Splitting of the Squashing Collective Mode of Superfluid ^{3}He-B by a Magnetic Field

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Pulse-time-of-flight sound propagation (group velocity) studies of the squashing mode of superfluid ^{3}He-B in a magnetic field have revealed four of the five expected Zeeman sublevels for this $J=2^-$ mode. Variation of the pressure at very low temperatures ($T/T_c \leq 0.35$) was used to sweep through the squashing mode at constant frequency (137.6 MHz). Observation of very low group velocities (down to ~ 6 m/s) and pulse-shape analysis enabled the identification of the $J_z=+2$, $+1$, 0, and -1 sublevels. The Landé g factor was found to be 0.042 near $T=0$ and $P=19$ bars as determined from the linear dependence of the $J_z=+2$ Zeeman component.

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Collective modes are among the most interesting features of the ^{3}He superfluids.[1] Perhaps the most powerful technique for probing these modes has been the study of sound propagation. Although the ^{3}He superfluids are complex many-body systems, their sound spectra display properties of simple atoms or molecules. In particular, it has been found in superfluid ^{3}He-B that the spectra of both the $J=2^+$ real squashing collective mode[2,3] and the recently observed $J=1^-$ mode[4] exhibit observable Zeeman splittings. Two factors combine to make resolution of the expected fivefold Zeeman splitting[5] of the $J=2^-$ (imaginary) squashing mode rather difficult. This mode is very strongly coupled to sound, so that the sound attenuation is extremely high in the neighborhood of the mode. Furthermore, there exists a broad evanescent region where the sound propagation is strongly inhibited in the immediate vicinity of this mode.[6,7] Adding further to the difficulty is the fact that the predicted g factor for the squashing mode is much smaller than that for the real squashing mode[5] and attains its smallest value as $T \to 0$. Previous attempts at observation of the splitting of the squashing mode were made by acoustic impedance experiments which were unsuccessful and are now believed to probe the $J_z=0$ mode only.[8]

In the present experiment, which involves the propagation of 137.6-MHz sound, we have employed a number of strategies to overcome these difficulties and have successfully observed the Zeeman splitting of the squashing mode. Four of the five expected sublevels have been directly observed in our experiment. The sound cell consists of two matched ~ 15-MHz quartz compressional sound transducers separated by a precision quartz spacer providing a sound path of 4.5 mm through the liquid ^{3}He. The high-frequency sound pulses for the time-of-flight technique were obtained by operating at the ninth harmonic. Cooling to the submillikelvin regime was achieved by adiabatic demagnetization of $PrNi_5$. The temperature of the cell was measured by a ^{3}He-melting-curve thermometer in zero magnetic field linked by annealed silver rods and sintered silver to the liquid ^{3}He. The variation of melting pressure, as determined by Osheroff and Yu,[9] is small below 1 mK, but the precision was sufficient for our measurements, since the gap and collective-mode variations with temperature are small in this region. Additional details of the experimental arrangement are given elsewhere.[10]

In order to maximize the magnitude of the splitting, magnetic fields ranging up to 0.46 T were applied to the sound cell. The temperatures during these measurements were held below 1 mK by the $PrNi_5$ refrigerant. These very low temperatures were necessary in order to avoid the mode broadening introduced by quasiparticle collisions.[11]

The squashing-mode frequency is proportional to the gap [$\omega(T) \cong (\frac{12}{5})^{1/2}\Delta_{BCS}(T)$ in weak-coupling theory]. The energy gap varies rather slowly with temperature below 1 mK and therefore it was not possible to sweep through the Zeeman spectrum by varying the temperature. Instead, the gap and squashing-mode frequency were varied by slowly reducing the pressure through the spectrum. Care was taken to depressurize slowly enough (less than 0.5 bar/h) to avoid significant viscous heating effects. The magnitude of the gap was obtained from the variation of T_c with pressure using $\Delta_{BCS}(0)=1.764k_BT_c$, and Greywall's[12] temperature scale. Note that with quartz transducers it was not feasible to sweep the frequency of the spectrometer except over a very narrow range.

Clean experiments required very low temperatures ($0.25 < T/T_c < 0.35$) to reduce quasiparticle broadening and to minimize corrections associated with the temperature variation of the gap. We operated at the highest harmonic of the quartz crystal for which the squashing mode was still accessible. The operating pressures then fell between 17 and 22 bars for 137.6-MHz sound. One of the prime tools in our experiment was group-velocity spectroscopy, a method first used by Calder *et al.*[7] to study the squashing mode. In this technique a measurement of the time of arrival of the peak of the propagating sound pulse is used to determine the group velocity. The group-velocity data are then analyzed to produce the frequency of the collective mode itself.

