Detector for the Temperaturelike Effect of Acceleration

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A single electron orbiting in a Penning trap may be used to demonstrate that radiation appears in the vacuum in an accelerated reference frame. As the electron accelerates centripetally in the trap's magnetic field, the vacuum radiation excites its motion along the trap axis, which is in turn measurable with present electronic techniques. A realizable example of such a detector is presented.

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The vacuum, as seen by an observer in an accelerated reference frame, has a spectrum of fluctuations greater than the zero-point spectrum seen in an inertial frame. A particle detector uniformly accelerated through the vacuum will detect radiation with the same spectrum as a detector at rest in a thermal bath of temperature $\hbar a/2\pi ck$, where a is the proper acceleration.[1] This effect is analogous to the Hawking radiation from a black hole, which also has a thermal spectrum.[2] A detector in circular motion through the vacuum also experiences radiation, which, however, deviates from a thermal spectrum.[3,4]

These intriguing effects are difficult to observe because of the extreme accelerations required to manifest a measurable temperature. A linear acceleration of 2.4×10^{22} cm/s^2 produces a 1-K equivalent background, with which the detector must come into equilibrium before traveling the length of the accelerating device. This is an impractical requirement for a laboratory experiment. However, similar accelerations may be achieved in circular motion. The incomplete polarization of electrons in storage rings has been attributed to the radiation seen in an accelerated frame.[5]

A single electron orbiting in a magnetic field in a cold enclosure would provide a decisive test that acceleration alters the electromagnetic vacuum state, provided that the effects of direct coupling of normal modes in the nonideal Penning trap do not mask the small effect being investigated. This system could also shed light on the quantum mechanical effects of strong gravitational fields, which are not available in the laboratory. In the model experiment described here, an electron in a Penning trap is constrained to move in a circular cyclotron orbit around the trap axis by a uniform magnetic field. Its centripetal acceleration depends on the radius of its orbit, which is controlled by driving the electron with a circularly polarized electromagnetic wave at its cyclotron frequency. The trap's quadrupole electric field creates a quadratic potential along the trap axis in which the electron can oscillate. The electron's axial motion will be excited by the vacuum radiation it sees in its own reference frame. When the Penning trap is surrounded by an electromagnetic cavity tuned to resonate at the electron's axial oscillation frequency $\omega_a=5.3\times10^{10}$ s^{-1}, the electron will transfer its energy of axial motion to the cavity electromagnetic mode. The presence of energy in the cavity mode at this frequency, detected by a microwave amplifier and spectrum analyzer, is the experimental signature of the vacuum radiation. The electron will also produce synchrotron radiation at multiples of its cyclotron frequency $\omega_0=2.1\times10^{12}$ s^{-1}, which can be identified by its frequency.

The spectrum seen by a detector in uniform circular motion depends on its velocity as well as its accelerator.[3,4] The spectrum is a density of states times the Fourier transform of the autocorrelation function, which is

$$\langle\phi(x)\phi(x')\rangle=\hbar c/\{4\pi^2[(\mathbf{r}-\mathbf{r}')^2-c^2(t-t')^2]\}$$

for a scalar field ϕ in free space. A useful parametrization in velocity $\beta=v/c$ and angular frequency ω_0 is used by Kim, Soh, and Yee,[4] who give a series expansion in β for the equivalent energy volume density per unit angular frequency of a massless scalar field. This energy density is

$$\frac{de}{d\omega}=\frac{\hbar}{\pi^2c^3}\left[\frac{\omega^3}{2}+\frac{\omega_0^3}{\gamma}\left(\frac{\omega}{\omega_0}\right)^2\sum_{n=0}^{\infty}\frac{\beta^{2n}}{2n+1}\sum_{k=0}^{n}(-1)^k\frac{n-k-\omega/\gamma\omega_0}{k!(2n-k)}\Theta\left(n-k-\frac{\omega}{\gamma\omega_0}\right)\right],$$

where $\gamma=(1-\beta^2)^{-1/2}$ and Θ is the step function. The spectra in Fig. 1 show the change in energy density with β, summing terms $n\le4$. The zero-point energy experienced by an observer at rest has been subtracted. For $\beta<0.6$, the series converges quickly.

