Longitudinal Response Functions and Sum Rules for Quasielastic Electron Scattering from ^{3}H and ^{3}He

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Longitudinal response functions for ^{3}H and ^{3}He have been obtained from inclusive electron scattering cross sections, for $200 \leq q \leq 550$ MeV/c. The response functions are compared with several theoretical calculations, all of which use exact three-body ground-state wave functions. The response functions are found to be more similar than the calculations predict. When a direct calculation of the Coulomb sum rule is compared with our data, the agreement is good and evidence for two-proton correlations in ^{3}He is clear.

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In inclusive inelastic electron scattering, there is a broad "quasielastic" peak whose centroid is located at approximately the same electron energy loss as that for electrons scattered elastically from a nucleon at rest.[1] The peak is shifted by the nucleon binding energy, and broadened by the Fermi motion. Experiments that have measured the longitudinal (R_L, scattering from charges) and transverse (R_T, scattering from spins and currents) contributions to the cross section on a number of nuclei, from carbon to uranium, are poorly described by calculations assuming a quasielastic model (plane-wave impulse approximation).[2] In particular, the longitudinal response functions are generally predicted to be about (20–40)% larger than the measured response. Approximate treatments of nucleon final-state interactions are not sufficient to produce agreement. Meson-exchange currents have little effect as they are purely transverse, to lowest order in q/M_N. Other suggested improvements include the use of more realistic ground-state and final-state wave functions, modifications of the hadronic current, and quark effects.[3]

Our understanding of the three-body bound states is much more complete than for heavier nuclei. The three-body nuclei also provide an additional constraint on theory, since they are mirror nuclei, and the bound states differ only in the third component of isospin. The nonrelativistic wave equation for the bound state has been solved with use of realistic two- and three-nucleon potentials; these wave functions describe the experimental elastic form factors reasonably well at the momentum transfers of interest here.[4] However, continuum final-state wave functions of comparable quality are not yet available. We present here the first measurement of the ^{3}H longitudinal response functions, and simultaneous measurement of the ^{3}He longitudinal response functions.

An elegant way of comparing data to theory, avoiding the difficulties associated with final-state wave functions, is to use the Coulomb sum rule (the integrated longitudinal strength at a given three-momentum transfer q).[5,6] This tests the initial-state wave functions, including correlation effects. These effects are not expected to be important in the Coulomb sum rule when q is larger than twice the Fermi momentum, where the sum rule should approach the target charge Z (plus a small contribution from the neutrons).[7] In contrast to the case for heavy nuclei, ^{3}He data taken at Saclay approximately saturate the Coulomb sum rule.[3,8] Although numerous theoretical interpretations of the lack of saturation have been suggested, there is still an experimental problem with much of the heavier-target data in the required extrapolation to high energy loss. For the three-body nuclei, the quasielastic peak is not as broad as for the heavier nuclei, and so the extrapolations are not as important. Experimental sum rules for both ^{3}H and ^{3}He with small systematic errors provide a more accurate test of the ground-state wave functions than is possible for heavier nuclei, and highlight the effects of proton-proton correlations in the ground state, which only contribute in ^{3}He.

Electron scattering measurements were performed on ^{3}H and ^{3}He at the MIT Bates Linear Accelerator Center, with use of the energy-loss spectrometer system described by Bertozzi *et al.*[9] Elastic and inelastic cross sections were measured at two angles (54° and 134.5°), over a wide range of incident and final energies. The elastic data have been reported earlier.[10] The threshold breakup data and the transverse response functions are being analyzed separately.

Identical cryogenic gas cells were used for ^{3}H and ^{3}He. The cells operated at 225 psi (absolute) and 45 K; the pressure and temperature were continuously measured *in situ,* and used to calculate the gas densities.[11] The background due to scattering in the target cell walls was subtracted with use of scattering measurements from an empty target cell. A correction for events due to showering in the target-defining slits was made based on a background seen in the measured scattering-angle spectrum. The contribution of all backgrounds was less than 2% in the region of the quasielastic peak. After subtraction of a calculated elastic radiative tail, continuum radiative corrections were performed with use of the techniques of Mo and Tsai,[12] and Miller.[12] The random uncertainty of all of the radiatively corrected cross sections was approximately 5%; the uncertainties due to counting statistics, target density measurement, and radiative corrections were the most important. The systematic uncertainty was also approximately 5%; this was primarily due to uncertainties in the target density and the radiative corrections.[13]