To help understand the experiment we show in Fig. 1 schematic dispersion curves and group-velocity curves for the crossing of the zero sound mode with *two* adjacent undamped squashing modes strongly coupled to sound, and also a weakly coupled mode, where the model of Ref. 6 was used. The frequencies of the collective modes correspond to vanishing group velocity ($V_g = d\omega/dk$).

Figure 2 displays the measurements of the group velocity when the pressure is swept. The group-velocity spectrum is clearly split by the applied magnetic field. Figure 2(a), in particular, shows the presence of two strongly coupled modes which can be related to Fig. 1. Since extremely low group velocities (down to ~6 m/s, compared to first-sound velocities of about 350 m/s) were obtained in our experiment, the points of intersection between a linear extrapolation of the group-velocity data and the ω/Δ_{BCS} axis were known quite precisely. The intersection occurring at the *higher* value of ω/Δ_{BCS} for each evanescent region is empirically chosen (see below) as the point corresponding to the location of the collective mode. For the extremely low group velocities found near the evanescent region the sound signals were quite weak and required signal averaging. Increasing the drive level significantly did not improve matters but instead simply destroyed the signal.

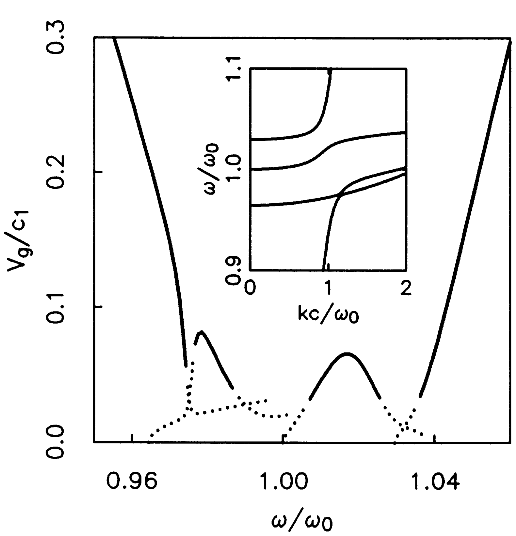

FIG. 1. Group-velocity spectrum calculated from model dispersion curves (Ref. 6) for the crossing of zero sound with two strongly coupled modes and one weakly coupled mode shown in the inset. The mode frequencies are 1.03, 1, and 0.964, and their coupling constants (Ref. 6) are 0.01, 0.01, and 0.00002, respectively. The squashing-mode velocity is $c_{sq}/c_1 = 0.125$. The dotted lines represent the regions of high attenuation and hence no received signal.

Two orientations, **q**∥**H** and **q**⊥**H**, were studied (**q** is the propagation vector of the sound). Figure 2(a) corresponds to data taken with **q**⊥**H**. The entire group-velocity spectrum showing all the observable Zeeman components cannot be obtained at a single magnetic field. The narrow peak on the far side of the figure indicated by the arrow did not emerge from the broad evanescent region of the adjacent peak until the field was greater than 0.4 T. On the other hand, if the field was more than about 0.4 T, the structure on the right-hand side was obliterated by the onset of strong pair-breaking absorption associated with the magnetic field.[5] To show as much of the structure as possible, we display the data for three magnetic fields in Fig. 2(a).

For the real-squashing-mode Zeeman effect, the strength of the J_z components is proportional to $[Y_{J_z}(\theta)]^2$, where $Y_{J_z}(\theta)$ is an associated Legendre polynomial corresponding to $J=2$ and θ is the angle between **q** and **H**.[2] Similar behavior is expected for the squashing mode.[5] For **q**⊥**H**, $J_z = \pm 2,0$ should be strong and $J_z = \pm 1$ should be quite weak, and, in fact, should be observable mainly because of possible misalignments from exact perpendicularity between **q** and **H**. Textural bending effects near walls and transducers may also be

