Thermal Shape Fluctuations, Landau Theory, and Giant Dipole Resonances in Hot Rotating Nuclei

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A macroscopic approach to giant dipole resonances (GDR's) in hot rotating nuclei is presented. It is based on the Landau theory of nuclear shape transitions and provides a unified description of thermal fluctuations in all quadrupole shape degrees of freedom. With all parameters fixed by the zero-temperature nuclear properties the theory shows a very good agreement with existing GDR measurements in hot nuclei. The sensitivity of the GDR peak to the shape of hot nuclei is critically examined. Low-temperature experimental results in Er show clear evidence for changes in the nuclear energy surface, while higher-temperature results are dominated by the fluctuations.

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Experimental information on the shapes of highly excited rotating nuclei is presently becoming available through studies[1-4] of spectra of giant dipole resonances (GDR's) built on nuclear excited states. Theoretically, the Landau theory of shape transitions was recently proposed,[5,6] and it predicts a universal pattern of the mean-field shape evolution as a function of temperature and angular momentum. Shape fluctuations around mean-field values are not negligible in finite nuclear systems. They were considered in Ref. 6. Their effects on the GDR cross sections were first investigated in Ref. 7 in the context of a particular microscopic model. The Landau theory offers a very general framework to include the thermal fluctuations of nuclear shapes since it predicts the general dependence of the nuclear free-energy surface $F(T,\omega,\alpha_{2\mu})$ on all five quadrupole deformation parameters $\alpha_{2\mu}$ for relevant values of the temperature T and the rotational velocity ω.

In this Letter we present a macroscopic approach to the GDR in hot nuclei in which the quadrupole shape parameters play the role of the macroscopic degrees of freedom. Based on the Landau theory, this approach offers a unified description of fluctuations in all quadrupole shape degrees of freedom. This of course means that the fluctuations of the nuclear orientation relative to the axis of rotation are also included. We use our approach to address the two important questions: (i) Is the theory able to describe the observed GDR systematics? (ii) How sensitive is the GDR as a probe of the nuclear shape at finite temperature? Below we summarize the main results of the approach; for details see Ref. 8.

We start with the expression for the cross section for the absorption of $E1$ quanta of energy ϵ by an equilibrated nucleus of energy E and spin J,

$$\sigma=\frac{4\pi^2\epsilon}{3\hbar c}\frac{1}{\rho(E,J)}\sum_{\substack{i,f\\ \mu,M}}|\langle f|D_\mu|i,J,M\rangle|^2\delta(E-E_i)\delta(E'-E_f)\,, \tag{1}$$

where $E'=E+\epsilon$. In (1) the sum over initial states is restricted to states of spin J and ρ is the density of states. Equation (1) can be expressed as

$$\sigma=\frac{2\pi\epsilon}{3c\hbar^2}\int_{-\infty}^{\infty}dt\,e^{i\epsilon t/\hbar}\sum_\mu\langle D_\mu^\dagger(t)D_\mu(0)\rangle_{E,J}\,, \tag{2}$$

where $D_\mu(t)$ is the dipole operator in the Heisenberg representation and the average is over the initial microcanonical ensemble, i.e.,

$$\langle\Theta\rangle_{E,J}\equiv\mathrm{Tr}[\delta(E-H)P_J\Theta]/\mathrm{Tr}[\delta(E-H)P_J]\,,$$

where P_J is the projection onto spin J.

We next replace the microcanonical averaging in $\langle D_\mu^\dagger(t)D_\mu(0)\rangle$ with given E and J by a canonical one with the corresponding temperature T and angular velocity ω. This is a common approximation and is discussed, e.g., in Ref. 5. In the absence of rotation ($\omega=0$), the dipole correlation tensor is diagonal in the principal frame and is assumed to be characterized by a natural frequency $E_i=E_i^0$ and decay width Γ_i along the ith semiaxis. The corresponding contribution to the photoabsorption cross section in (2) is then proportional to a Lorentzian

$$f_i(\epsilon)=\frac{\Gamma_i\epsilon^2}{(\epsilon^2-E_i^2)^2+\Gamma_i^2\epsilon^2}\,. \tag{3}$$