This equivalent energy density has been calculated with the assumption that the observer is in free space. Within a cavity with conducting walls, the density of states and average zero-point energy density are approximately the same if two conditions are satisfied: the cavity has an opening through which it is coupled to free space and the dimensionless

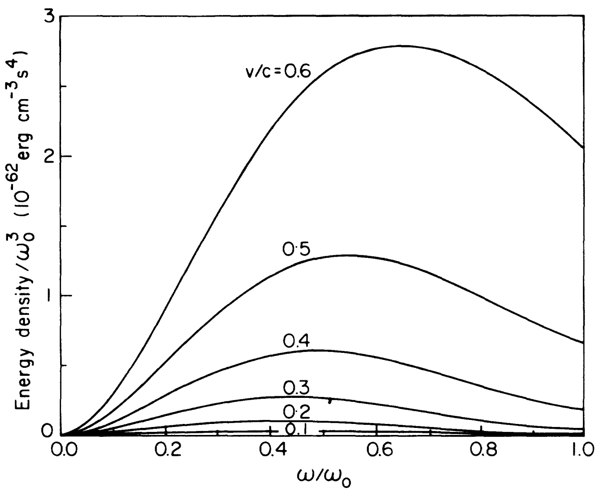

FIG. 1. Normalized energy density spectra $(de/d\omega)\omega_0^{-3}$ in a reference frame undergoing circular acceleration in the vacuum. The zero-point energy density has been subtracted.

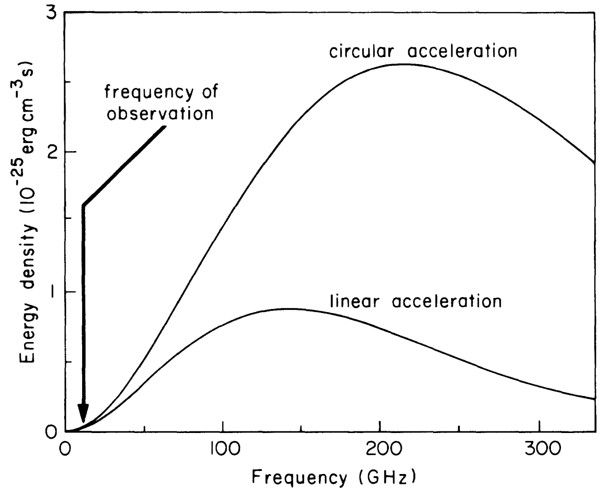

FIG. 2. Energy density spectrum $(de/d\omega)$ seen by the electron orbiting with velocity $\beta=0.6$ in the model detector described in the text. Also shown is the energy density seen by an observer moving linearly with the same acceleration, 5.9×10^{22} cm s^{-2}. The frequency under observation is $\gamma\omega_a/2\pi$ $=10.5$ GHz.

coupling κ is greater than the fractional mode spacing $\Delta\omega(\omega)/\omega$; and $\omega\gg c/L$, where L is a characteristic cavity dimension. The modes at frequencies $\omega_0\pm\omega_a$, $2\omega_0\pm\omega_a$, $3\omega_0\pm\omega_a,\ldots,$ contribute to the excess spectral density which excites the electron's motion at ω_a. It will be shown that at these frequencies, the two conditions necessary to assume a free-space spectral density are satisfied for the model experiment.

A semiclassical analysis by Boyer[6] of the electromagnetic vacuum spectrum seen by a linearly accelerated observer shows the same energy density as in the scalar vacuum, but with an additional positive term. One may speculate that this also occurs in the case of circular motion.