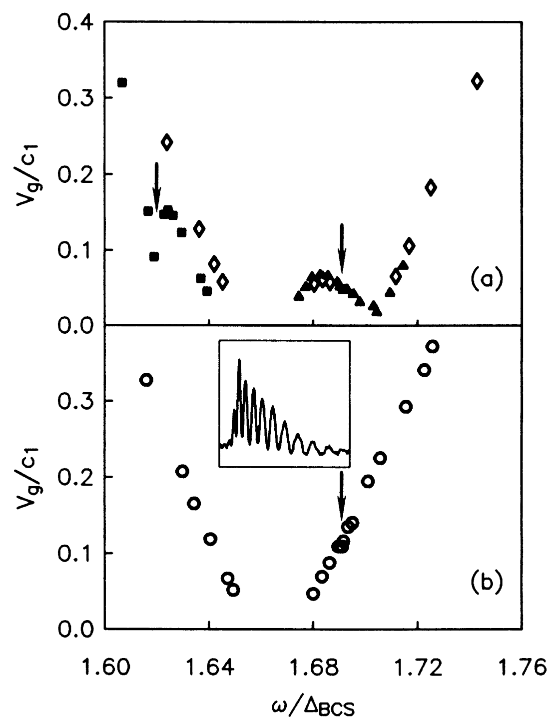

FIG. 2. (a) Observed group-velocity spectra for **q**⊥**H**: ◇, 0.36 T; ▲, 0.38 T; ■, 0.46 T. Arrows indicate locations of $J_z = \pm 1$ as explained in the text. (b) Group-velocity spectrum for **q**∥**H**, for $H = 0.38$ T. Only the $J_z = 0$ mode is strongly coupled. The position of $J_z = +1$ for $H = 0.38$ T is indicated by the arrow. Inset: broken-up pulse shape corresponding to the signature of $J_z = \pm 1$.

important, especially for the case of $\mathbf{q}\perp\mathbf{H}$.[1] The broad evanescent region on the far right is identified with the strongly coupled $J_z=+2$ component. The other broad feature is taken to be a combination of strongly coupled $J_z=0$ and $J_z=-2$ which are too close together to be resolved. As in the case of the real squashing mode,[3] we expect that at these high magnetic fields, the Zeeman splitting should exhibit a highly nonlinear field dependence which makes overlap between the evanescent regions of $J_z=0$ and $J_z=-2$ permissible. In fact, the Zeeman components $J_z=2,1,0,-1,-2$ need no longer be consecutively arranged since some of the levels can cross one another at high fields. Thus, we identify the narrow dip in the group-velocity data for 0.46 T at $\omega/\Delta_{\rm BCS}=1.620$ with $J_z=-1$. The narrow evanescent region for this line is symptomatic of weaker coupling to the mode. For the perpendicular orientation, the $J_z=\pm 1$ components are only weakly coupled to the sound since $Y_1(\theta)\rightarrow 0$ for this case. Finally, the very narrow feature at $\omega/\Delta_{\rm BCS}=1.691$ and 0.38 T is the $J_z=+1$ Zeeman component. This feature has only a small group-velocity signature and is mainly characterized by a sharp drop in peak pulse amplitude. We emphasize here that the observation of the weakly coupled $J_z=\pm 1$ modes is possible only at high magnetic fields for which the modes are well separated and low temperatures where the widths due to quasiparticle broadening of the neighboring strongly coupled modes are reduced.

Observations in the parallel orientation serve to support further the above identifications. For $\mathbf{q}\parallel\mathbf{H}$, it is expected theoretically that $J_z=0$ is strong, $J_z=\pm 1$ are weak, and $J_z=\pm 2$ are very weak.[5] Typical group-velocity versus pressure $(\omega/\Delta_{\rm BCS})$ sweeps, portrayed in Fig. 2(b) for the $\mathbf{q}\parallel\mathbf{H}$ orientation at 0.38 T, show a broad evanescent region for the $J_z=0$ component. This is the only component coupling strongly to sound for this orientation. The location of the $J_z=+1$ component for 0.38 T is also indicated in Fig. 2(b). The main experimental manifestation of the two weakly coupled modes $J_z=\pm 1$ for this orientation is a dramatic breakup of the received pulse into many spikes, as shown in the inset of Fig. 2(b) and discussed previously for the real squashing mode in the context of precursory sound pulse propagation.[6] These data were obtained by recording the received pulses on a transient recorder. No pronounced effect on the group velocity was observed for the $J_z=\pm 1$ components, which are weakly coupled (via nonexact parallelism) at this orientation. The $J_z=\pm 1$ modes were so narrow at this orientation that they could be completely traversed by sweeping the *frequency* of the sound transducer over a few hundred kilohertz through the narrow quartz resonance.