The resonance parameters E_i^0 and Γ_i depend on the deformation in a way specified in Eq. (8) below. In the rotating case ($\omega\neq0$) the Coriolis force shifts the normal frequencies from E_i^0 to ω-dependent $E_i(\omega)$ and makes the correlation tensor nondiagonal in the intrinsic frame. To calculate $E_i(\boldsymbol{\omega})$ we assume harmonic vibrations of the dipole in the intrinsic principal frame rotating with angular velocity $\boldsymbol{\omega}$ and thus solve for the normal modes of $\sum_i E_i^0 D_i^\dagger D_i - \boldsymbol{\omega}\cdot\mathbf{J}$ in this frame. In the normal mode variables the dipole correlation tensor is assumed to be diagonal and characterized by equal strengths. Transforming the dipole correlation function from the intrinsic to the laboratory frame one finds the following expression for the photoabsorption cross section of a nucleus with fixed $\alpha_{2\mu}$, i.e., fixed deformation β and γ and fixed orientation whose Euler angles are $\Omega=(\psi,\theta,\phi)$:

$$\sigma(\epsilon;\beta,\gamma,\Omega)=\frac{4\pi e^2\hbar}{mc}\frac{ZN}{A}\sum_{j=1}^{3}[S_j^{(0)}f_j(\epsilon)+S_j^{(-)}f_j(\epsilon+\hbar\omega)+S_j^{(+)}f_j(\epsilon-\hbar\omega)]\,. \tag{4}$$

Here $S_j^{(0,\pm)}$ are the resonance strengths (in units of the classical sum rule). These strengths depend on E_i^0 and $\boldsymbol{\omega}$ and are given elsewhere.[8]

In the mean-field approximation the β, γ, and Ω to be used in (4) should be found from the minimization of the free-energy surface $F(T,\boldsymbol{\omega},\alpha_{2\mu})$. In order to account for the thermal fluctuations away from the mean-field values we average (4) over all possible $\alpha_{2\mu}$,

$$\sigma(\epsilon;T,\omega)=\int D[\alpha_{2\mu}]P[\alpha_{2\mu}]\sigma(\epsilon;\beta,\gamma,\Omega)\,. \tag{5}$$

Here

$$D[\alpha_{2\mu}]=\prod_\mu d\alpha_{2\mu}=\beta^4|\sin 3\gamma|\,d\beta\, d\gamma\, d\Omega \tag{6}$$

is the unitary invariant measure and $P[\alpha_{2\mu}]$ is the probability to find the nucleus in a "state" of deformation $\alpha_{2\mu}$.[6] The averaging technique in (5) is justified when the macroscopic quadrupole shape variables are slow, i.e., in the so-called adiabatic approximation.[9] Treating the latter as classical variables we have in the thermodynamical fluctuation theory of Einstein[10] and Callen[11]

$$P[\alpha_{2\mu}]=Z^{-1}\exp[-F(T,\boldsymbol{\omega},\alpha_{2\mu})/T]\,,$$
$$Z(T,\omega)=\int D[\alpha_{2\mu}]\exp(-F/T)\,. \tag{7}$$

Expression (5) takes into account fluctuations not only in the intrinsic shape parameters β,γ but also in the orientation Ω of the intrinsic shape with respect to the rotation axis. This was commonly neglected in the past.[7]

We have adopted the following expressions for the resonance parameters in Eq. (3):

$$E_j^0=E_0\frac{R_0}{R_j}=E_0\exp\left[-\left(\frac{5}{4\pi}\right)^{1/2}\beta\cos\left(\gamma+\frac{2\pi}{3}j\right)\right],$$
$$\Gamma_j=\Gamma_0(E_j/E_0)^\delta\,. \tag{8}$$

The first relationship follows from the nuclear hydrodynamics model.[12] The second is consistent with the experimental dependence of the ground-state GDR widths on deformation in heavy nuclei.[13] It can also be derived via one-body dissipation theory[8] for an arbitrary triaxial deformation (where $\delta=1.6$). The three parameters, E_0, Γ_0, and δ were assumed temperature independent. They were fixed for each nucleus from its zero-temperature properties, i.e., by comparing the experimental ground-state GDR cross section to that predicted by the collective quadrupole plus giant dipole Hamiltonian[12] spread according to the width model (8) above.

