Modulational Instability and Its Consequences for the Beat-Wave Accelerator

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The modulation instability caused by the coupling of a Langmuir wave to the ion motion is investigated in the domain of large $v_L/v_{te} > 1$ ratios, where v_L and v_{te} denote the pump Langmuir-wave quiver velocity and the electron thermal velocity, respectively. A convenient approximate expression for the growth rate is given for $v_L/v_{te} < 50(A/Z)^{1/6}$. The limitation of the beat plasmon growth due to the modulational instability is studied in the context of plasma beat-wave experiments and the maximum beat plasmon amplitude is determined numerically.

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It has been recently suggested[1] that the large electric field of a relativistic ($v_\phi \approx c$) electron plasma wave may be used for ultrahigh gradient acceleration of particles. The plasma beat-wave accelerators (PBWA) and the plasma wake-field accelerators are the two main proposals.[2] In the PBWA, the beat wave grows linearly until it saturates because of a mismatch (linear or nonlinear) between the Langmuir wave frequency and the beat frequency.[3] However, it is well known that the coupling of the Langmuir waves to the ion motion can give rise either to the modulational or to the decay instabilities.[4] For this reason several authors[5,6] have pointed out that such instabilities might have a dramatic effect upon the growth of the beat plasma wave in the context of PBWA. It was, however, pointed out that the latter conclusions have a narrow applicability range since they were derived within the usual framework $v_L/v_{te} < 1$ in which the decay and the modulational instabilities have mainly been investigated in the past[7]; here $v_{te} \equiv (k_B T_e/m_e)^{1/2}$ denotes the electron thermal velocity and $v_L \equiv eE_L/m_e\omega_{pe}$ is the peak oscillation velocity in the Langmuir wave electric field; the latter is characterized by a frequency $\omega_0 \approx \omega_{pe}$ and a wave number $k_0 \approx \omega_{pe}/c$ (ω_{pe} denotes the electron plasma frequency and c the speed of light). Henceforth the results which are usually derived in the domain $v_L/v_{te} < 1$ are referred to as "weak-field regime."

In the context of PBWA, the quantity v_L/v_{te} is easily related to the density amplitude of the Langmuir wave δn_L; one finds $v_L/v_{te} = 23(\delta n_L/n_0) T_e^{-1/2}$, where n_0 is the electron density and T_e the electron temperature in units of keV. In PBWA experiments, Langmuir wave levels $(\delta n_L/n_0)$ as large as 10^{-1} are expected; since the plasma temperature of present day experiments can be very low, of the order of a few tens of eV, v_L/v_{te} could reach values significantly larger than unity. One thus clearly sees the need for a careful investigation of the instability domains for low-temperature plasma. For the reader's convenience, we represent in Fig. 1 part of the results obtained previously in Ref. 5 as well as the results derived in this Letter. In this figure are drawn the domains corresponding to the different types of instabilities in $(\rho I\lambda^2, T_e)$ space; here I is the flux of the higher frequency laser, λ is its wavelength, and ρ is the amplitude ratio of the two lasers. Domains (1) and (2) have been considered in Ref. 5 and they correspond to the regions where the standard modified decay [domain (1)] and standard modulational instabilities [domain (2)] reduce the beat plasmon growth as compared with the relativistic predictions. In these two domains the weak-field results apply. Region (3) represents the domain investigated in this Letter where the beat plasmon is still limited by the modulational instability, although the limitation is less severe than what could be expected from a naive extrapolation from the results of domain (2): as shown later on, the ion instabilities' limiting role decreases in domain (3) because of a reduction of the modulational growth rate caused by a large ratio $v_L/v_{te} \gg 1$. Domain (3) will be referred to as "strong-field regime." Lastly, region (4) corresponds to the domain