For the model accelerated detector, an electron orbiting in a uniform magnetic field **B**, a velocity $\beta=0.6$ is chosen. For $|\mathbf{B}|=150$ kG, a field attainable with superconducting solenoid magnets, the angular frequency of the electron is $\omega_0=e|\mathbf{B}|/\gamma m_e c=2.1\times10^{12}$ s^{-1}. The electron's proper acceleration is $a=\gamma^2 v\omega_0=5.9\times10^{22}$ cm s^{-2}. Figure 2 shows the energy density in a hypothetical massless scalar field seen by the orbiting electron and by an observer moving linearly with the same acceleration, which corresponds to a 2.4-K temperature.[7]

The electron is confined within a Penning trap, shown in Fig. 3. The static quadrupole electric field of the trap creates a quadratic potential wall along the trap's axis, in which the electron can oscillate. With an electrode separation $2z_0=2$ mm and electrode potentials $\pm\Phi_0$ $=\pm10$ kV, the axial oscillation frequency, $\omega_a=(2e\Phi_0/\gamma m_e z_0^2)^{1/2}$, is 5.3×10^{10} s^{-1}. By the placement of the Penning trap in a microwave cavity, the electron's energy of axial motion can be coupled to an electromagnetic cavity mode. Two conditions must be met for this coupling to occur: The cavity mode frequency must coincide with ω_a and the electric field in the cavity mode at the electron's position must be parallel to $\hat{\mathbf{z}}$, the trap axis. The coupling time of the electron to the cavity mode is

$$\tau_a=\frac{m_e\omega_a}{4\pi e^2 Q}\,\frac{\int_{\text{cavity}}|\mathbf{E}(\mathbf{x})|^2 d^3x}{|\mathbf{E}(\mathbf{x}_e)\cdot\hat{\mathbf{z}}|^2}\,,$$

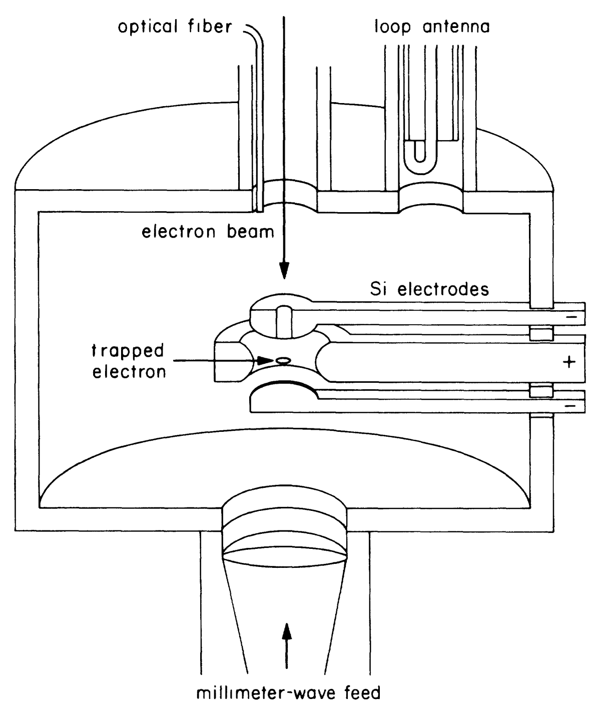

FIG. 3. The model detector, a Penning trap within a microwave cavity. A uniform magnetic field **B** points along the trap axis.

where Q is the cavity quality factor and $\mathbf{E}(\mathbf{x})$ is the electric field in a single mode. For a cavity of length 1 cm, radius 1.36 cm, and $Q \approx 10^4$, in the TM_{010} mode, $\tau_a = 520$ μs. The approximation of a free-space spectrum of vacuum fluctuations holds when a tube of radius R couples the cavity to the outside space, and $R > (2\pi^2)^{1/3} c/(\omega_0 - \omega_a) = 0.4$ mm.

