New Mechanism for Superconductivity in Cosmic Strings

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We point out the existence of reasonable models in which cosmic strings contain charged-vector-boson fields, and show that these give rise to vector-meson superconductivity analogous to Witten's scalar-boson superconductivity. In addition, in such models the electromagnetic field is distorted within the string in such a way that it couples to the otherwise neutral Higgs field, giving a new mechanism for scalar superconductivity.

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In some models containing cosmic strings[1] the strings exhibit superconductivity[2]; if this actually occurs in nature, it can have very important cosmological consequences.[3-5] Witten, in Ref. 2, discussed two mechanisms, involving, respectively, charged-scalar-boson and charged-fermion fields, which can produce superconductivity. Both mechanisms are model dependent, and hence they do not guarantee that cosmic strings, even if they exist, are superconducting. As we shall see below there is a class of reasonable models in which there are strings with charged-vector-boson (CVB) fields. This raises the question of whether CVB's, like charged scalars, give rise to superconductivity. (Preskill[6] has remarked on the possibility of CVB superconductivity, though without discussing specific models or details.) We shall find that superconducting CVB currents can indeed be induced. Moreover, in these models the electromagnetic (EM) field within the string is distorted in such a way that it couples to the scalar Higgs field of the string, which is electrically neutral outside the string core; this can also give rise to electric currents in the string. Thus we find that there is a plausible class of models in which cosmic strings exhibit superconductivity due to a new mechanism.

As a specific example of such a model, consider the following chain of possible spontaneous symmetry breaking[7]:

$$\mathrm{spin}(10) \rightarrow SU(5) \otimes Z_2 \rightarrow SU(3) \otimes SU(2) \otimes U(1) \otimes Z_2 \rightarrow SU(3) \otimes U(1) \otimes Z_2 \,,$$

where Z_2 is an unbroken discrete symmetry group of two elements. Strings are formed in the first phase transition, when the discrete symmetry appears in the unbroken subgroup, which we will call H, and in which a scalar Higgs field acquires a vacuum expectation value, ϕ; ϕ stands for a vector, whose dimensionality is that of the representation of G in which ϕ belongs. We let η be the value of $|\phi|$ at which the Higgs potential takes on its minimum value. Along a circle parametrized by the angle θ and centered on a string, ϕ is given by

$$\phi(\theta) = g(\theta)\phi(0) = \exp(i\tau^s\theta)\phi(0) \,, \tag{1}$$

where τ^s is the matrix of S, one of the generators of G; S is not a generator of the unbroken subgroup H, and thus acts nontrivially on ϕ. For the string to be topologically stable and ϕ to be single valued, τ^s must satisfy

$$\exp(2\pi i\tau^s) = P \,, \tag{2}$$

where P is the nontrivial element of Z_2. The string has a radius of order $r_0 \approx (\eta e)^{-1}$ within which $|\phi|$ differs significantly from its asymptotic value of η, where e is the gauge coupling constant; we are assuming, for simplicity, that the gauge- and Higgs-boson masses are approximately equal.

In addition to ϕ there are also gauge-boson fields associated with the string. Consider a single string loop. We adopt an approximately cylindrical coordinate system with coordinates r, θ, and z with the z axis along the string, which for most purposes can be regarded as straight since the radius of curvature, R, is very large. We may choose a gauge in which ϕ is constant asymptotically and also on the surface $\theta=0$; τ^s, and therefore ϕ, will then be independent of z. One can then write the coupled field equations for ϕ and the gauge fields A^i_μ, where i ranges over the generators of G. With the exception of A^s_θ, the azimuthal component of the field coupled to S, $A^i_\mu=0$ in vacuum since the current j^i_μ to which it couples vanishes,[8] while

$$A^s_\theta = A^s_\theta(r) \sim -1/er \quad (r_0 \ll r \ll R) \,. \tag{3}$$

We will adopt the notation $A_\mu(r)$ for the vector belonging to the adjoint representation of G whose components are $A^i_\mu(x_a)$.