The free-energy surfaces F were constructed with use of a standard Nilsson Hamiltonian with a cranking term and a Strutinsky renormalization. The cranking calculations were performed only for $\boldsymbol{\omega}$ parallel to a principal axis, and the free energy for a general orientation Ω was determined from the Landau expansion.[5]

The simple theory presented above provides a very good description of the available data. Examples of this are presented in Fig. 1 in a direct comparison of calculated GDR strengths with experimental results for ^{166}Er and ^{160}Er at various temperatures and spins. The resonance parameters obtained from the ground-state GDR cross section of ^{166}Er were $E_0=14.4$ MeV, $\Gamma_0=3.64$ MeV, and $\delta=1.6$. The same parameters were also used for ^{160}Er where the ground-state data do not exist. We

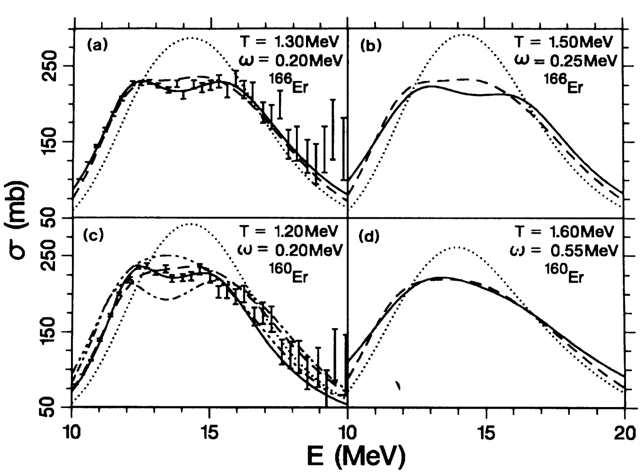

FIG. 1. Comparison of the calculated photoabsorption GDR cross sections (dashed lines) with the results of CASCADE code fits to the experimental ones (solid lines). (a) ^{166}Er at $T=1.30$ MeV, $\omega=0.20$ MeV (corresponding to an excitation energy $E_x=49.2$ MeV in Ref. 2). (b) ^{166}Er at $T=1.50$ MeV, $\omega=0.25$ MeV ($E_x=61.5$ MeV in Ref. 3). (c) ^{160}Er at $T=1.20$ MeV, $\omega=0.20$ MeV ($E_x=43.2$ MeV in Ref. 2). (d) ^{160}Er at $T=1.60$ MeV, $\omega=0.55$ MeV ($E_x=90.3$ MeV in Ref. 4). The dotted lines represent the calculated GDR with the $\beta d\beta d\gamma$ metric (Ref. 7). The other lines in (c) are as in Fig. 2. Note that the error bars plotted in (a) and (c) are only suggestive of the accuracy of the experimental measurements and do not represent the actual data.

TABLE I. Comparison of two-Lorentzian fits to ^{166}Er cross sections at $T=1.3$ MeV, $\omega=0.2$ MeV.

	E_1 (MeV)	Γ_1 (MeV)	S_1	E_2 (MeV)	Γ_2 (MeV)	S_2
Experiment[a]	12.15 ± 0.09	3.69 ± 0.23	0.43 ± 0.07	15.77 ± 0.17	5.75 ± 0.71	0.74 ± 0.11
Theory	12.25	3.57	0.38	15.56	5.54	0.67

[a]Reference 2.

see from Fig. 1 that the agreement between the experimental CASCADE fits of the photoabsorption cross section (solid lines) and our theoretical calculation (dashed lines) is remarkable considering that there are essentially no free parameters in the theory. A two-component Lorentzian fit to the theoretically calculated GDR (Table I) agrees well with the CASCADE fit—especially when the systematic uncertainty ($\sim 20\%$) in the experimental strength is considered.

The dotted lines in Fig. 1 show the GDR calculated when the metric $\beta d\beta d\gamma$ of Ref. 7 is used instead of (6) but with the same resonance parameters E_0, Γ_0, and δ. If other parameters are used one may fit with this metric the low-temperature ($T\sim 1$ MeV) but not the higher-temperature measurements. Thus the experimental results suggest that the metric (6) (which follows the "unified" approach) should be used.

Having established the ability of our theory to provide a quantitative account for the experimental situation we wish to address the second question posed at the beginning of this article, i.e., how sensitive is the GDR as a probe of the shape of hot nuclei. In Fig. 2 we show the systematics of the calculated GDR cross sections for ^{166}Er at several temperatures and angular velocities. At $T=1$ MeV and low ω (0 and 0.35 MeV) the nucleus has a prolate equilibrium shape with relatively small shape fluctuations and the cross section clearly shows the splitting of the two modes of vibration. At higher temperatures ($T\gtrsim 2$ MeV) or at higher ω's the nucleus becomes oblate. However, the fluctuations are large and favor triaxial shapes which result in an asymptotic line shape with a long high-energy tail.

