$(e,2e)$ Collisions in the Presence of a Laser Field

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(Received 15 April 1988)

We study the influence of a laser field on the dynamics of fast $(e,2e)$ collisions on atomic hydrogen, in the asymmetric coplanar geometry. We find that the triply differential cross sections are strongly dependent on the "dressing" of the atomic target by the laser.

PACS numbers: 34.80.Qb, 32.80.−t

In this Letter, we present a theoretical treatment of fast $(e,2e)$ reactions in a laser field, and report a number of new results concerning the modifications of the angular distributions of the ejected electrons due to the presence of the laser. In particular, we have found that dramatic changes in the triply differential cross sections (TDCS) can occur because of the "dressing" of the atomic target states by the laser.

In order to keep the discussion simple, we shall assume that the laser is treated classically as a spatially homogeneous electric field, linearly polarized and single mode, $\mathcal{E}(t)=\mathcal{E}_0\sin(\omega t)$, and that the target consists of atomic hydrogen. The $(e,2e)$ kinematical arrangement which we have selected is the Ehrhardt asymmetric coplanar geometry,[1] such that a fast electron of momentum $\mathbf{k}_i$ is incident on the target, and a fast ("scattered") electron of momentum $\mathbf{k}_A$ is detected in coincidence with a slow ("ejected") electron of momentum $\mathbf{k}_B$, the three momenta $\mathbf{k}_i$, $\mathbf{k}_A$, and $\mathbf{k}_B$ being in the same plane. Moreover, the scattering angle θ_A of the fast electron is fixed and small, while the angle θ_B of the slow electron is varied. The reasons for our choice of the Ehrhardt geometry are that (i) at high incident energies most of the $(e,2e)$ collisions occur in this kinematical regime and (ii) accurate experimental[2] and theoretical[3-5] results are available in this geometry for the corresponding field-free $(e,2e)$ reaction $e^-+\mathrm{H}(1s)\rightarrow \mathrm{H}^+ +2e^-$.

Remembering that in the Ehrhardt geometry exchange effects between the projectile and target electrons are small, and that a perturbative treatment of the (direct) interaction between the fast projectile electron and the target atom is justified,[3,4] we start from the first Born ionization S-matrix element, which for the present laser-assisted $(e,2e)$ reaction is given (in atomic units) by

$$S_{\mathrm{ion}}^{\mathrm{BI}}=-i\int_{-\infty}^{+\infty}dt\langle\chi_{\mathbf{k}_A}(\mathbf{r}_0,t)\Phi_{\mathbf{k}_B}(\mathbf{r}_1,t)\,|\,1/r_{01}-1/r_0\,|\,\chi_{\mathbf{k}_i}(\mathbf{r}_0,t)\Phi_0(\mathbf{r}_1,t)\rangle, \tag{1}$$

where $\mathbf{r}_0$ and $\mathbf{r}_1$ are respectively the coordinates of the projectile and target electrons, and $r_{01}=|\mathbf{r}_0-\mathbf{r}_1|$. The wave functions $\chi_{\mathbf{k}_i}(\mathbf{r}_0,t)$ and $\chi_{\mathbf{k}_A}(\mathbf{r}_0,t)$ are Volkov wave functions of the form

$$\chi_{\mathbf{k}}(\mathbf{r}_0,t)=(2\pi)^{-3/2}\exp[i(\mathbf{k}\cdot\mathbf{r}_0-\mathbf{k}\cdot\boldsymbol{\alpha}_0\sin\omega t-E_k t)], \tag{2}$$

where $E_k=k^2/2$ and $\boldsymbol{\alpha}_0=\mathcal{E}_0/\omega^2$, ω being the laser angular frequency.

The wave functions $\Phi_0(\mathbf{r}_1,t)$ and $\Phi_{\mathbf{k}_B}(\mathbf{r}_1,t)$ in Eq. (1) are the "dressed" states of the hydrogen atom embedded in the laser field, the first one corresponding to the initial (bound) state and the second one to the final (continuum) state in which a slow electron of momentum $\mathbf{k}_B$ has been ejected. In what follows we shall consider laser fields such that $\mathcal{E}_0\ll 5\times10^{11}$ V m^{-1} (the atomic unit of field strength). The dressed atomic bound states can then be obtained by use of first-order, time-dependent perturbation theory.[6] In particular, the dressed ground-state wave function $\Phi_0(\mathbf{r}_1,t)$ is given by[7]