Rosenbluth separations were made for $200 \leq q \leq 550$ MeV/c, with use of the measurements at two angles, and the standard expression for the cross section:

$$\frac{d^2\sigma}{d\Omega\, dE_f} = \sigma_{\mathrm{Mott}} \left[\left(\frac{Q^2}{q^2} \right)^2 R_L(q,\omega) + \left(\frac{1}{2} \left| \frac{Q^2}{q^2} \right| + \tan^2\frac{\theta}{2} \right) R_T(q,\omega) \right],$$

where q and ω are the three-momentum and energy lost by the electron in the scattering, Q is the four-momentum transfer, and θ is the electron scattering angle. The coefficients of R_L and R_T are such that the random uncertainty for R_L (7% at best and 18% at worst) is much larger than that of the cross sections; the systematic uncertainty is of similar magnitude. The experimental response functions for ^{3}He are in good agreement with previous data taken at Saclay.[8] Representative longitudinal response functions are shown (random uncertainties only) in Fig. 1, along with theoretical calculations by Meier-Hajduk *et al.*[14] (Hannover group), Ciofi degli Atti, Pace, and Salmé[15] (Rome group), and Schiavilla and Pandharipande[16] (Illinois group). Both the Hannover and Rome calculations assume a reaction mecha-

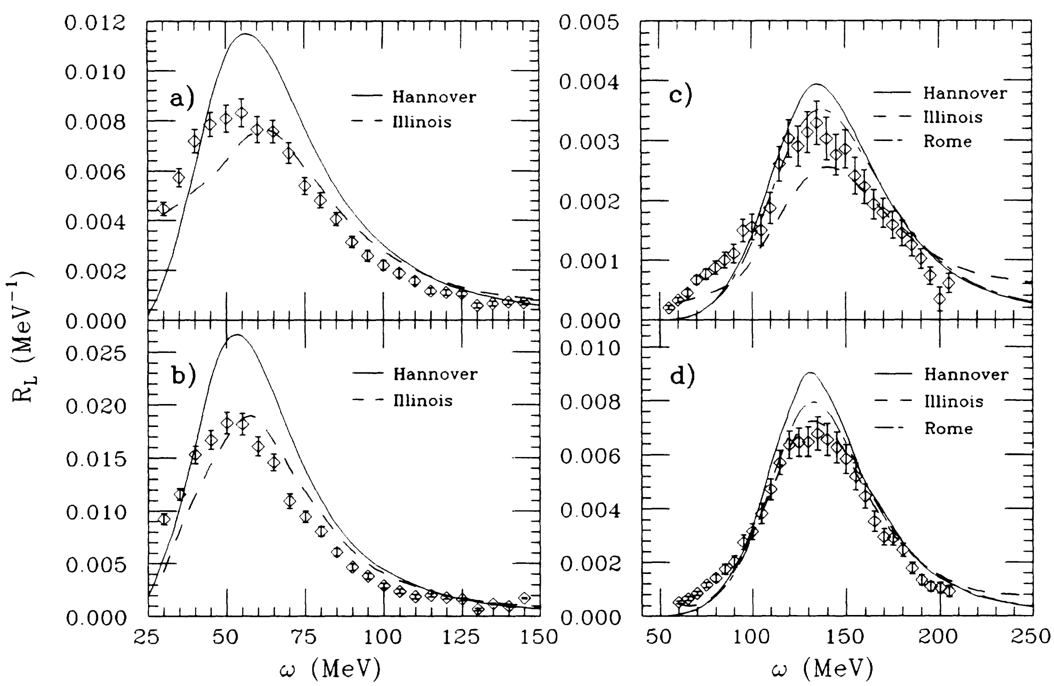

FIG. 1. Longitudinal response functions compared with theoretical calculations at $q = 300$ MeV/c for (a) ^{3}H and (b) ^{3}He, and at $q = 500$ MeV/c for (c) ^{3}H and (d) ^{3}He.

