PHYSICAL REVIEW

LETTERS

VOLUME 61 | 31 OCTOBER 1988 | NUMBER 18

Excited Atoms in Strong Microwaves: Classical Resonances and Localization in Experimental Final-State Distributions

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(Received 1 July 1988)

Highly excited hydrogen atoms have been exposed to an intense pulse of microwaves with frequency near the initial classical electron orbit frequency. Measurements were made of the probability distribution for different bound states after the pulse. Structure in such distributions is found to be near classical, and arises from the presence of a classical resonance island in action-angle space. The measured distributions also yield values of electron localization length in action space that are in agreement with quantum localization theory and with numerical values based on quantum mechanics.

PACS numbers: 03.65.Bz, 32.80.Rm, 32.90.+a, 42.50.Hz

Many predictions of the mathematical theory of nonlinear dynamics have been confirmed by experiments on macroscopic systems described by classical physics. However, much of modern physics is in the microscopic realm described by quantum mechanics. Here there are outstanding questions, such as to what features of quantum systems correlate in the classical limit with interesting phenomena such as bifurcations and deterministic chaos. Thus there is considerable present interest in semiclassical quantum systems with nonintegrable underlying classical nonlinear dynamics.[1] One such system is the highly excited hydrogen atom in a partially ionizing microwave field. This system has classical electron dynamics involving both regular and chaotic trajectories.

Past evidence for near-classical behavior in our system has included the relationship between the microwave field strength "thresholds" for classical chaos and for ionization. Experiments working with atoms of principal quantum number n_0 and with a statistical distribution of angular momentum quantum number values have provided microwave field strength values for 10% apparent ionization probability, for the case of a microwave exposure time of about 300 microwave oscillations.[2] The measured probabilities included contributions from final bound states with principal quantum numbers n above a value as high as 97. Theoretical chaos threshold field values have been based on classical models. For a given initial value of classical action, these values typically have been determined by the condition that 1% of the trajectories within an ensemble uniform in initial value of the angle variable be classically chaotic. When the microwave frequency ω is less than the initial value of the bound electron's classical orbit frequency $1/n_0^3$, the experimental ionization and classical chaos "thresholds" are found to be the same typically to within 15%.[3] As a function of the classically scaled frequency $\omega_0 = n_0^3\omega$, both thresholds reveal structure that arises from classical resonances that occur when the ratio of microwave frequency to the classical electron orbit frequency is close to a rational number.

The classically chaotic trajectories produce a growth in time of the mean values of classical action and of energy. This evolution can be described by a Fokker-Planck equation,[4] a numerical finding valid for both one- and two-dimensional models.[5,6] As a consequence, the evolution of the distribution of classical action values $I = n\hbar$ generated by an ensemble of chaotic trajectories with initial $I_0 = n_0\hbar$ is believed to be a diffusion process. After a finite microwave exposure time t, this distribution can be approximated by an exponential dependence with a characteristic length in action space that we call the classical distribution length $l_C(t)$. This definition is in keeping with that for the quantum distribution length $l_Q(t)$ obtained from numerical quantum calculations,[5] and we write

$$P_i(n) = P_0 \exp[-2|n-n_0|/l_i], \quad n > n_0, \quad i = C, Q.$$

For $\omega_0 < 1$, our system when treated quantum mechanically is found to be near classical for all times less than that for almost 100% ionization probability.[5] The length $l_Q(t)$ continuously grows during these times, and has values close to $l_C(t)$.[5,7] However, for higher frequencies and some field strengths, numerical quantum calculations find a suppression of the diffusive classical time evolution.[5,7] The quantum and classical evolutions are then found to depart from one another after a "break time" and the quantum evolution of the distribution length stops after a "freezing time" that can be tens of hundreds of microwave oscillation periods. After the freezing time, $l_Q(t)$ remains constant at a value l_Q called the localization length, while $l_C(t)$ continues to grow for orders of magnitude longer times.

A possible explanation for the quantum suppression is based upon the theoretical finding that the problem of Anderson localization of a quantum electron in a lattice with random on-site potentials can be mapped[8,9] into the problem of quantum localization in action space for the quantum kicked rotator, another driven system also exhibiting classical deterministic chaos. Numerical quantum calculations for the kicked-rotator and hydrogen-in-microwave problems given qualitatively similar results, suggesting that quantum suppression in both problems arises from destructive wave interference produced by the randomness of the chaotic underlying classical trajectories.[5-10]

In the present paper we present the first experimental evidence for strong classical resonance effects in final-bound-state quantum number distributions, which correspond in the classical limit to classical action distributions. We also present first values for experimental distribution lengths at frequencies in the fundamental resonance region $\omega_0 \cong 1$ (when corrected for an applied static electric field, up to 1.2), where quantum suppression is predicted to produce observable effects.[11] We compare values of experimental distribution length l_e with previously calculated numerical values of the quantum localization length l_Q obtained for parameter values in the region of quantum suppression.

