Resistive Transition of High-Temperature Superconductors

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The model proposed recently by Yeshurun and Malozemoff to explain the "irreversibility line" in high-temperature superconductors is extended to account for the experimentally measured width and shape of the resistive transition in a magnetic field, without invoking material inhomogeneity. It is argues that high T_c and H_{c2} are necessary, but not sufficient, conditions for such materials to show zero resistance at room temperature in substantial magnetic fields.

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Yeshurun and Malozemoff[1] have recently proposed a new interpretation of the "irreversibility line" in the high-temperature superconductors, first reported by Müller, Takashige, and Bednorz.[2] This line separates the region near T_c in the (H,T) plane in which the sample shows a unique reversible magnetization $M(H,T)$, from the region in which $M(H,T)$ depends on the previous path in the (H,T) plane. The form of this irreversibility line was found to be

$$1-t \propto H^{2/3}, \tag{1}$$

where $t=T/T_c$. This behavior resembled known phenomena in spin-glass physics, which led to calling the irreversibility line also a "quasi de Almeida-Thouless line." The interpretation offered by Yeshurun and Malozemoff was in the more conventional language of pinning, creep, and flow of fluxons (quantum flux lines) in the superconducting medium. In their view, the irreversibility line identifies the limiting conditions under which nonequilibrium supercurrents can "persist" over the duration of an experiment, giving an irreversible contribution to the magnetization. Starting from this "giant flux creep" point of view, the IBM group also explained apparent differences in $T_c(H)$ measured on the same set of samples as being a consequence of different measuring frequencies relative to flux creep rates.[3] This led them to suggest that the values of $H_{c2}(T)$ inferred from such dissipative measurements are serious underestimates of the true thermodynamic values.

In this Letter, we extend this attractive model to account for the *width* and *shape* of the *resistive transition* as a function of field. The good agreement which we find with experimental results provides further support for the model of Ref. 1. In addition, we point out some of its more general implications.

The key ingredient that we take from Ref. 1 is the estimate of an activation energy U_0 which must be overcome to allow flux motion and hence resisitance. By a heuristic scaling argument, Yeshurun and Malozemoff argue that U_0 should have a form that we can write as

$$U_0=\beta H_c^2 \xi \phi_0/B. \tag{2}$$

Here H_c is the thermodynamic critical field, ξ the coherence length, ϕ_0 the flux quantum $hc/2e$, B the flux density in the sample, and the parameter β (presumed ~ 1) is introduced to absorb all numerical factors. Although we consider (2) to be an empirically justified bit of phenomenology, we offer the following possible rationale for this curious formula in terms of thermally activated vortex lattice shear: For $H<H_{c1}$, $B=0$, and no fluxons are present to move. For $H>H_{c1}$, the fluxons overlap as soon as $B>H_{c1}$, because their separation is then less than the penetration depth λ; as a result, their motions become energetically correlated. This correlation energy goes to zero again at H_{c2}, as the order parameter does. For these high κ materials, there is a wide intermediate filed range, where $H_{c1} \ll H \ll H_{c2}$ and $B \approx H$. In this range, the vortex-lattice-dependent term in the Gibbs free-energy density

$$\Delta G \approx -(H_{c2}-H)^2/[8\pi(2\kappa^2-1)\beta_A]$$

can be approximated by $-H_c^2/8\pi\beta_A$, since $H_{c2}=\sqrt{2}\kappa H_c$. The Abrikosov parameter β_A has the value 1.16 for the equilibrium triangular lattice and 1.18 for the metastable square lattice. To estimate the energy density barrier δG opposing the motion of a row of fluxons past neighbor rows (which may be pinned), we take $\delta(1/\beta_A) \sim 0.02$, based on the square-triangle lattice difference. To convert this energy density to an absolute energy barrier U_0, we must multiply by a volume. The elemental movable volume in the flux lattice has the cross-sectional area of the Abrikosov unit cell ϕ_0/B and length ξ. This volume element must be multiplied by the number of adjacent fluxons that effectively move together to avoid large vortex compressional energies. Similarly, to avoid the energy cost of undue elongation of fluxons by sharp bends, the jump must extend over a fluxon length which is a numerical multiple (>1) of ξ. [Alternatively, the jumping length might be related to the fluxon spacing $(\phi_0/B)^{1/2}$ instead of ξ, or perhaps to the layer spacing d in very strongly layered materials. Such situations would change detailed quantitative dependences, but not the qualitative conclusions of this Letter.] Combining these considerations, and absorbing numerical factors in β, we

