Possible New Form of Spontaneous T Violation

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In (2+1)-dimensional models with a Chern-Simons term, there is a periodicity in the parameter $e^2/2\mu$, where e is the smallest charge in the theory and $\mu/2$ is the coefficient of the Chern-Simons term. Since $\mu \rightarrow -\mu$ under the action of time reversal T, in such a theory the question of T invariance becomes involved with the dynamical issue of what are the allowable values of the charge. We show how cross sections develop T-violating terms for "forbidden" values of the charge. Connections are made with some recent ideas concerning the mechanism of high-temperature superconductivity.

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(1) Introduction.—In (2+1)-dimensional electrodynamics, the possibility arises of a term

$$\Delta L = \tfrac{1}{2}\mu\epsilon_{abc}A^a\partial^b A^c, \tag{1.1}$$

the Chern-Simons term. This is of the standard form for the coupling of the photon to a conserved current, $\Delta L = \frac{1}{2}\mu A_a \tilde{j}^a$, but the current

$$\tilde{j}_a = \epsilon_{abc}\partial^b/A^c \tag{1.2}$$

is a little unusual. Initial interest in this term is centered on its interpretation as a gauge-invariant photon mass.[1] In this connection, μ combines with an ordinary photon mass, $\Delta L = \frac{1}{2}m^2A^2$, to produce spin-up and spin-down mass eigenstates of masses $(m^2+\mu^2/4)^{1/2} \pm \mu/2$.[2] The inequality of these masses is an unmistakable sign of P and T violation, which of course is no surprise since from (1.1) we see that $\mu \rightarrow -\mu$ under P or T.

More recently interest has focused largely on theories in which the field A has no kinetic-energy term F^2. These theories[3] have appeared as a general way of implementing fractional statistics,[4,5] and in proposals for effective Langrangian descriptions of the fractional quantum Hall effect[6,7] and resonating-valence-bond states.[8] In such pure Chern-Simons theories, the sole appearance of A is in the Chern-Simons term itself, and in couplings of the form j_aA^a to other conserved matter currents. For definiteness, let us suppose that there is just one matter current, so that the complete Lagrangian for A is

$$L_A = ej_aA^a + \tfrac{1}{2}\mu\epsilon_{abc}A^a\partial^b A^c. \tag{1.3}$$

Then the field equation for A is simply

$$ej_a = -\tfrac{1}{2}\epsilon_{abc}F^{bc}. \tag{1.4}$$

This implies that F is completely determined by the matter degrees of freedom, and has no independent dynamics. Indeed, the implication of (1.4) is that charged point particles of charge e are also point flux tubes for A flux, of magnitude $\Phi = e/\mu$, while F vanishes everywhere else. Thus all independent local dynamics have been squeezed out of A, leaving only global gauge-invariant degrees of freedom. Charge particles moving around flux tubes acquire Aharnonov-Bohm phases,[9] and thus (also taking into account the field contribution from the second term) (1.3) implements θ statistics for the particles of charge e,[10] where

$$\theta = e^2/2\mu, \tag{1.5}$$

in the sense that the slow interchange of distant particles is accompanied by a phase $e^{i\theta}$. The flux and charge carrying particle states so constructed (anyons) have statistics which continuously interpolate between bosons and fermions.

(2) Periodicity, T invariance, and spontaneous breaking.—The statistical interaction is the only manifestation of the gauge field A, in these pure Chern-Simons theories. As a consequence, they are periodic under

$$e^2/2\mu \rightarrow e^2/2\mu + 2\pi n. \tag{2.1}$$

What happened to the highly nonperiodic dependence of the photon mass on μ? The point is that by removing the kinetic energy we have essentially taken $m \rightarrow \infty$. This also explains the lack of local excitation of A.

P and T transformations have the action $\mu \rightarrow -\mu$, and because of (2.1) we find that they may be good symmetries if $e^2/2\mu = n\pi$. These values, not surprisingly, correspond to fermions or bosons depending, respectively, on whether n is odd or even. This condition must be satisfied by the charges of all states in the Hilbert space of the theory for P and T to be good symmetries.

The question of the allowed values of charge in a theory is, however, a dynamical one. Let us suppose, for example, that a Higgs condensate of charge e particles forms. Then there will be finite-energy vortices with flux $\Phi = 2\pi/e$ (indeed, at the classical level one can support vortex asymptotics with field configurations of arbitrarily small energy). Reading the field equation (1.4) backwards, we find that the vortices carry a charge $q = \mu\Phi$, and so their statistical parameter is

$$\theta_v = q\Phi/2 = 2\pi^2\mu/e^2 . \qquad (2.2)$$

Clearly periodicity in $e^2/2\mu$ does not enforce periodicity in this statistical parameter. Nor, therefore, does T invariance in the unbroken phase ensure T invariance in the Higgs phase. For example, if we take the solution $e^2/2\mu = 2\pi$, then $\theta_v = \pi/2$.