The generator S is not uniquely determined by the condition (2), and one must appeal to dynamics to determine it. In the $\mathrm{spin}(10) \rightarrow SU(5) \otimes Z_2$ example, it is shown in Ref. 8 that for most, and probably all, values of the Higgs potential parameters the lowest mass per unit length for strings is given, up to a gauge transformation, by the choice $S=t^1_R$, where t^1_R is the first component of the right-handed weak isospin; hence this choice of S, which we assume from now on, is likely to describe energetically stable strings. Since $[t^1_R,Q]\neq 0$, where Q is the EM charge, A^s_θ for this choice of S is a charged field.

For $S=t_R^1$, the generators of H, in particular Q, depend on θ, with

$$\tau^q(\theta)=g(\theta)\tau^q(0)g^{-1}(\theta). \tag{4}$$

Let $A_\mu^q(x_a)$ be the EM vector potential and $u(0)$ a unit vector in internal space in the direction of Q at $\theta=0$ and belonging to the adjoint representation. $A_\mu^q(x_a)$ is the component of $A_\mu(x_a)$ in the direction of $u(\theta)$ where

$$u(\theta)=\exp(iT^s\theta)u(0), \tag{5}$$

with T^s the matrix of S in the adjoint representation. We take a set of unit basis vectors in internal space which are eigenvectors of T^s and write

$$u(0)=\sum\nolimits_n c_n u_n, \tag{6}$$

where $T^s u_n = n u_n$ with all n integral, as they will be in models where A_μ^q is single valued. In our model, $Q=Y_R+t_R^3$, where Y_R, the right-handed analog of the weak hypercharge, is an $SU(2)_R$ invariant, which means that n in Eq. (6) takes on the values 0, ± 1. Equations (5) and (6) now give

$$A_\mu^q=\sum\nolimits_n c_n^* e^{-in\theta}A_\mu^n. \tag{7}$$

Compare this case with the case of a charged-scalar field as discussed in Ref. 2. Let q be the electric charge carried by the scalar field and ϕ_c be its expectation value within a string. Global EM gauge invariance allows the replacement $\phi_c \rightarrow e^{iq\alpha}\phi_c$, where α is a constant phase angle. One then generalizes to solutions with $\alpha=\alpha(z)$, where $\alpha(L)-\alpha(0)=2N\pi$ for a string loop of length L; the topologically conserved winding number N may have any integer value. For an isolated loop, solutions with $N=0$ describe states carrying a persistent nonzero electric current. The present case differs in a significant way; because Q depends on θ there is no such thing as a *global* electromagnetic gauge transformation. The analogous transformation under which the interaction between ϕ and A_θ is invariant is

$$A_\theta^s\tau^s \rightarrow \exp[i\tau^q(\theta)\alpha][A_\theta^s\tau^s+(\alpha/er)\,\partial_\theta\tau^q(\theta)]\exp[-i\tau^q(\theta)\alpha]. \tag{8}$$

Even with α constant, the θ dependence of Q forces the presence of a nonvanishing inhomogeneous term in the gauge transformation (8), without which (8) would not yield a static solution of the field equations. If $\alpha\neq 0$, this term is singular at $r=0$ where θ, and hence Q, is undefined; hence $\alpha=0$ at $r=0$. Thus, if one attempts to generalize by letting $\alpha=\alpha(z)$, one finds that the winding number $N=0$ at $r=0$. Thus $N=0$ at all r, since if $N(r)\neq 0$, A_θ^s must vanish at some $r_1<r$ to avoid a discontinuity in N at temperatures below that at which H develops a U(1) factor and becomes multiply connected, and it will be energetically favorable for r_1 to relax toward $r=\infty$ with time, leaving $N=0$ within the string in equilibrium. Since $N=0$, we can, and shall, choose an EM gauge, which we call the z gauge, with A_θ^s given by Eq. (3) and independent of z (and θ). The fact that $N=0$ means that a totally isolated string loop will carry no current. The Maxwell equations will be homogeneous in A_μ^q, and, apart from possible gauge artifacts, the only nonsingular solution satisfying the boundary condition of vanishing at infinity will be the trivial one $A_\mu^q=0$.