nism with only single-nucleon ejection, and so use a spectral function to describe the probability distribution of the initial momentum and energy of the struck nucleon. Both groups use ground-state wave functions which are solutions of the nonrelativistic three-body Faddeev wave equation, using either Reid soft-core or Paris potentials (which give nearly equivalent spectral functions). The spectral function is integrated over the two- and three-body hadronic final states to generate the curves shown. The difference between these two calculations lies in the choice of the free-nucleon form factors and/or in the model for the spectral function. Neither calculation includes final-state interactions, meson-exchange currents, or effects of relativistic order. The third calculation was provided by the Illinois group. This more complete calculation includes the elastic channel and orthogonalizes the final-state channels. This gives part of the contribution due to final-state interactions, but does not include effects due to rescattering of the struck nucleon. A much better description of ^{3}He is achieved, except close to the inelastic threshold where the effects due to final-state interactions should be most apparent. On the other hand, the predicted quasielastic peak in ^{3}H is suppressed and shifted compared to the data. The Illinois calculation still does not treat the three-body final state exactly; the approximations affect ^{3}H much more than ^{3}He, as a result of the relative importance of the two-body final states.[16]

Figure 2 shows a comparison of the experimental and theoretical longitudinal responses for ^{3}H and ^{3}He at 400 MeV/c, divided by Z. The response for ^{3}H is slightly wider than that for ^{3}He. To first order, this reflects the different proton momentum distributions in the two nuclei, suggesting a higher probability for high-momentum components in ^{3}H than ^{3}He; this is expected since ^{3}He has the larger rms charge radius. The normalized data are more similar in peak height than any of the calculations. However, the data do not necessarily imply that the theoretical momentum distributions are wrong, since proper final states are also important in determining the shape and magnitude of the responses.

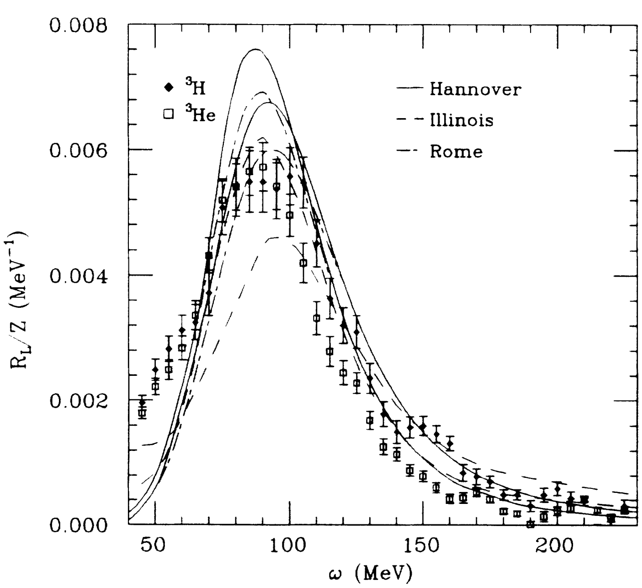

FIG. 2. Measured and theoretical longitudinal response functions divided by Z at $q=400$ MeV/c. The calculated response for ^{3}H is always lower than ^{3}He at the peak.

The theoretical and experimental strengths may be further examined by forming the Coulomb sum rule Σ_L. The longitudinal response is divided by the square of the proton charge form factor G_E^p, to obtain the "point-nucleon" response, and then an integration is performed over all ω:

$$\Sigma_L(q)_{\rm expt} = \sum_{\omega_{\rm min}}^{\omega_{\rm max}} \frac{R_L(q,\omega)\Delta\omega}{[G_E^p(Q^2)]^2(1+Q^2/4M_N^2)(1+Q^2/2M_N^2)^{-1}},$$

where $Q^2=q^2-\omega^2$ and M_N is the nucleon mass. The correction to the proton form factor is suggested by deForest,[17] in order to account for relativistic effects due to the motion of the proton in the nucleus. This decreases the denominator by as much as 7%. Höhler's parametrization 8.2 has been used for the proton form factor[18]; the use of the dipole form factor would decrease Σ_L by 3% at most.