(3) Cross sections.— How does P and T violation manifest itself? The most direct effect is an asymmetry in scattering cross sections. Our greatest interest is in the scattering of identical anyons, although T violation also occurs in other cross sections. Suppose we have identical anyons both of statistical parameter $\theta_v = q\Phi/2$. The associated scattering, in the nonrelativistic limit, is easily related to the problem solved by Aharnonov and Bohm in which we scatter a pure charge q off an uncharged vortex of flux Φ. Indeed, studying the two-body Schrödinger equation for the anyons in Coulomb gauge, it is found that the contribution of the scalar potential due to the motion of the anyons, duplicates that of the vector potential $\mathbf{A}$. Therefore in center-of-mass coordinates we are led to precisely the equation studied by Aharnonov and Bohm. The correspondence with the parameter α used in Ref. 9 is $\theta_v = -\pi\alpha$. Working in the gauge $A_r = 0$, $A_\varphi = \Phi/2\pi r$, the wave function describing a particle incident from the right is $\psi = \exp[+i(\theta_v\varphi/\pi - kr\cos\varphi)]$, where k is the wave vector of the relative motion. The solution to the scattering problem in this case is

$$\psi = e^{+i(\theta_v\varphi/\pi - kr\cos\varphi)} - \frac{ie^{ikr}}{(2\pi ikr)^{1/2}} \sin|\theta_v| \frac{e^{\mp i\varphi/2}}{\cos(\varphi/2)} , \qquad (3.1)$$

where the $\mp$ sign is for $\theta_v > 0$ and $\theta_v < 0$, respectively, and thus

$$f(\varphi) = \frac{-i\sin|\theta_v| e^{\mp i\varphi/2}}{\cos(\varphi/2)} \qquad (3.2)$$

is the scattering amplitude. The appropriate scattering amplitude in the identical particle case is just $f_{\text{tot}}(\varphi) = f(\varphi) + f(\varphi - \pi)$. The absence of a relative phase is due to the fact that our anyons are just bosons with an additional "statistical" interaction. We will comment further on this below. For a reason that will soon become obvious, we also wish to include the effect of some additional, nonstatistical interaction, in the amplitude. The simplest way to accomplish this is to put in a phase shift by hand, modifying the asymptotics of $(-i)^{|\theta_v|/\pi} \times J_{|\theta_v|/\pi}(kr)$ to

$$\frac{e^{-i|\theta_v|/2}}{(2\pi kr)^{1/2}} (e^{i(kr - \pi/4 - |\theta_v|/2 + \delta)} + \text{incoming wave}) . \qquad (3.3)$$

Converting to the true scattering angle, $\rho = \pi - \varphi$, the center-of-mass cross section is therefore

$$\frac{d\sigma}{d\rho} = \frac{4(1-\cos\delta)}{\pi k} + \frac{\sin^2\theta_v}{2\pi k \sin^2(\rho/2)} + \frac{\sin^2\theta_v}{2\pi k\cos^2(\rho/2)} + I , \qquad (3.4)$$

where I is the P- and T-violating interference term that we are interested in:

$$I = -\frac{8\sin\theta_v \sin(\delta/2)}{\pi k \sin\rho} \sin(|\theta_v| - \delta/2 + \text{sgn}(\theta_v)\rho) . \qquad (3.5)$$

It may seem, at first glance, that (3.4) cannot possibly be correct, since for the choice $\theta_v = n\pi$, n = odd, corresponding to the scattering fermions, the cross section is independent of scattering angle. However, it must be recalled that in two dimensions, Fermi statistics do *not* imply the vanishing of the cross section at $\pi/2$. Indeed, in two dimensions the entire angular dependence of the mth partial wave is $e^{im\rho}$, and apart from the terms in the cross section that result from interference between different partial waves, the cross section is isotropic. In particular, the first term of (3.4) describes both "s-wave" and "p-wave" scattering. Furthermore, the cross section (3.4) is invariant under the combined transformation $\rho \to -\rho$, $\theta_v \to -\theta_v$ corresponding to a "complete" P or T transformation (i.e., transforming the angles and performing a compensating P or T transformation on the coupling constants), as well as under the coordinate rotation $\rho \to \rho + \pi$. These necessary properties would not have been maintained if we had introduced a relative phase into the identical particle scattering amplitude.

Concerning the T noninvariance of our result, we should remark that introducing phase shifts generically breaks T "kinematically," in the sense that the phase shift, $\delta_{m-\theta_v/\pi}$, does not have the T-transformed partner, $\delta_{-(m-\theta_v/\pi)}$, as $-(m-\theta_v/\pi)$ is not in the spectrum. Amusingly the interference term vanishes together with the P- and T-violating asymmetry as we turn off the nonstatistical interaction. Also, note that the part of the cross section due to the statistical interaction vanishes for both bosons and fermions.