$$\Phi_0(\mathbf{r}_1,t)=\exp(-iE_0t)\exp(-i\mathbf{a}\cdot\mathbf{r}_1)\left[\psi_0(r_1)+\frac{i}{2}\sum_n\left(\frac{\exp(i\omega t)}{E_n-E_0+\omega}-\frac{\exp(-i\omega t)}{E_n-E_0-\omega}\right)M_{n0}\psi_n(\mathbf{r}_1)\right], \tag{3}$$

where ψ_n is a target state of energy E_n in the absence of the laser field, $M_{n0}=\langle\psi_n\,|\,\mathcal{E}_0\cdot\mathbf{r}\,|\,\psi_0\rangle$ is a dipole-coupling matrix element, and the summation runs over the discrete and continuum hydrogen-atom p states. Moreover, $\mathbf{a}=\omega^{-1}\mathcal{E}_0\times\cos(\omega t)$, the factor $\exp(-i\mathbf{a}\cdot\mathbf{r}_1)$ ensuring gauge consistency between the Volkov wave function (2) and the dressed target wave function (3).

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For the dressed continuum wave function $\Phi_{\mathbf{k}_B}(\mathbf{r}_1,t)$ we have used the expression

$$\Phi_{\mathbf{k}_B}(\mathbf{r}_1,t)=\exp(-iE_{k_B}t)\exp(-i\mathbf{a}\cdot\mathbf{r}_1)\exp(-i\mathbf{k}_B\cdot\boldsymbol{\alpha}_0\sin\omega t)$$
$$\times\left[\psi^{(-)}_{C,\mathbf{k}_B}(\mathbf{r}_1)+\frac{i}{2}\sum_n\left(\frac{\exp(i\omega t)}{E_n-E_{k_B}+\omega}-\frac{\exp(-i\omega t)}{E_n-E_{k_B}-\omega}\right)M_{n,\mathbf{k}_B}\psi_n(\mathbf{r}_1)+i\mathbf{k}_B\cdot\boldsymbol{\alpha}_0\sin\omega t\,\psi^{(-)}_{C,\mathbf{k}_B}(\mathbf{r}_1)\right], \quad (4)$$

where $\psi^{(-)}_{C,\mathbf{k}_B}$ is a Coulomb wave function with incoming spherical wave behavior, corresponding to momentum $\mathbf{k}_B$ and energy $E_{k_B}=k_B^2/2$, normalized to a δ function in momentum space, and $M_{n,\mathbf{k}_B}=\langle\psi_n|\mathcal{E}_0\cdot\mathbf{r}|\psi^{(-)}_{C,\mathbf{k}_B}\rangle$. The wave function given by Eq. (4) is a generalization of that proposed by Jain and Tzoar[8] and used by Cavaliere, Ferrante, and Leone[9] to study laser-assisted $(e,2e)$ reactions. This generalization, which takes into account (to first order in $\mathcal{E}_0$) the role of all the target states in "dressing" the ejected-electron wave function, has been obtained by the use of time-dependent perturbation theory in a way analogous to the low-frequency analysis of Banerji and Mittleman.[10]

Using Eqs. (2)–(4), one finds that the S-matrix element (1) can be rewritten as

$$S^{\mathrm{B1}}_{\mathrm{ion}}=(2\pi)^{-1}i\sum_{l=-\infty}^{+\infty}\delta(E_{k_A}+E_{k_B}-E_{k_i}-E_0-l\omega)f^{\mathrm{B1},l}_{\mathrm{ion}} \quad (5)$$

where $f^{\mathrm{B1},l}_{\mathrm{ion}}$, the first Born approximation to the $(e,2e)$ scattering amplitude with the transfer of l photons, is given by

$$f^{\mathrm{B1},l}_{\mathrm{ion}}=f_{\mathrm{I}}+f_{\mathrm{II}}+f_{\mathrm{III}} \quad (6)$$

with

$$f_{\mathrm{I}}=-2K^{-2}J_l(\lambda)\langle\psi^{(-)}_{C,\mathbf{k}_B}|\exp(i\mathbf{K}\cdot\mathbf{r})|\psi_0\rangle, \quad (7a)$$