Low-energy electrons are produced in a trap through ejection from residual gas atoms by an electron beam. Dehmelt and co-workers[8] have trapped single electrons by applying sufficient excitation at ω_a to evaporate all but one electron out of the trap. The electron's cyclotron motion is rapidly cooled by synchrotron radiation with a characteristic time $\tau_c \approx 3m_e c^3/4e^2\omega_0^2 = 18$ ms. To increase its velocity to $0.6c$ and to replace the 2.0×10^{-5} erg/s of power that it radiates at that velocity, it is irradiated with a circularly polarized wave of frequency ω_0 and flux 1.3×10^{-9} W/cm^2 normal to the plane of its orbit. The cyclotron damping time is affected by the coupling of the cyclotron motion to the cavity modes and may vary from the calculated free-space value by an order of magnitude,[9,10] necessitating a larger flux at ω_0. The electron's axial motion is damped in its coupling time to the cavity. The magnetron motion of the electron, a precession of the center of the cyclotron motion around the trap axis with angular velocity ω_m, where $2\omega_m(\omega_0 - \omega_m) - \omega_a^2 = 0$, is diminished[8] by pumping the trap at the sum frequency $\omega_m + \omega_a$.

After the electron reaches its final velocity, a low-noise cryogenic amplifier and spectrum analyzer coupled through an isolator to the microwave cavity measure the spectral density in the cavity mode. The presence of the amplifier halves Q and doubles τ_a to 1.0 ms. The power spectral density at the amplifier due to the electron's axial excitation in its bandwidth $1/2\pi\tau_a = 150$ Hz is

$$\frac{dP}{df} = \frac{\pi^3 c^3}{2\omega^2}\frac{de}{d\omega} .$$

The energy density $de/d\omega$ is evaluated at a frequency $\omega = \gamma\omega_a = 6.6\times 10^{10}$ s^{-1} to account for the relativistic frequency shift. Then $dP/df = 0.47\times 10^{-22}$ W/Hz. By comparison, cryogenic GaAs field-effect transistor amplifiers[11] are capable of $dP/df = 1.5\times 10^{-22}$ W/Hz, which implies a signal-to-noise ratio of $S/N = 0.3$. Through signal averaging, the electron's axial motion can be distinguished from the amplifier noise in a time $\tau_a(S/N)^{-2} = 12$ ms. Figure 4 shows the expected spectral density at the amplifier.

The temperature and vacuum requirements of this model experiment are stringent. The energy density seen by the accelerating electron at frequency $\gamma\omega_a$ corresponds to a temperature of 6.8 K. Ideally, the physical temperature of the cavity should be well below this. The cavity temperature can be easily reduced to 1.3 K in a low-pressure liquid-He bath. A possible refinement is the use of a dilution refrigerator to reach 50 mK.

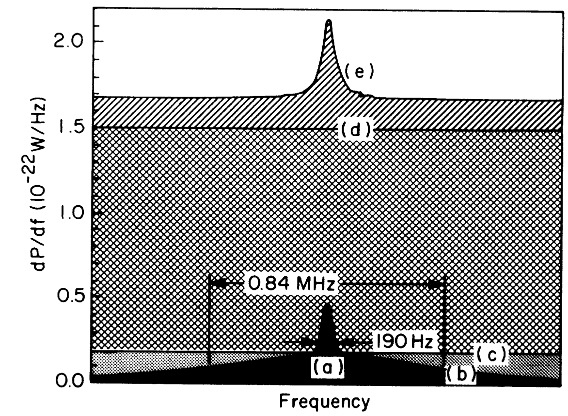

FIG. 4. The expected power spectral density dP/df at the amplifier vs frequency. The width of the expected signal has been exaggerated for clarity. Displayed is the spectral density due to (a) the electron's axial motion excited by vacuum radiation, (b) thermal noise in the empty cavity, (c) the sum of thermal noise from the cavity and isolator, (d) amplifier noise, and (e) the observed total spectral density.

Collisions of the orbiting electron with residual gas molecules would excite its axial motion. A pressure of less than 10^{-13} Torr is necessary to maintain a collision frequency of less than 1 s^{-1} at 1.3 K. This is attainable, and is aided by cryopumping at low temperatures.