obtain (2).

We now transform (2) by substitution of the Ginzburg-Landau relation[4] $\phi_0 = 2\sqrt{2}\pi H_c \xi\lambda$ and the intrinsic Ginzburg-Landau depairing critical current density $J_{c0} = cH_c/3\sqrt{6}\pi\lambda$, obtaining

$$U_0 = (3\sqrt{3}\phi_0^2\beta/2c)J_{c0}/B. \quad (3)$$

(Note that this J_{c0} is *not* the measured J_c; it is the value of J_c at $B=0$ and *without* reduction by thermal fluctuations.) We prefer formula (3) over (2) because it involves only one material parameter J_{c0} instead of two (H_c and ξ); more importantly, J_{c0} has a relatively clear operational meaning even in the presence of granularity, while the other parameters become more ambiguous.

Since thermally activated processes depend exponentially on U_0/k_BT, we define a normalized barrier height by

$$\gamma_0 = U_0/k_BT = [CJ_{c0}(0)/T_cB]g(t), \quad (4)$$

where

$$C = (3\sqrt{3}\phi_0^2\beta/2ck_B = 80.7\beta \ \ (\text{G K cm}^2/\text{A}) \quad (4a)$$

if B is expressed in G, T in K, and J_c in A/cm^2. Here $J_{c0}(0)$ is the critical current density at $T=0$ and $B=0$, evaluated along the crystal direction of the *field* in the resisitance measurement; the function

$$g(t) = 1 - t^2(1-t^4)^{1/2}/t \approx 4(1-t)^{3/2}, \quad (4b)$$

builds in the "two-fluid" empirical approximations to the temperature dependences of H_c and λ. [It is important to recognize that fluctuation effects reduce the *measured* J_c essentially to *zero* between T_c and the irreversibility temperature. When this $J_c(T)$, or the resulting irreversible magnetization $M(T)$, is forced to fit a form $(1-t)^n$, large values of $n \approx 2$–8 are found because of this finite range of $J_c \approx 0$ near T_c. This does not reflect the temperature dependence of J_{c0}, which, by definition, is not reduced by fluctuations, and is described by $g(t)$.] The final expression in (4b) is the limiting form of $g(t)$ near T_c, which is actually accurate to better than $\pm 4\%$ all the way from $t=1$ down to $t=\frac{1}{2}$. In this region, we can write (4) as

$$\gamma_0 = A(1-t)^{3/2}/B, \quad (4c)$$

where $A = 4CJ_{c0}/T_c$. For YBCO, with $T_c = 92$ K, the factor $4C/T_c \approx 3.5\beta$, so that A can be simplified further to $A \approx 3.5\beta J_{c0}(0)$, where β is expected to be of order unity.

In the work of Ref. 1, the parameter U_0 was used to estimate the temperature at which thermally activated phase slippage becomes slow enough to leave nominally persistent currents, with decay measurable only as flux creep over a period of time. We now use this U_0 to describe the *visible resistive width* of the transition, in which the phase-slip rate $d\theta/dt$ is so high that one sees its effect as a time-average dc voltage $V = (\hbar/2e)d\theta/dt$. To do this, we treat the resistance of the sample as arising from phase slippage at a complicated network of channels, where fluxons are slipping past one another over barriers between local energy minima; the passage of each vortex gives a phase slip of 2π. We then argue that the kinetics of this driven, highly damped thermally activated process involves essentially the same 2π-phase-slip physics as the case of thermally activated phase motion in a *single* heavily damped current-driven Josephson junction. The latter problem was worked out in detail by Ambegaokar and Halperin,[5] and we simply apply their work in the present context. Even if not rigorously applicable to this case, their results at least provide a plausible semiquantitative model for the dependence of R/R_n upon γ_0.