$$f_{\mathrm{II}}=iK^{-2}\sum_n\langle\psi^{(-)}_{C,\mathbf{k}_B}|\exp(i\mathbf{K}\cdot\mathbf{r})|\psi_n\rangle M_{n0}\left[\frac{J_{l-1}(\lambda)}{E_n-E_0-\omega}-\frac{J_{l+1}(\lambda)}{E_n-E_0+\omega}\right], \quad (7b)$$

and

$$f_{\mathrm{III}}=iK^{-2}\sum_n\langle\psi_n|\exp(i\mathbf{K}\cdot\mathbf{r})|\psi_0\rangle M^*_{n,\mathbf{k}_B}\left[\frac{J_{l-1}(\lambda)}{E_n-E_{k_B}+\omega}-\frac{J_{l+1}(\lambda)}{E_n-E_{k_B}-\omega}\right]$$
$$-K^{-2}\mathbf{k}_B\cdot\boldsymbol{\alpha}_0[J_{l-1}(\lambda)-J_{l+1}(\lambda)]\langle\psi^{(-)}_{C,\mathbf{k}_B}|\exp(i\mathbf{K}\cdot\mathbf{r})|\psi_0\rangle. \quad (7c)$$

In these equations, J_l is a Bessel function of order l, $\mathbf{K}=\mathbf{k}_i-\mathbf{k}_A$ is the momentum transfer, and we have defined $\lambda=(\mathbf{K}-\mathbf{k}_B)\cdot\boldsymbol{\alpha}_0$. The first Born TDCS corresponding to the laser-assisted $(e,2e)$ reaction accompanied by the transfer of l photons is given by

$$\frac{d^3\sigma^{\mathrm{B1},l}_{\mathrm{ion}}}{d\Omega_A\,d\Omega_B\,dE}=\frac{k_Ak_B}{k_i}|f^{\mathrm{B1},l}_{\mathrm{ion}}|^2, \quad (8)$$

and we note that the results of Cavaliere, Ferrante, and Leone[9] can be recovered from the foregoing treatment if only the first term f_{I} is kept in Eq. (6).

In the case of the absorption of one photon ($l=1$) we have also derived the lowest-order time-dependent perturbative version of the scattering amplitudes. These results can be recovered from Eqs. (7) by retaining only the first term (lowest order in the field strength) in the expansion of the Bessel functions.

The amplitudes f_{II} and f_{III} contain infinite sums running over the whole atomic spectrum. These sums have been accurately computed with the help of the Dalgarno method,[11] which we found to be more tractable for the present problem than other techniques we had used previously in similar instances.[7,12–15] The implementation of Dalgarno's method presented some technical difficulties caused by the fact that the argument of the Coulomb Green's function G_C can become positive in cases of physical interest (typically if $\omega<k_B^2/2$). In such a situation the argument of G_C is on its cut in the complex plane and one has to determine the analytically continued values of the matrix elements. This difficulty was overcome by generalization of the technique of Zernik and Klopfenstein,[16] originally devised to compute two-photon–ionization dipole matrix elements, to the case of second-order amplitudes containing the $\exp(i\mathbf{K}\cdot\mathbf{r})$ operator.[17]

Our results clearly demonstrate the strong influence of the laser parameters on the dynamics of laser-assisted $(e,2e)$ reactions. Figure 1 displays the effects of the field strength $\mathcal{E}_0$ at a fixed frequency (here $\omega=1.17$ eV, corresponding to a Nd-doped yttrium aluminum garnet laser). We have presented the angular distributions of the ejected electrons at two different field strengths, $\mathcal{E}_0=5\times10^9$ V m^{-1} [Fig. 1(a)] and $\mathcal{E}_0=10^8$ V m^{-1} [Fig. 1(b)], which correspond respectively to a nonperturbative regime and a perturbative regime. For comparison we also show angular distributions obtained when the dressing of the target is neglected; the resulting curve is then homothetic to the first Born field-free result.