The 336-GHz plane wave necessary to drive the electron's cyclotron motion can be supplied by frequency tripling the output of a phase-locked Gunn-effect source with a GaAs Schottky-barrier varactor diode.[12] The wave is focused onto the electron by quasioptical techniques. As the electron is brought from rest to its final energy, the solenoid's magnetic field must be ramped from 120 to 150 kG to maintain a constant ω_0. To accomplish this in the roughly 1-s interval before an electron-gas molecule collision, a low inductance modulation coil with suppressed mutual inductance may be used within a persistent-field solenoid.[13]

Excitation of the electron's axial motion at ω_a by cyclotron drive noise at 8.4 GHz or at 336 ± 8.4 GHz can be avoided. The tube coupling the 336-GHz drive into the cavity will attenuate noise below its cutoff frequency exponentially along its length, eliminating direct 8.4-GHz noise. The phase noise spectral density of a Gunn-effect source at frequencies more than 100 MHz from the carrier is less than its thermal noise. When the source is attenuated by 4 to 5 orders of magnitude to supply the necessary cyclotron drive, this thermal noise will be reduced to a negligible level of 3 to 30 mK.

The relativistic mass increase of a single electron has been observed by Gabrielse, Dehmelt, and Kells,[14] but phase fluctuations in their microwave source, a crystal oscillator followed by a chain of multipliers, limited the kinetic energy to 10 eV. The rms phase fluctuation $(\langle\phi^2\rangle)^{1/2}$ of an oscillator is the double sideband phase noise integrated over frequency times the square of the multiplication factor[15]; for the tripled Gunn oscillator $(\langle\phi^2\rangle)^{1/2}=0.13$ in 100 ms,[16] which is less than the $\pi/2$ phase fluctuation needed to decelerate the electron. The short-term frequency fluctuation in the source changes ω_0 which, in turn, alters ω_a through the anharmonicity parameter C_4 of the nonideal trap,[10] broadening its natural linewidth. The fractional change in the axial frequency is

$$\Delta\omega_a/\omega_a=(\omega_a/\omega_0)^2(3m_ec^2/4e\Phi_0)C_4(\Delta\omega_0/\omega_0)$$
$$=0.024C_4(\Delta\omega_0/\omega_0)\,.$$

We require $\Delta\omega_a/\omega_a<10^{-8}$ to avoid increasing the axial linewidth. Typically[16] $\Delta\omega_0/\omega_0\approx10^{-7}$, implying the trap anharmonicity is not a problem even for C_4 of order unity, so that compensation electrodes,[10] while desirable, are not necessary.

The perturbation of the cavity modes by the Penning trap electrodes can be almost eliminated by constructing them of silicon, which is transparent to microwave radiation at low temperatures. As the electron is brought to is final energy, the electrode's potentials must be increased to keep ω_a fixed on the cavity's resonant frequency to maintain axial damping. Photoconductivity can be temporarily induced by illuminating the $0.3\times1\times7\text{-mm}^3$ Si electrodes with 1 μW of 1.06-μm light. This results in a resistance of approximately 10^5 Ω along the electrode,[17] and a charging time of 1 ns into the electrode's 0.01-pF capacitance, sufficiently small to allow a 1-s ramp in electron energy.

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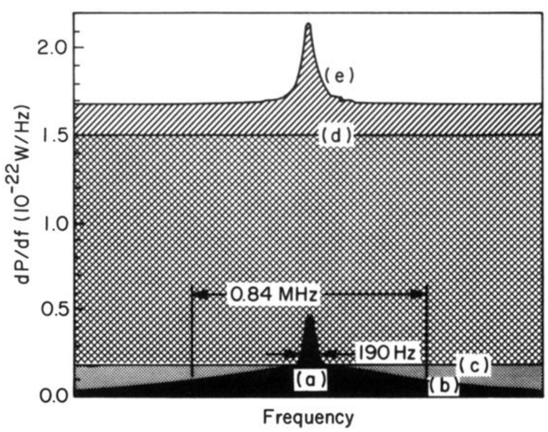

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