The lower limit of integration $\omega_{\rm min}$ has been taken to be the energy loss for the two-body breakup threshold ($E_{\rm exc}=5.5$ MeV), since Σ_L is an inelastic sum rule. $\omega_{\rm max}$ was chosen such that $R_L(q,\omega_{\rm max})=0.01R_L(q,\omega_{\rm peak})$. At some momentum transfers, an exponential tail was fitted to a section of the high-ω side of the quasielastic peak in order to reach the $\omega_{\rm max}$ limit. The uncertainty on the ex-

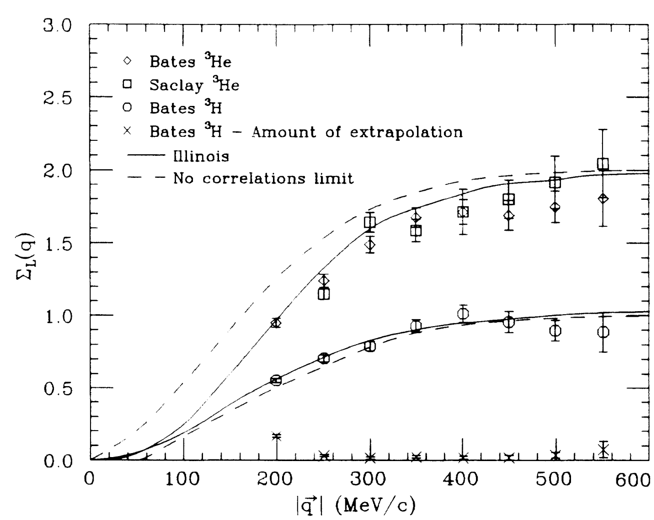

FIG. 3. Experimental and theoretical Coulomb sum rules for ^{3}H and ^{3}He.

perimental sum rule is the average of the fractional systematic uncertainties on the response-function points. The random uncertainty on the integral is negligible.

The experimental Coulomb sum rules for ^{3}H and ^{3}He are shown in Fig. 3. The amount of strength in the extrapolated tail for ^{3}H is shown; the fractional contribution for ^{3}He is similar. The Saclay ^{3}He data were also summed by the same method; the systematic uncertainties were assumed to be equal to the random uncertainties. For the Saclay data, extrapolation was needed only at $q=250$ and 300 MeV/c.

The curves labeled "Illinois" are exact nonrelativistic calculations performed by the Illinois group, using only ground-state wave functions.[6] Finally, the curves labeled "no correlations"[6] were calculated with use of the measured elastic charge form factors $F_{\rm ch}$ (Ref. 10):

$$\Sigma_L^{\rm nc}(q)=Z[1-|F_{\rm ch}(q)|^2/G_E^p(Q^2)^2],$$

where $Q^2=q^2-\omega_{\rm peak}^2$. This is the inelastic sum rule in the absence of correlations.

Several observations can be made. For both $A=3$ systems, the data approximately saturate at high momentum transfer (the no-correlation limit) and appear to approach zero at zero momentum transfer (where only elastic scattering can occur). Although the Illinois calculations agree well with both sets of data, integrating the Hannover and Rome longitudinal response functions yields strengths in excess of the data, differing from Z by small amounts. The Illinois prediction and the data for ^{3}H are both quite close to the no-correlation limit. The presence of two-proton correlations is only clear for ^{3}He. This is a very interesting test of three-body wave functions, and other possible correlation functions should be used to calculate the sum rule.

In summary, quasielastic electron scattering data have been obtained for ^{3}H and ^{3}He. Theoretical calculations using exact ground-state wave functions and simple assumptions about the final state overestimate the longitudinal response; a calculation which attempts to include some of the final-state-interaction effects agrees with the ^{3}He response at 500 MeV/c, where the ignored effects should be small. However, this same calculation fails to reproduce the ^{3}H data. This substantial disagreement between theory and experiment may result from shortcomings in the theoretical treatment of continuum wave functions, from physics beyond the scope of the available calculations (e.g., modified nucleon currents in the medium), or possibly from both. The agreement of the data with the Illinois calculation of the Coulomb sum rule clearly implies that the first alternative should be pursued vigorously before one reaches more sweeping conclusions.

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