According to Ambegaokar and Halperin, in the limit of very small currents, the resistance of each link is reduced by the ratio

$$R/R_n = [I_0(\gamma_0/2)]^{-2}, \quad (5)$$

where I_0 is the tabulated modified Bessel function. (For large values of γ_0, R/R_n falls as $\gamma_0 e^{-\gamma_0}$, but for $\gamma_0 < 1$, it only falls quadratically with γ_0.) This has the consequence (insofar as γ_0 is the same for all the links) that the resistance of the macroscopic network, however complex, will display this same dependence. Combining (4c) and (5), we can write

$$R/R_n = \{I_0[A(1-t)^{3/2}/2B]\}^{-2}. \quad (6)$$

This is the central result of this paper. From it, we make the following observations:

(a) The temperature width of the transition (at *any* R/R_n level) should scale as

$$\Delta T \propto B^{2/3}. \quad (7)$$

This dependence not only accounts *qualitatively* for the ubiquitous broadening of the resistive transition in a field *without* needing to invoke an inhomogeneous sample as done in most previous explanations, but also the specific predicted $B^{2/3}$ dependence fits quite well with a variety of published data.[6–8] We also point out that the result (7) would hold even if the functional form (5) were replaced by some other similar function of U_0/k_BT, so long as the form of (4c) holds.

The result (7) can be viewed as a generalization of the expression (1) for the irreversibility line, which in fact corresponds to choice of a level of R/R_n sufficiently low (or of γ_0 sufficiently high, ~ 20) that apparently persistent currents flow.

(b) Another scaling consequence of the dependence of R/R_n on the single variable γ_0 in (4c) is that, at any chosen R/R_n level, the *resistively measured* "upper critical field" near T_c will vary as $(T_c - T)^{3/2}$, *not* linearly with $(T_c - T)$ as in classic superconductors; in the latter, the resistive transition occurs at the same (H,T) as the thermodynamic one because these activated processes

are negligible. $H_{c2}(T)$ data in the literature on high-temperature superconductors often display an "upward curvature" behavior near T_c, qualitatively similar to this dependence. In many cases, such as the data on single crystals of YBCO by Iye *et al.*,[6] and also the data on epitaxial films of Oh *et al.*,[7] it appears that there is a good *quantitative* fit to this predicted $(T_c - T)^{3/2}$ dependence. This fit was noted by Oh *et al.*, but interpreted in terms of critical fluctuations.

(c) We now go beyond scaling arguments to compare the experimental data and the specific functional form of (6). In Fig. 1, we reproduce some particularly clean data from the work of Iye *et al.*[6] on a high-quality YBCO single crystal with H parallel to the c axis, together with a similar family of curves computed from (6) with the single fitting parameter $A = 1.2\times10^7$ G. For $\beta \approx 1$, this number corresponds to the value $J_{c0}(0) \approx 3\times10^6$ A/cm^2 in (4). This is not an unreasonable value for an excellent crystal, even though it refers