The situation is different, however, if the loop responds to an external EM field, whose sources provide inhomogeneous terms in the field equations. This difference reflects the fact that the current in a superconducting loop depends both on the winding number N, which determines the total magnetic flux through the loop, and on the external magnetic flux. We look for solutions to the field equations for a string in the presence of a plane EM wave polarized along the z axis. We take the wave to have frequency $\omega\ll\eta$ so that for most purposes we can consider the fields as static and neglect time derivative terms. In the z gauge the wave will be described by an A_μ^q whose only nonzero component is A_z^q, which is independent of z. This follows because one can easily see that there is a contribution to the current j_θ^s coupled to A_θ^s which depends on A_μ^q; this current is similar in structure to the EM current which we obtain below in Eq. (17) but will be quadratic in one of the components of the EM vector potential rather than in A_θ^s, and will be a small correction to the principal contribution to j_θ^s coming from the Higgs field ϕ.[8] Thus in a gauge in which A_μ^q depends on z, A_θ^s will also.

The definition of the u_n together with the fact that $T_{jk}^s=if^{jsk}$ where the f^{jsk} are the structure constants of G, implies that $f^{msn}=-in\delta^{mn}$ in our basis. With use of this, the Yang-Mills equation for A_z^n in the presence of the gauge field A_θ^s of the string can be written (repeated indices are summed over)

$$(\partial^\mu+ienA^{s\mu})F_{\mu z}^n=\partial^\mu F_{\mu z}^n-J_z^n=J_z^{n\mathrm{H}}, \tag{9}$$

where the field tensor $F_{\mu z}^n$ is given by, taking account of the z-gauge condition,

$$F_{\mu z}^n=(\partial_\mu A_z^n+ienA_\mu^s A_z^n). \tag{10}$$

In Eq. (9) J_z^n and $J_z^{n\mathrm{H}}$ are, respectively, the gauge-boson and Higgs-meson contribution to j_z^n. Since $\partial_z\phi=0$, the only contributions to $J_z^{n\mathrm{H}}$ arise from the terms in the Hamiltonian which give mass to the vector bosons. Let M^2 be the vector-boson mass contribution to the Hamiltonian; we can write

$$M^2=(m^2)^{ij}A^{\mu i}A_\mu^j/2=J^{i\mathrm{H}\mu}A_\mu^i/2, \tag{11}$$

where the $(m^2)^{ij}$ are the elements of the mass-squared matrix, which are quadratic in ϕ. Then

$$J_z^{n\mathrm{H}}=-\partial M^2/\partial A_z^n. \tag{12}$$

Outside the string, if the only external field present is the EM field satisfying Eq. (7), the fact that the photon mass is zero means, of course, that $M^2=0$, and since this is necessarily a minimum of M^2, it follows from (12) that $J_z^{n\mathrm{H}}=0$ outside the string; however, we shall see that $J_z^{n\mathrm{H}}$ does play a role within the string core.

Let $a_z(x_\mu)$ be a solution of the Abelian-Maxwell equations describing, in z gauge, an EM wave in free space polarized in the z direction. Then a solution to Eqs. (9) and (10) outside the string is given by

$$A_z(x_\mu)=a_z(x_\mu)u(\theta)\,, \tag{13}$$

and thus $A_z^q=a_z$. That is, one obtains a solution in the presence of the string in which, outside the string, the Abelian solution simply rotates in internal space as one goes around the string to follow the direction of Q.[9] To verify that Eq. (13) does provide a solution, note that, combined with Eqs. (5) and (6), it yields

$$A^n=c_n e^{in\theta}a_z\,, \tag{14}$$

which in turn gives

$$F_{\mu z}^n=e^{in\theta}\partial_\mu a_z\,, \tag{15}$$

since, taking Eq. (3) into account, the contribution from the azimuthal component of the gradient acting on $e^{in\theta}$ just cancels $ienA_\theta^s A_z^n$. A similar cancellation then implies that Eq. (15) substituted into Eq. (9) reduces, for $J_z^{n\mathrm{H}}=0$, to one of the Maxwell equations for a_z, which is satisfied by hypothesis.

