Quantum Limitations on Measurement of Magnetic Flux

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If the magnetic flux trapped in a superconducting quantum-interference device has observable values $\pm h/2e$, which alternate in time, and if the value at a given moment is unknown, it is in general impossible to determine that value without affecting the results of subsequent observations.

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There has recently been a controversy on whether the magnetic flux trapped in an rf SQUID (apparently, a macroscopic object) can be measured noninvasively, that is, without affecting the results of subsequent observations. Leggett and Garg[1] showed that the combined assumptions of *macroscopic realism* and *noninvasive measurability* lead to a conflict with some predictions of quantum mechanics, which can in principle be tested experimentally in a macroscopic setup.[2] Leggett and Garg further argued that under these macroscopic conditions, noninvasive measurability was "extremely natural and plausible" and their paper therefore suggests a contradiction between macroscopic realism and quantum mechanics.

This conclusion was criticized by Ballentine,[3] who showed by an explicit (albeit grossly oversimplified) quantum-mechanical model that a dichotomic variable could not in general be measured noninvasively. (This result is not surprising. It is well known[4] that any measurement of a quantum-mechanical variable will necessarily change the state of the observed system, unless that state is already an eigenstate of the variable being measured.) Ballentine therefore concluded that, even at this seemingly macroscopic level, it was noninvasive measurability that failed, while "the metaphysical principle of realism" could not be experimentally tested.

I shall refrain from discussing the issue of "realism" which has at least as many different definitions as there are authors.[5] On the other hand, "measurability" seems to be a legitimate, reasonably well defined physical concept. Leggett and Garg[1] attempt to give credibility to the idea of noninvasive measurability by introducing the notion of an "ideal negative measurement" whereby information is obtained from the *lack* of response of a detector. However, it was shown by Dicke[6] that the absence of detector response does not imply that there was no interaction, and it cannot be considered as a proof that the quantum state was not affected. As a trivial example, if we replace, in a Stern-Gerlach experiment, one of the detectors by a hole through which the outgoing particle may pass unhindered, the lack of response of the remaining detector does not imply in general that the spin state of the particle was not affected. It thus appears that noninvasive measurements are in general incompatible with quantum theory (except in the trivial case where the quantum-mechanical system is already in an eigenstate of the variable being measured).

Nevertheless, the issue raised by Leggett and Garg is not trivial because, as suggested in another article,[7] there is a possibility that "quantum mechanics is not the right theory to describe the world at the macroscopic level —that qualitative changes in the basic laws of physics occur when matter becomes sufficiently complex." There have indeed been speculations[8] that the laws of physics may reduce to quatnum mechanics for very small systems, to classical mechanics for very large ones, and have a complicated hitherto unknown form for systems of intermediate size such as an rf SQUID. In that context, what can be said about the measurability of a very small magnetic flux?

The absence of a detailed theory does not preclude the possibility of semiquantitative estimates. Heisenberg's microscope[9] can be discussed in quasiclassical terms without the use of formal quantum theory. Likewise, Bohr and Rosenfeld[10] investigated the measurability of the electromagnetic field in terms of semiclassical test bodies following the laws of classical mechanics, but with their positions and momenta subject to Heisenberg's uncertainties. At the same level of discussion, it may be argued that in order to measure the magnetic flux through a loop, one must use its mutual inductance with another loop (the flux meter) where an electromotive force (emf) is induced. This emf causes the flow of a current in the secondary loop. The latter generates a magnetic flux causing in turn an emf in the primary loop, which then perturbs the flux that had to be measured. Note that the emf is, in *classical* electrodynamics, the dynamical variable canonically conjugate to the magnetic flux through a loop, apart from a geometrical factor having the dimensions of a length. This follows from the classical Poisson brackets between components of **E** and **B**. In the particular case under consideration, this geometrical factor simply is the capacitance of the circuit, so that the variable canonically conjugate to the flux Φ is the charge Q. This result is a *classical* property of any *macroscopic* circuit.