Our experimental approach has been described previously.[12,13] Hydrogen atoms in a monoenergetic fast atomic beam were collinearly laser excited while in static electric fields to a selected quantum state with parabolic quantum numbers $n, n_1, m = n_0, 0, 0$. A monodirectional static electric field of 6.25 V/cm was present during the entire history of each atom. During the experiment time of a few microseconds, such a field does not ionize $n,0,0$ atoms with n less than 100. The laser-excited atoms passed through a microwave waveguide region, resulting in their exposure to a pulse of microwave electric field linearly polarized along the static electric field direction. The pulse time was determined by the atom's velocity and the length of the waveguide region. The distribution of final quantum states of the atoms leaving the waveguide region was ascertained by means of differential state-selective static electric field ionization. As in our earlier work,[12,13] the response of the atoms to the microwave pulse was found experimentally to change n but not n_1 or m by more than one unit. That m should not change is a standard electric dipole selection rule; for n_1, this is understood in terms of an approximate selection rule for electric dipole multiphoton transitions between pairs of atomic Stark quantum states,[13] and justifies the comparison of the experimental data with quantum and classical predictions that are based upon one-dimensional models.

An important feature of our apparatus was a new waveguide system, permitting an increase in the microwave frequency up into the range 12 to 18 GHz and reducing the microwave exposure pulse time to 7.5 ns. The atoms were passed through holes (electric-discharge machined) in opposite narrow sides of a standard WR-62 rectangular waveguide to be operated in the fundamental TE_{10} mode. For this mode, the microwave electric field is zero at the narrow sides. The envelope of the microwave pulse was then half a sine wave, modified by the presence of the holes. Machining of the holes produced a measured fractional change in microwave power transmission through the waveguide of only 4%. This and other measures of the loss of microwave power out through the holes indicate that one effect of the holes is to produce long tails on the microwave pulse envelope with a field strength approximately 1% of that of the peak of the pulse envelope.

To investigate the possibility that laser excitation might be occurring within the waveguide, background final-state distributions were measured with all atoms that were laser excited before the waveguide field being ionized away. These background contributions were found to be fractionally small; correcting for them did not significantly change any of our results.

Spectrum-analyzer and power-meter studies were carried out in search of any broad-band microwave noise emitted by the two different microwave oscillators that we used. Most measurements used the unamplified output of a Hewlett Packard model 8690B frequency synthesizer. An upper bound on its total rms broad-band noise power over the 12–18 GHz nominal bandwidth of the system was 4 orders of magnitude lower than typical sine-wave rms microwave power levels used in the experiments. A tunable filter for the frequency range 16–18 GHz was available to reduce the possible noise bandwidth by 2 orders of magnitude. Its addition to the microwave system was found to not change our data obtained at 18 GHz.

Figure 1 shows experimental final-bound-state quantum number distributions for $n_0 = 72$, for three frequen-

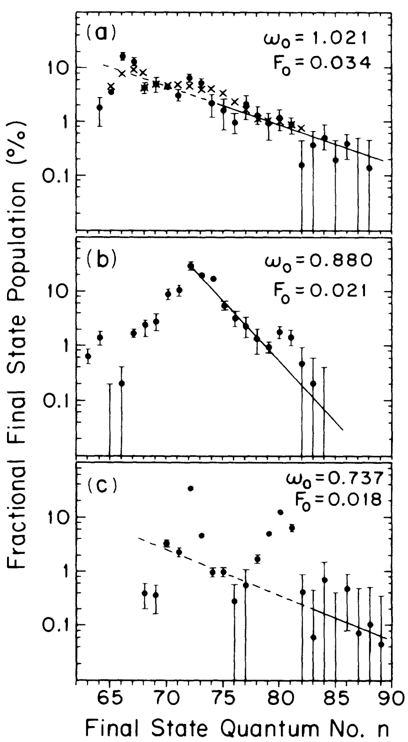

FIG. 1. Experimental final-bound-state quantum number distributions for three pairs of values of classical scaled microwave frequency $\omega_0 = n_0^3\omega$ and peak microwave field strength $F_0 = n_0^4 F$. The error bars are an rms measure of overall data reproducibility. Values of experimental distribution length are obtained from the solid straight lines. For case (a), the crosses are numerical predictions assuming classical physics, from Ref. 11.