However, within the string Eq. (3) fails. Near the string the angular-dependent terms in the Fourier series for a_z vanish at least as fast as ωr,[9] so that a_z is isotropic at small r. Then, using Eq. (14) for the exterior solution and imposing continuity at the string boundary, we can write within the string

$$A_z^n=A^n(r)e^{in\theta}\,. \tag{16}$$

Reading off J_z^n from Eq. (9) then gives for the gauge-boson currents

$$J_z^n=en^2A^{s\theta}A_z^n(1/r+eA_\theta^s)\,. \tag{17}$$

From Eq. (3) $J_z^n=0$, as expected, for $r\gg r_0$. Within the string, however, $J_z^n\neq 0$ for $n\neq 0$. Suppose we put the string in an external EM wave, one whose source is other than the string, given near the string by a constant, $a_{z,\mathrm{ext}}$. The total field a_z at small r is reduced by a factor $\gamma\approx[1+\ln(\eta/\omega)]^{-1}$ because of interference between the incident and scattered waves[1,9]; in realistic models, γ is expected to lie in the range from about 0.01 to 1. The field equations force both A_θ^s and A_z^n to vanish as r^n at the origin; for simplicity, we confine ourselves to the case where $n=0,\pm 1$, as in our model. We evaluate J_z^n by approximating A_θ^s and A_z^n by the first terms in their power series in r and setting them equal to their respective asymptotic values, $-1/er$ and $c_n\gamma a_{z,\mathrm{ext}}$, at $r=r_0$. This gives

$$J_z^n=c_n\gamma n^2r^2a_{z,\mathrm{ext}}(1/r-r/r_0^2)e^{in\theta}/r_0^3\approx c_n\gamma n^2 r a_{z,\mathrm{ext}}e^{in\theta}/r_0^3\,. \tag{18}$$

Thus $J_z^0=0$. For $n\neq 0$, $J_z^N\neq 0$; I^n, the total gauge-boson current obtained by integration of J_z^n over the cross section of the string does vanish because of the factor $e^{in\theta}$. However, I^n is not gauge invariant, since the value of the integral can be changed by a local gauge rotation which alters the relative signs of J_z^n at different values of θ, and the vanishing of I^n is a gauge artifact, as we shall see later.

There will also be a nonzero contribution to $J_z^{n\mathrm{H}}$ within the string. It vanishes only when one has a pure photon field, which means, from Eq. (14), $A^n/A^0=c_n/c_0$. This fails within the string because $A^n\sim r^n$ for $r\to 0$. In particular, consider $J_z^{0\mathrm{H}}$; this is independent of θ and hence gives a nonvanishing $I^{0\mathrm{H}}$ when integrated over angle. The θ independence follows from Eq. (11) and the invariance of M^2 under gauge rotations generated by S; thus $J_z^{0\mathrm{H}}$, like A_z^0, must also be invariant under such gauge rotations. To find $J_z^{0\mathrm{H}}$, we take the difference between its value for the actual fields, given by Eq. (16), and its value ($=0$) for the same A^0 and $A^n=(c_n/c_0)A^0$ as in a photon field. This yields

$$J_z^{0\mathrm{H}}=(m^2)^{0n}[A^n(r)-c_nA^0/c_0]\,. \tag{19}$$

We estimate $J_z^{0\mathrm{H}}$ as we did J_z^n. We take $A^0\approx\gamma c_0a_{z,\mathrm{ext}}$. A_z^0 is not a mass eigenstate, so the off-diagonal matrix elements $(m^2)^{0n}\neq 0$, and their magnitude becomes of order $e^2\eta^2$ for $r\gtrsim r_0$. Since ϕ vanishes as r at the origin, $(m^2)^{0n}\sim r^2$, so within the string we can approximate $|(m^2)^{0n}|\approx e^2\eta^2r^2/r_0^2$. Making these approximations and calculating the current $I^{0\mathrm{H}}$ by integrating over r form 0 to r_0 gives