In a highly idealized scheme, a measurement of Φ can be described by our introducing in the Hamiltonian an interaction term $g(t)\Phi x$, where x is the position of a "pointer" and $g(t)$ is an externally controlled function such that $g(t)\neq 0$ only for a brief time (I am not concerned here with the technical difficulty of realizing such a coupling). Let $G=\int g(t)dt$. The brief interaction changes the momentum p of the "pointer" by the amount $G\Phi$. This change is used to measure the value of Φ, and the result has an uncertainty $\Delta\Phi=\Delta p/G$, where Δp is the initial uncertainty on the value of p. On the other hand, the coupling $g(t)\Phi x$ also causes Q, the variable conjugate to Φ, to change by Gx. Therefore Q becomes uncertain by $\Delta Q=G\Delta x$, where Δx is the initial uncertainty on x. We thus obtain

$$\Delta\Phi\Delta Q\simeq\Delta x\Delta p\,.$$

It follows that if the initial state of the "pointer" is subject to Heisenberg's minimum uncertainty, the induced charge Q is uncertain by at least $\Delta Q\gtrsim\hbar/\Delta\Phi$.

This semiclassical result seems not to depend on the particular oversimplified model which was used to derive it. In any realistic setup, the measuring apparatus *must* interact with the object being measured. If we admit the existence of any uncertainty in the initial preparation of the measuring instrument (which is described in a classical phase space) a similar uncertainty will appear in the measured object, as a consequence of Liouville's theorem.

How does this uncontrollable perturbation affect the subsequent motion of the rf SQUID? As it is sufficient to measure Φ with a precision $\Delta\Phi\simeq h/2e$, it may seem that the resulting $\Delta Q\simeq e/\pi$ is negligible. It is not so, however, because of the very narrow range of parameters allowing macroscopic quantum coherence.[2] The dynamical properties of an rf SQUID can be schematically represented as the motion of a particle in a one-dimensional double-well potential.[1] As shown in Fig. 1, the wells must be very shallow, with two nearly equal energy levels lying below the central hump, and all the other levels well above it. If it is not so, the quantum tunneling time becomes inordinately long. That time is governed by a factor $\exp[\int P(\Phi)d\Phi/\hbar]$, where $P(\Phi)$ is the momentum missing to pass over the hump—the equivalent of $[2m(V-E)]^{1/2}$ in a mechanical problem. The hump should therefore not be too high above the energy levels, if we want to have a reasonable tunneling time. Qualitatively, since the distance between the bottoms of the two wells is h/e, it follows that P should not be much more than $\hbar/(h/e)=e/2\pi$.

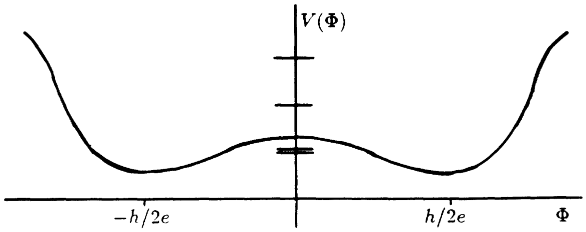

FIG. 1. The double well which simulates an rf SQUID. The four lowest energy levels have been indicated on the V axis.

Now consider the effect of the uncontrollable perturbation caused by a measurement of Φ. We have seen that the resulting uncertainty in the momentum is at least $\Delta Q\simeq e/\pi$. This is the same order of magnitude as the missing momentum P. Therefore, *the perturbation is not small*. Even without a detailed knowledge of the hypothetical theory which interpolates between classical mechanics and quantum mechanics, it is clear that a measurement of Φ cannot in general be noninvasive, if that measurement has to be performed during an experiment of the type discussed in Refs. 1 and 2.

The point is that a magnetic flux $\pm h/2e$ is *not* macroscopic, even if it is trapped in an rf SQUID manufactured to industrial specifications. Likewise, an antenna for gravitational waves may weigh several tons, but its fundamental vibration mode is so well decoupled from all the other degrees of freedom that it is expected to behave as a *quantum* harmonic oscillator over a time scale of several seconds.[11] As I explained elsewhere,[12] the distinction between microscopic and macroscopic systems is not in their sheer size, but in whether they can, or cannot, be perfectly isolated from their environment (or else placed in a perfectly controlled environment) over a long period of time compared to the duration of the experiment. A system is microscopic if dissipation is negligible. The stringent conditions required[2] to observe the so-called "macroscopic quantum coherence" of an rf SQUID are such that its magnetic flux must be considered as microscopic.

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