Finally, experiments performed in this parallel $(\mathbf{q}\parallel\mathbf{H})$ orientation revealed no evidence of the $J_z=\pm 2$ Zeeman components. This can be understood in terms of the exceedingly weak coupling of the $J_z=\pm 2$ modes for this orientation,[5] mentioned previously, which varies as $\sin^4\theta$ for small θ in contrast to the $\sin^2\theta$ behavior for $J_z=\pm 1$.

All the reduced frequencies $(\omega/\Delta_{\rm BCS})$ corresponding to the various squashing-mode Zeeman components for *both* parallel and perpendicular orientations are plotted against magnetic field in Fig. 3. For strongly coupled modes, these data were derived from group-velocity spectroscopy. For weakly coupled modes, pulse breakup was the preferred signature for the parallel orientation, and the attenuation feature indicated the mode frequencies for the perpendicular orientation.

A value of the Landé factor g $(\Delta\omega=gJ_z\Omega$, where Ω is the renormalized[5] Larmor frequency) can be derived from the linear dependence of the $J_z=+2$ component on field. The raw data of Fig. 3 were used in conjunction with our experimentally determined pressure variation of the zero-field squashing-mode frequency to obtain a value of $g=0.042\pm 0.002$, in agreement with the theoretical value of $g=0.0435$ calculated using expressions of Sauls and Serene[13] for the weak coupling plus energy gap[14] and the f-wave pairing parameter $X_3^{-1}=0$. As expected, the data for the $J_z=0$ component shown in Fig. 3 display no linear dependence with field. The quadratic dependence is characterized by a parameter Γ according to the relation $\omega-\omega_0=-\Gamma H^2$. Again, with the appropriate pressure corrections, $\Gamma=4.5\pm 1.5$ MHz/T^2 and is found to be reasonably consistent with the acoustic impedance studies of Shivaram *et al.*[3] and the prediction of Schopohl, Warnke, and Tewordt.[15] The data for $J_z=0\pm 1,+2$ observed in the nonlinear regime did not appear to fit the equation

$$\omega(P,T,H,J_z)=\omega_0(P,T)+\alpha J_z H+\beta J_z^2 H^2-\Gamma H^2$$

in contrast with the case of the real squashing mode.[3] Since we are working at high fields it is possible that

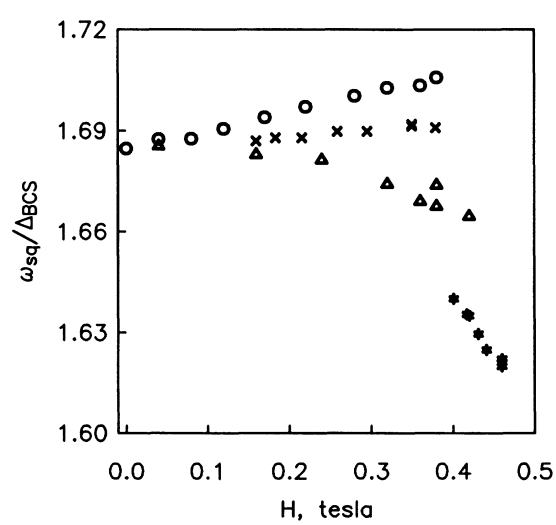

FIG. 3. Zeeman sublevels as a function of magnetic field: ○, $J_z=+2$; ×, $J_z=+1$; △, $J_z=0$; ∗, $J_z=-1$.

terms of order higher than quadratic become important. In addition, two potentially serious corrections are necessary for a detailed analysis of the data. First, our determination of the exact locations of the $k=0$ collective modes by a simple linear-extrapolation procedure should be replaced by a more refined model of mode crossings, including a proper description of the coupling between zero sound and the collective modes as well as a treatment of collision damping. Secondly, the values of the $J_z=\pm 1$ frequencies should be corrected because dispersion effects shift the observed frequencies at finite k away from the $k=0$ value for nonvanishing squashing-mode velocities, as depicted in Fig. 1.

In conclusion, we have shown using the pulse-time-of-flight technique that Zeeman splitting for the squashing mode occurs as predicted in 1979 by Tewordt and Schopoh.[5] Nonlinear splitting is clearly evident but considerable additional work will be required to map out and understand quantitatively all five Zeeman components.

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