The increase of the thermal fluctuations with increasing T is due to two factors in our theory: the explicit appearance of T in the denominator of the exponent in (7) and the T dependence of the free-energy surface $F(T,\omega,\alpha_{2_\mu})$ there. It is the last factor which is associated with the shape changes caused by heating and rotating a deformed nucleus. In order to see the sensitivity of the GDR results to these changes, we show in Fig. 2 the results of the calculations with "wrong" surfaces F substituted in (7); the $T=0$ (the "dot-dot-dashed-dashed" lines) and the $T=3$ MeV (the "dot-dot-dot-dashed" lines) surfaces. At $T=1$ MeV and $\omega=0$ these cross sections are significantly different from the one calculated with the "correct" surface. At $T\gtrsim 2$ MeV the GDR becomes less sensitive to surface variations especially at higher ω.

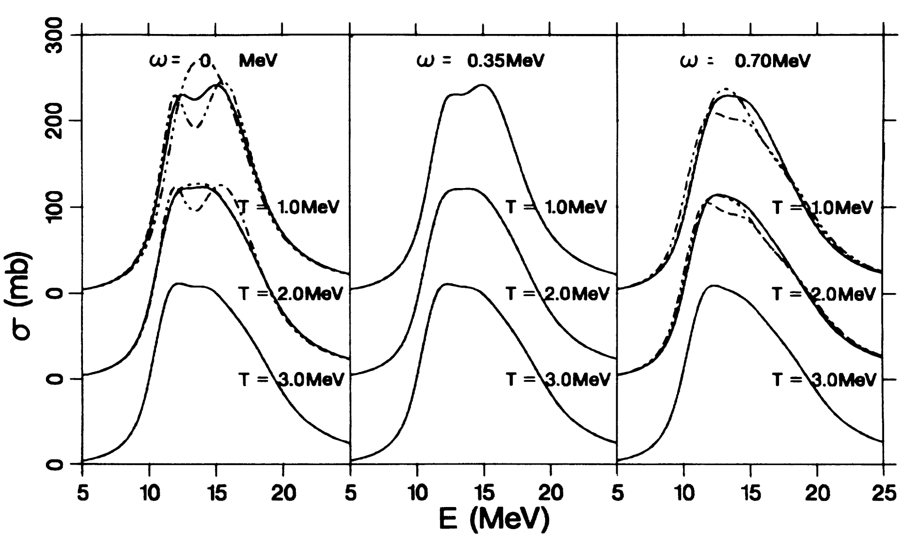

FIG. 2. Calculated ^{166}Er GDR cross sections at various temperatures and angular velocities (solid lines). The dot-dot-dashed-dashed lines represent the result of the use of the $T=0$ MeV free-energy surface and the dot-dot-dot-dashed lines represent the use of the $T=3$ MeV surface (see text).

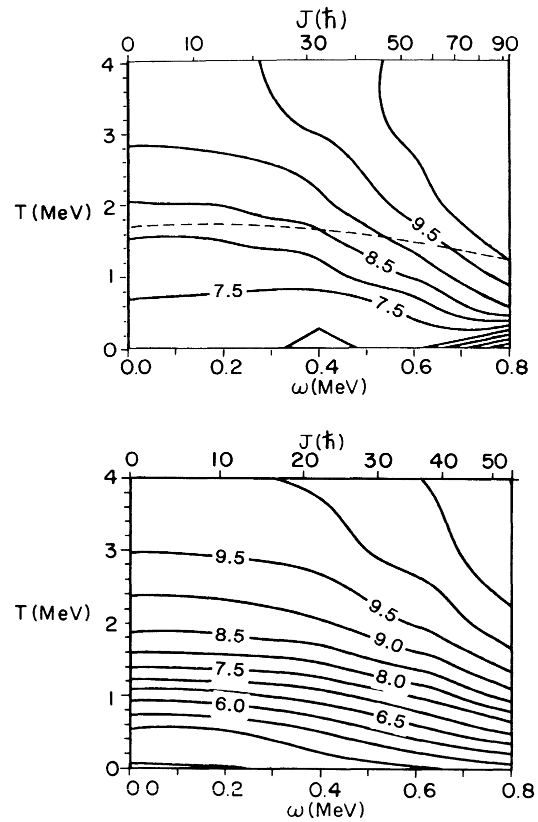

FIG. 3. FWHM contour lines (MeV) in the T-ω (or T-J) plane of GDR for ^{166}Er (top) and ^{140}Ce (bottom). The shape transition line is shown dashed in the upper figure.