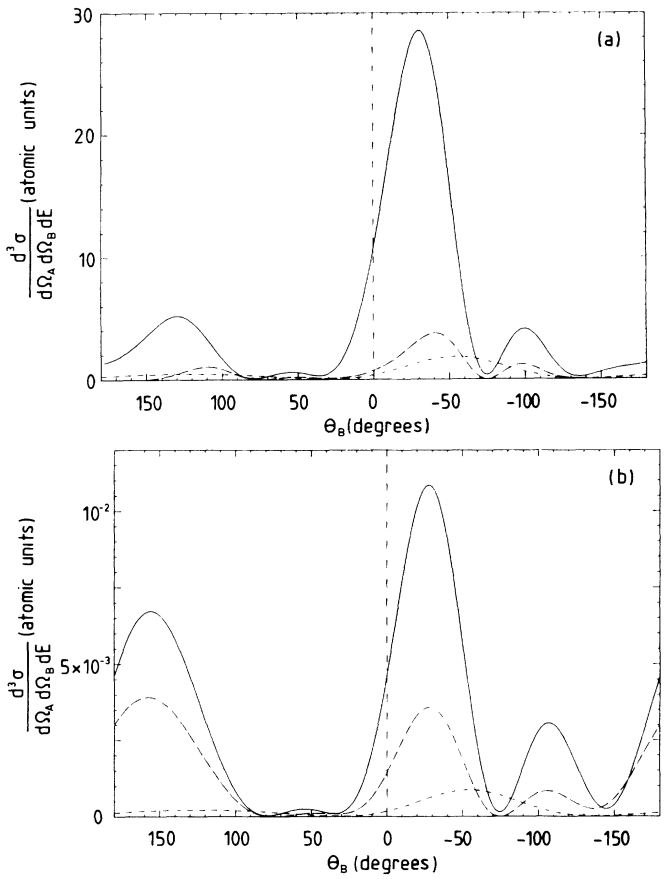

FIG. 1. The TDCS (in atomic units) for the ionization of atomic hydrogen from the ground state by electron impact in the presence of a laser field, as a function of the ejected electron angle θ_B. The incident electron energy is $E_i = 250$ eV, the ejected electron energy is $E_B = 5$ eV, and the scattering angle is $\theta_A = 3°$. The electric field $\mathcal{E}$ is taken to be parallel to the incident momentum $\mathbf{k}_i$ and the laser photon energy is $\omega = 1.17$ eV. (a) corresponds to a nonperturbative regime, the electric field strength being $\mathcal{E}_0 = 5\times10^9$ V m^{-1}, while (b) corresponds to a perturbative regime with $\mathcal{E}_0 = 10^8$ V m^{-1}. Solid line, complete calculation by use of the scattering amplitude given by Eq. (6). Dashed line, results obtained by use of the simplified approach of Cavaliere, Ferrante, and Leone (Ref. 9). Dotted line, results obtained when the dressing of the target is neglected.

Also included in Fig. 1 are the variations of the TDCS as obtained from the simplified analysis proposed by Cavaliere, Ferrante, and Leone,[9] which takes into account the dressing of the ejected electron in an approximate way, via Jain and Tzoar's *Ansatz*.[8] The comparison with our results clearly demonstrates the importance of the dressing of the target.

Such atomic structure effects can also be dramatically enhanced by suitable adjustment of the laser frequency and/or the kinetic energy of the slow (ejected) electron. Indeed, the amplitudes [7(b) and 7(c)] exhibit reso-