$$I^{0\mathrm{H}}=-Kc_0\gamma a_{\mathrm{ext}}^z\,, \tag{20}$$

where K is a positive constant of ~ 1 which depends on the details of the c_n and $(m^2)^{0n}$. The sign of $I^{0\mathrm{H}}$ is given by the observation that, from Eq. (11), $M^2=-J_z^{n\mathrm{H}}A_z^n$; since $M^2>0$, it follows that $J_z^{n\mathrm{H}}$ has the opposite sign from A_z^n for all n. Let $i^n=I^n+I^{n\mathrm{H}}$, B be the azimuthal component of the total EM magnetic field due to the string, and B^0 the contribution to B from $n=0$. For $r\ll\omega^{-1}$, Stokes' theorem applied to Eq. (9) gives

$$B^0=i^0/2\pi r=I^{0\mathrm{H}}/2\pi r\,, \tag{21}$$

since $I^0=0$, while from Eq. (15)

$$B^n=c_ne^{in\theta}B^0/c_0 \tag{22}$$

outside the string where only the massless component of

A_z exists, and this together with Eq. (7) and the fact that $\sum_n |c_n|^2=1$ then gives

$$B=-K\gamma a^z_{\rm ext}/2\pi r\,, \tag{23}$$

the same field that would be produced by an ordinary current-carrying wire with an effective current

$$I_{\rm eff}=-K\gamma a^z_{\rm ext}\,. \tag{24}$$

The theory indeed exhibits superconductivity, since

$$dI_{\rm eff}/dt=-K\gamma da^z_{\rm ext}/dt=K\gamma E^z_{\rm ext}\,,$$

where $\mathbf{E}_{\rm ext}$ is the external electric field. Thus a current produced by a unidirectional pulse of electric field will persist after the pulse has passed the string. The magnitudes of $I_{\rm eff}$ and $dI_{\rm eff}/dt$ are comparable, in a given external EM field, to those found by Witten for the superconducting mechanisms in Ref. 2. The gauge dependence of the result, reflected in the appearance of a^z in Eq. (24), is only apparent and results from our use of the z gauge.

As in Ref. 2 the current will saturate when $I_{\rm eff}\approx\eta$, at which point the interaction energy between A^s and A^q (or, equivalently, the energy in the magnetic field of the induced current) becomes comparable to other components of the string energy. It then becomes energetically favorable for any further increase in $a^z_{\rm ext}$ to be accompanied by a distortion of A^s_θ within the string so that no further increase in current results. A likely form of such distortion would be for the $-1/er$ behavior of A^s_θ to persist to smaller values of r than for $I_{\rm eff}=0$, which, recalling Eq. (16), decreases the effective current-carrying area of the string; alternatively, A^s_θ might develop a zero for some $r<r_0$, allowing N to change for some r within the string.

With the choice of gauge defined by the second equality in Eq. (1), the integral of $\mathbf{B}^n$ around a closed curve C enclosing the string vanishes for $n\neq 0$ because of the factor $e^{in\theta}$ in B^n; this is consistent with the vanishing of I^n noted earlier. We can, however, carry out a θ-dependent gauge rotation generated by S such that on C the phase of B^n is constant except within an arbitrarily small range of angles $\delta\theta$ in coordinate space within which the entire phase variation occurs. The gauge-invariant magnitude of B^n is unaffected, and the contribution to the integral from $\delta\theta$ is negligible. In the new gauge one has, using Eqs. (20)-(24),

$$i^n=\int_C \mathbf{B}\cdot d\mathbf{r}=2\pi r B^n=-c_n I_{\rm eff}\,, \tag{25}$$

so all the I^n are unidirectional and the string carries a total EM current $i=\sum_n c_n^* I^n=I_{\rm eff}$.

Thus we conclude that there is a class of plausible models in which cosmic strings contain gauge vector-boson fields having charged components. External electric fields can induce electric currents in such strings whose time derivative is proportional to the field strength, and which are persistent once established, as in a superconductor. The potential cosmological effects of such currents are the same as for currents produced by the mechanisms of Ref. 2.

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