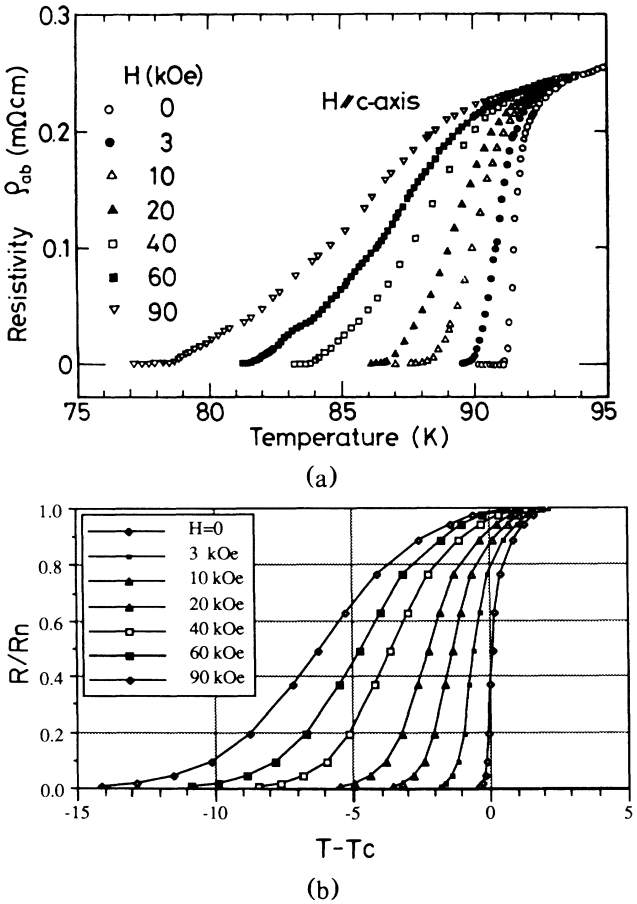

FIG. 1. (a) $\rho_{ab}(T)$ of YBCO crystal for various values of H as reported in Ref. 6. (b) Computed curves from Eq. (6) with single fitting parameter A. At each level of R/R_n, the theoretical downshift $T_c - T$ is plotted relative to the *experimental* $R(T)$ curve for $H = 0$. No adjustment has been made for the general linear slope of $R_n(T)$.

to the direction parallel to the c axis. The agreement between the two sets of curves is quite impressive for a single parameter fit. Comparable agreement is found with the data of Iye *et al.*, taken with H perpendicular to the c axis; in that case, the fitted A is about 6×10^7 G, corresponding (for $\beta = 1$) to $J_{c0}(0) \approx 1.8\times10^7$ A/cm^2 in the ab-plane direction. When a similar analysis is made of the data of Oh *et al.*[7] polycrystalline-oriented epitaxial films, the A values are found to be 7×10^6 and 4×10^7 G for the two directions. The anisotropy ratio of these values is 6, in good agreement with the single-crystal data of Iye *et al.*, but the absolute magnitudes are only about $\frac{2}{3}$ as large, perhaps indicating slightly poorer quality crystals. Direct measures of J_c suggest values of the order of 4×10^5 and 3×10^6 A/cm^2 in the two directions for an anisotropy ratio of ~ 7, but the published data[9] are rather meager. Thus our model gives good consistency for the anisotropy *ratio* of J_{c0}; the absolute magnitudes suggest that the value of β in (4a) may be ~ 5, or less if the measured J_c values seriously underestimate $J_{c0}(0)$ because of fluctuation effects.

Data on ceramic "pellet" material must be expected to be more variable. However, the field-sensitive part of the resistive transition (for $H \leq 50$ Oe) reported by Dubson *et al.*[8] is quite well described by our model, with $A \approx 1.2\times10^5$ G. This value is about 100 times smaller than found in the crystalline samples, corresponding to a $J_{c0}(0)$ value lower by a similar factor. This does not seem unreasonable, if we consider the presumed weak links between grains in the ceramic.