cies 13.00, 15.51, and 18.00 GHz. Microwave field strengths were chosen that produce apparent ionization (experimentally defined to include excitation to n above 93 by an applied static electric field not present in the microwave region) in the 5% to 70% range. In all cases there is a central maximum in the distribution near the initial value of 72. At 13 GHz there is a large second maximum near $n = 80$, while at 18 GHz there is one near $n = 66$ that actually is larger than the central maximum. Also shown are the results of classical calculations available for the 18-GHz case,[11] which are in good agreement with the experimental results. In particular, the structure near $n = 66$ is reproduced, and this maximum is found to be the largest. Calculations of action-angle phase-space pictures for field strengths equal to that at the maximum of our microwave pulse for this case have been carried out, and display the $\omega_0 = 1$ resonance island of regular trajectories embedded in a chaotic region.[14] In action the island covers the range from $n = 66$ to 77. Thus our data support expectations based on classical physics that some structure around n_0 in final-state distributions can be associated with islands of regular classical electron trajectories in action-angle space.

TABLE I. Experimental values of distribution length l_e in principal quantum number space, obtained for the given microwave frequencies, classically scaled frequencies $\omega_0 = n_0^3\omega$, and classically scaled peak microwave field strengths $F_0 = n_0^4 F$. The value of the initial atom quantum number n_0 was 72.

Frequency (GHz)	ω_0	F_0	l_e
15.512	0.880	0.021	4 ± 2
15.512	0.880	0.042	6 ± 3
13.000	0.737	0.018	9 ± 6
18.000	1.021	0.039	11 ± 7
18.015	1.021	0.034	14 ± 4
13.000	0.737	0.029	17 ± 10

Also shown in Fig. 1 are straight lines drawn through the data for $n > n_0$ that extend to the highest values observed. Following the previous theoretical work mentioned above, such lines are directly related to experimental values of distribution length via the equation for $P_i(n)$. We suspect that near n_0 the data reflect unknown mixtures of regular and chaotic classical trajectories, while about $n = 81$ or so all the distributions reflect only chaotic ones. Thus sets of straight lines were drawn that contain various reasonable possibilities, omitting obvious structure away from n_0. For each set of data shown, and for three other sets, a typical line was chosen to give a value of the distribution length, and the most extreme of other lines were taken to provide an uncertainty in the length value. The case of Fig. 1(c) was the most difficult one to assess. Table I lists the results. Figure 2 compares these results with numerical quantum results for quantum localization lengths present after the "freezing" sets in, for a wide choice of parameters that all place the system in the quantum suppression regime.[5,15] The axes in Fig. 2 are scaled values of the microwave field strength and of the localization length, the scaling chosen to connect these values with a localization model

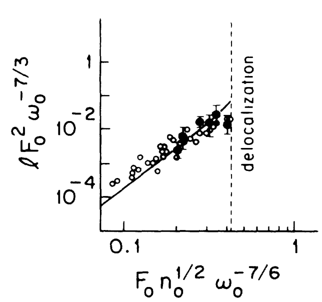

FIG. 2. Scaled values of localization length l, following Refs. 5 and 15. The open circles are numerical results based on quantum mechanics, the filled circles are the experimental data of Table I, and the solid line follows from a model localization theory. The vertical dashed line is the theoretically predicted quantum delocalization border (Refs. 5 and 15).

that also describes the quantum kicked rotator.[5,15] (In both systems, the localization length is approximately proportional to the classical diffusion constant.) Our data are found to agree with the quantum numerical results and appear to confirm the solid curve shown, which follows from the localization model, except very near the vertical dashed line which is the so-called quantum delocalization border. There, both several numerical points and one experimental one suggest a deviation from the model that might be a smoothing out of the border.

The one presently available quantum calculation for one of our 18-GHz cases shows that for that case our experimental microwave pulse time is just long enough for the "freezing" of the quantum suppression to set in.[11] In this situation classical distribution lengths and quantum localization lengths are comparable. At present we cannot conclude on the basis of experiment alone that the freezing actually occurs. It may require new experiments with a much longer microwave pulse time to establish this.

The authors thank the National Science Foundation for continued support of their research.

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