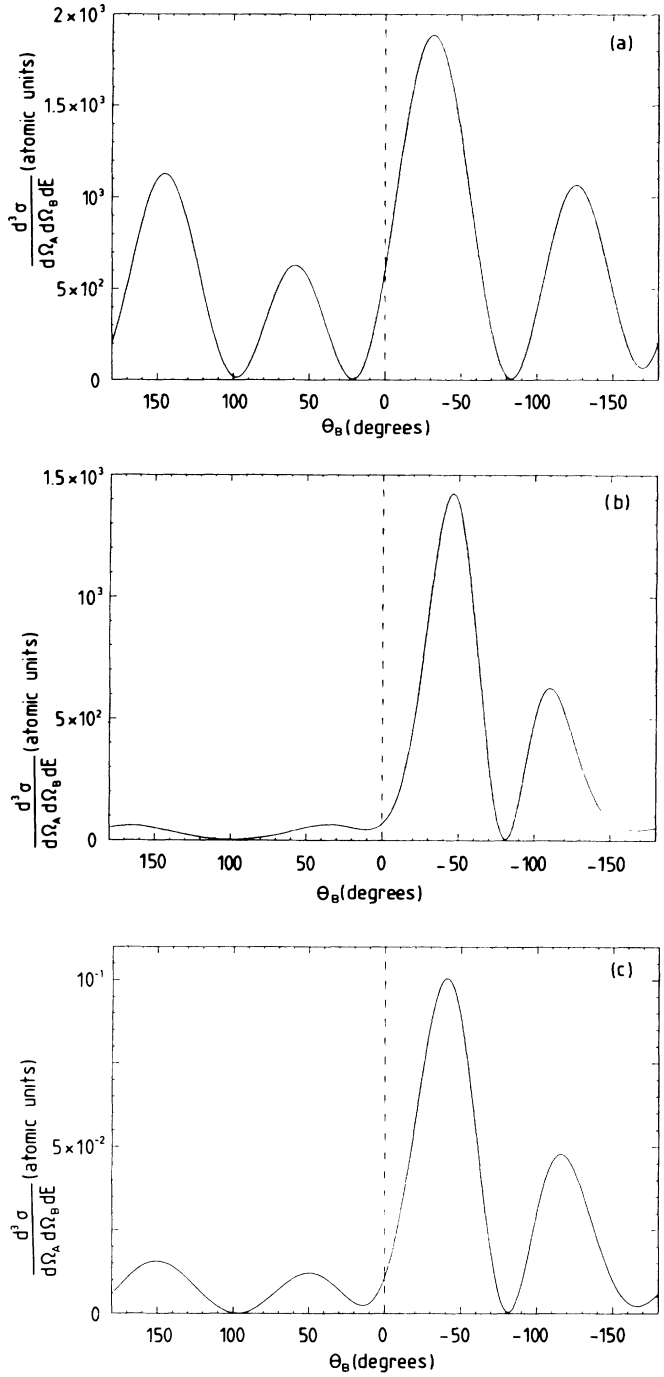

FIG. 2. The TDCS (in atomic units) for the ionization of atomic hydrogen from the ground state by electron impact in the presence of a laser field, as a function of the ejected electron angle θ_B. The incident electron energy is $E_i = 250$ eV, the ejected electron energy is $E_B = 5$ eV, and the scattering angle is $\theta_A = 3°$. The electric field $\mathcal{E}$ is oriented along the incident momentum $\mathbf{k}_i$ and the electric field strength is $\mathcal{E}_0 = 10^9$ V m^{-1}. (a) The laser photon energy is $\omega = 8.4$ eV and corresponds to the resonance condition of Eq. (10). (b) The laser photon energy $\omega = 10.2$ eV and corresponds to the resonance condition of Eq. (9). (c) The laser photon energy is $\omega = 9.3$ eV and corresponds to a typical nonresonant situation.

nances, the resonance conditions being

$$\omega = E_n - E_0 \tag{9}$$

for the photon energy and

$$E_{k_B} = E_n + \omega \tag{10}$$

for the energy of the ejected electron (at a fixed laser frequency ω).

Figure 2 displays the important modifications of the angular distribution of the ejected electron, observed when we vary the laser energy between 8.4 and 10.2 eV for a fixed ejected-electron kinetic energy of 5 eV. At $\omega = 8.4$ eV, Eq. (10) shows that the process is close to resonance on the state $n=2$ ($E_2 = -3.4$ eV). This means that the ground state is dominantly coupled to the $n=2$ manifold via the operator $\exp(i\mathbf{K}\cdot\mathbf{r})$. We note that these states are, in turn, coupled to s, p, and d continua through the dipole interaction. The corresponding angular distribution, shown in Fig. 2(a), displays split binary and recoil peaks. Another extreme situation occurs when $\omega = 10.2$ eV. The process is again nearly resonant, the ground state being dominantly coupled to the $2p$ state via the dipole interaction $\mathcal{E}_0\cdot\mathbf{r}$. The TDCS is then strongly modified with respect to the previous case, the recoil peak being almost completely suppressed [see Fig. 2(b)]. Finally, Fig. 2(c) corresponds to a typically nonresonant situation such that $\omega = 9.3$ eV, so that none of the resonance conditions Eq. (9) or Eq. (10) is fulfilled. The general shape of the angular distribution appears as a mixture of the two previous cases though the magnitude of the cross section is smaller by several orders of magnitude.

This work was partially supported by the grant from the Centre National de la Recherche Scientifique-CGRI agreement. One of us (P.F.) is supported by the IRSIA Institute. The Laboratoire de Chimie Physique de l'Université Pierre et Marie Curie is "Unité Associée au Centre National de la Recherche Scientifique No. 176."

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