglio, and G. Paladin, preceding Comment [Phys. Rev. Lett. **61**, 2156 (1988)].
[3] N. Pitsianis, G. Bleris, and P. Argyrakis, to be published.
[4] A. Coniglio and F. di Liberto, Phys. Rev. Lett. **57**, 1016 (1986).
[5] D. Ben-Avraham and S. Havlin, J. Phys. A **15**, L691 (1982).
[6] P. Argyrakis and R. Kopelman, Phys. Rev. B **29**, 511 (1984).
[7] R. Kopelman, J. Stat. Phys. **42**, 185 (1986); J. Prasad and R. Kopelman, Phys. Rev. Lett. **59**, 2103 (1987).
[8] A. I. Khinchin, *Mathematical Foundation of Information Theory* (Dover, New York, 1957).
[9] D. V. Lindley, *Introduction to Probability and Statistics* (Cambridge Univ. Press, London, 1969).