(4) Comments.— Recently Kalmeyer and Laughlin[11] proposed a variational wave function for spin liquids that is closely related to Laughlin's earlier and highly successful variational wave function for the fractional quantized Hall effect.[12] Among the candidates to be described by such spin liquids are Mott insulators, including the CuO layers crucial to high-temperature superconductivity. Laughly also found that the quasiparticle excitations around this state were half fermions, and a mechanism of superconductivity, based on this property, was proposed. These ideas were forcefully elaboratored in Ref. 13.

Kivelson and Rokhsar[14] have criticized Laughlin's proposals on the following grounds. The ground-state wave function is found to be real, indicating that T invariance is satisfied for this state. Now in T-invariant systems, the only possible statistics are those corresponding to bosons and fermions. Related to this is the fact that the statistical phase may be derived by an application of the refined adiabatic theorem (Berry's phase).[15] If the relevant wave functions, for a single quasiparticle localized at different places, may be taken real, then the only possible accumulated Berry phase is ± 1.

It seems to us that the considerations above provide a class of field theory models quite closely related to Laughlin's proposals, and show how the objections of Kivelson and Rokhsar may be met. Namely, we have found a class of models for which all observables that do not involve globally nontrivial excitations (vortices) possess a T symmetry, but this T symmetry is broken by the interactions of these nontrivial excitations. The ground-state wave function, expressed in particle coordinates, will respect T invariance and is therefore real. However, we expect that the quasiparticle wave functions, unlike the ground state, cannot be taken to be real. Note that Laughlin's quasiparticles are indeed vortices. Further, in our language the half-filling-fraction wave function he uses corresponds to $e^2/2\mu = \pi/\frac{1}{2} = 2\pi$, and from (2.2) we do indeed have $\theta_v = \pi/2$.

This suggests that we should take quite seriously the possibility that T invariance is spontaneously broken in the high-temperature superconducting state and possibly in two-dimensional Mott insulators more generally. Fortunately, this idea is subject to experimental test. T invariance has the important macroscopic consequence that it leads to the Onsager reciprocity relations among transport coefficients.[16] A classic example of an Onsager relation is the symmetry of the thermal conductivity tensor $\sigma_{xy} = \sigma_{yx}$, and we expect the spontaneous breaking of T invariance to lead to a nonsymmetric conductivity tensor. Another possibility is a violation of the equality of the thermoelectric coefficients. It would be most interesting to check for deviations from these, and other symmetry relations. It should be remembered, however, that our mode of T violation applies to a single layer of, for example, CuO, and the consequences of such effects may be rather subtle for a bulk sample.

Deviations from this and other Onsager reciprocity relations can be computed explicitly for a model of interacting anyons at low density (of course, the relevance of this model to the condensed, superconducting state is questionable). Details of such calculations will be presented elsewhere.

In closing, let us emphasize our belief that, independent of any detailed model, the question of the *symmetry properties* of the new states is a most important, and experimentally accessible one.

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[1]S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) **140**, 372 (1982).

[2]S. K. Paul and A. Khare, Phys. Lett. B **174**, 420 (1986).

[3]Such theories were first considered explicitly by C. Hagen, Ann. Phys. (N.Y.) **157**, 342 (1984), who, however, did not draw the connection with fractional statistics, and in fact later rejected them [Phys. Rev. D **31**, 2135 (1985)].

[4]Fractional statistics: F. Wilczek, Phys. Rev. Lett. **48**, 1144 (1982), and Phys. Rev. Lett. **49**, 957 (1982).

[5]Field theory construction: F. Wilczek and A. Zee, Phys. Rev. Lett. **51**, 2250 (1983); D. P. Arovas *et al.*, Nucl. Phys. **B251** [**FS13**], 117 (1985).

[6]S. Girvin and A. MacDonald, Phys. Rev. Lett. **58**, 1252 (1987).

[7]N. Read, unpublished.

[8]I. Dzyaloshinskii, A. Polyakov, and P. Wiegmann, Phys. Lett. **127**, 112 (1988).

[9]Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).

[10]A. Goldhaber, R. MacKenzie, and F. Wilczek, Harvard University Report No. HUTP-88/A044, 1988 (to be published).

[11]V. Kalmeyer and R. B. Laughlin, Phys. Rev. Lett. **59**, 2095 (1987).

[12]For the classic theory of the quantized Hall effect the all-purpose reference is *The Quantized Hall Effect,* edited by E. Prange and S. Girvin (Springer-Verlag, New York, 1987).

[13]R. B. Laughlin, Phys. Rev. Lett. **60**, 2677 (1988), and "Why I Think High-Temperature Superconductivity and the Fractional Quantum Hall Effect are Related" (to be published).

[14]S. A. Kivelson and D. S. Rokhsar, "Quasiparticle Statistics in Time-Reversal Invariant States" (to be published).

[15]D. Arovas, J. Schrieffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722 (1984).

[16]See, for example, S. R. de Groot and P. Mazur, *Non-Equilibrium Thermodynamics* (North-Holland, Amsterdam, 1962).