For ^{160}Er at $T=1.2$ MeV and $\omega=0.2$ MeV we also show in Fig. 1 the GDR cross sections obtained by using the "wrong" surfaces at $T=0$ and 3 MeV (as in Fig. 2). The experimental results clearly indicate that at $T=1.2$ MeV ^{160}Er, though still prolate, has a softer energy surface than at $T=0$. Our theoretical calculations[8] indicate that between cases 1(c) and 1(d) in the figure ^{160}Er should make a transition to an oblate shape. However, at $T=1.6$ MeV and $\omega=0.55$ MeV the GDR becomes much less sensitive to variations in the energy surface. Within the present experimental accuracy, the GDR data are consistent with both the $T=1.2$ and 1.6 MeV surfaces (when used with the correct temperature of 1.6 MeV). From the results of Fig. 1 we observe that the decrease in the GDR sensitivity is smaller when only T increases than when both T and ω increase [as is the case in Fig. 1(d)]. Thus one may be able to learn more about the predicted shape changes by measuring case 1(d) at $\omega=0.2$ MeV.

In Fig. 3 we have plotted the cross sections' full width at half maximum (FWHM) as contour lines in the T-ω (or T-J) plane for ^{166}Er (a typical deformed nucleus) and for ^{140}Ce (a typical spherical nucleus). The latter nucleus shows a more dramatic change with temperature: Its width at $T=2$ MeV is about twice that at $T=0$ MeV (in accord with experiment[1]). This occurs because its free-energy surface "softens" as T rises from 0 to 2 MeV and because as a spherical nucleus it has only a single Lorentzian component at $T=0$. Erbium has a deformed ground state and its FWHM changes more slowly for $T \leq 2$ MeV. However, near the phase transition line of ^{166}Er the width increases more rapidly than elsewhere (from 8 to 8.5 MeV). This phenomenon may be detectable as the resolution of current experiments improves in the future.

Another interesting phenomenon is that for both nuclei the widths increase much more slowly at temperatures above 3 MeV. This is associated with the disappearance of shell effects at this temperature region. It will be interesting to see if this effect is observed experimentally.

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[1]K. A. Snover, Annu. Rev. Nucl. Part. Sci. **36**, 545 (1986).

[2]C. A. Gossett, K. A. Snover, J. A. Behr, G. Feldman, and J. L. Osborne, Phys. Rev. Lett. **54**, 1486 (1985).

[3]J. J. Gaardhoje, C. Ellegaard, B. Herskind, and S. G. Steadman, Phys. Rev. Lett. **53**, 148 (1984).

[4]D. R. Chakrabarty, M. Thoennessen, S. Sen, P. Paul, B. Butsch, and M. G. Herman, to be published.

[5]Y. Alhassid, S. Levit, and J. Zingman, Phys. Rev. Lett. **57**, 539 (1986), and Nucl. Phys. **A469**, 205 (1987).

[6]S. Levit and Y. Alhassid, Nucl. Phys. **A413**, 439 (1984).

[7]M. Gallardo, M. Diebel, T. Dossing, and R. A. Broglia, Nucl. Phys. **A443**, 415 (1985); M. Gallardo, F. J. Luis, and R. A. Broglia, Phys. Lett. B **191**, 222 (1987).

[8]Y. Alhassid and B. Bush, Center for Theoretical Physics, Yale University, Report No. YCTP-N16-88, 1988 (to be published).

[9]A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1975), Vol. 2, pp. 451–455.

[10]L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, Oxford, 1980), Part 1, Chap. XII.

[11]H. B. Callen, *Thermodynamics and Introduction to Thermostatistics* (Wiley, New York, 1985), Chap. 19.

[12]J. M. Eisenberg and W. Greiner, *Nuclear Theory* (North-Holland, Amsterdam, 1975), Vol. 1, Chap. 10 and 11.

[13]P. Carlos, R. Bergere, A. Beil, A. Lapetre, and A. Veyssiere, Nucl. Phys. **A219**, 61 (1974).