(d) In plotting the fitted curves to the resistive transitions in Fig. 1 for $H > 0$, we took the *measured* slightly rounded transition curve for $H = 0$ and, at each value of R/R_n, displaced it downward in temperature by the value of $T_c - T$ inferred from (6). Note that this amounts to the assumption that there is *no* downward shift of T_c itself by the magnetic field. Even with $\xi_{ab}(0)$ of only 12 Å, a field of 90 kOe parallel to the c axis should depress $T_{c2}(H)$ by ~ 3.5 K. This is about the magnitude of the downshift of the top of the resistive transition, as extrapolated from the linearly rising portion, but in the fitted curves this results from the *broadening* of the resistive transition, without any *shift* of the *end point* at T_c. The absence of a perceptible decrease in T_c is consistent with the precise magnetization data of Athreya *et al.*[10] They find a reversible magnetization proportional to $(T_c - T)^2$, with the fitted $T_c(H)$ apparently *unshifted* in a field of 20 kOe, at which one might expect a shift of 0.8 K for the same assumed $\xi_{ab}(0)$. A possible resolution of this puzzle is provided by the data of Fang *et al.*,[11] showing that the thermodynamic H_{c2} determined from the onset of reversible superconducting magnetization rises as $(1 - t)^{1/2}$ near T_c. (The unusual exponent of $\frac{1}{2}$ may arise from fluctuation effects near T_c, rather than from the two-dimensional model suggested by Fang *et al.*) According to this mea-

sured dependence, T_c would only be depressed by 0.4 K at 20 kG, which is probably near the limit of detectability in the data of Ref. 10. At the very least, the viewpoint put forward here accounts naturally for the fact that any shift in $T_c(H)$ in these data is *much* less than would be expected if $R/R_n = 0$ or 0.5 indicated the thermodynamic H_{c2}. By the same token, a small shift in $T_c(H)$ would also be inconspicuous in fitting the larger broadening of the resistive transition analyzed here.

(e) Finally, let us explore the broader implications of this model, extending the general thrust of our earlier publication[12] on the importance of flux creep in the high-temperature superconductors. The following remarks are somewhat speculative, in that they assume the expression (3) for the barrier U_0, although it has been validated empirically only in a limited range of conditions.

We note that a minimum requirement for a nonzero critical current at zero voltage is that R/R_n be exponentially small. According to (5), this requires that the ratio $\gamma_0 = U_0/k_B T$ exceed some threshold value $\gamma_{\min}$ of the order of 20. Given the form of (4), this requirement can be translated into the requirement that the product of B and T satisfy the inequality

$$BT < CJ_{c0}(t)/\gamma_{\min} = DJ_{c0}(0)(1-t^2)(1-t^4)^{1/2}. \quad (8)$$

The constant C, given by (4a), should be of order 200, while we expect $\gamma_{\min} \sim 20$, so $D \sim 10$ might be a reasonable estimate, for J_{c0} expressed in A/cm^2. For the representative value $J_{c0} \approx 10^7$ A/cm^2, (8) then becomes

$$BT < 10^8(1-t^2)(1-t^4)^{1/2}, \quad (8')$$

regardless of the values of H_{c2} and T_c.

At 4 K, the condition (8') is no serious constraint, although as B increases, making the zero-resistance condition less generously satisfied, it presages limited critical currents. The real significance of (8') is the constraint it puts on high-temperature operation. For example, if taken literally, it would exclude zero resistance at room temperature and ~ 300 kG, *no matter how high T_c and H_{c2} might be!* More concretely, with a 400-K superconductor operating at room temperature, zero resistance would be limited to $B < 100$ kG, again, no matter how much higher H_{c2} might be. One must bear in mind that these statements are based on the minimal criterion of *zero resistance,* not much stricter requirement of a *usefully large I_c.* On the other hand, the numbers represent a rather crude estimate; new materials might have significantly higher intrinsic $J_{c0}(0)$ values and/or different detailed dependences on parameters. The basic point, however, is that useful supercurrent properties in the presence of substantial magnetic fields will be difficult to achieve at room temperature. It is not enough just to raise T_c and H_{c2}; unless $J_{c0}(0)$ is of order 10^7 A/cm^2, it appears that the materials will be *resistive* at room temperature even if they are, thermodynamically speaking, in the "superconducting" state.

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Note added.—In fact, recent data by Palstra *et al.*[13] on the Bi superconductor show thermally activated resistance qualitatively similar to that described here, but with U_0 having a more complicated H dependence.

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