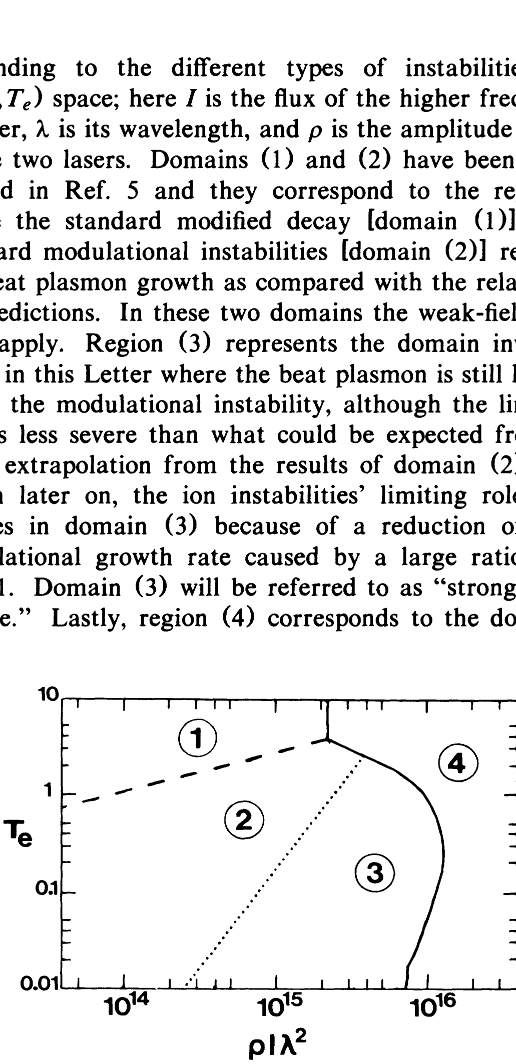

FIG. 1. Domains corresponding to the different saturation mechanisms for the beat plasmon growth in the case $Z/A = 1$. $\rho I\lambda^2$ is in W cm^{-2} μm^2 and the electron temperature T_c is in keV. Domain (1) corresponds to a saturation caused by the modified decay instability, domain (2) by the modulational instability in its usual weak-field regime, domain (3) by modulational instability in its strong-field regime, and domain (4) by relativistic detuning.

where the plasma-wave growth is not affected by the ion instabilities, i.e., the domain where the nonlinear relativistic phase slip of the Langmuir wave relative to the beat of the two lasers attains $\pi/2$ before the ion instabilities have sufficient time to grow.

The dotted line separating domains (2) and (3) represents the boundary at which the effects caused by a large ratio v_L/v_{te} have to be taken into account in the computation of the instability growth rate. One can see that in our present context of PBWA this boundary concerns the modulational instability only; for this reason we restrict our analysis to the two following questions: (i) On the one hand the modulational instability is investigated without the constraint $v_L/v_{te} < 1$; it is found that the usual weak-field expression for the growth rate is valid as long as the particle excursion in the plasma wave is smaller than the wavelength of the perturbation, i.e., $kv_L < \omega_{pe}$, corresponding to a maximum value for v_L/v_{te} of the order of $(\omega_{pe}/\omega_{pi})^{1/3}$; we compute the growth rate in the opposite regime, $kv_L > \omega_{pe}$, and find that it cannot exceed $1.8(A/Z)^{1/6}\omega_{pi}$ (here ω_{pi} is the ion plasma frequency and A and Z are the ion mass and charge number, respectively). (ii) On the other hand, we estimate the maximum amplitude of the plasma wave excited by the beating of two lasers of constant intensity when the limiting mechanism for the plasma-wave growth is the loss of coherency due to the e folding of the modulationally unstable fluctuations. We confirm that for moderate intensities this latter limiting mechanism dominates the nonlinear relativistic detuning effect studied by Rosenbluth and Liu.[3]

Let us first consider the modulational instability of a one-dimensional large amplitude Langmuir wave of wave number k_0 and natural frequency $\omega_0 \approx \omega_{pe}$. In the case $v_L/v_{te} > \omega_{pi}/\omega_{pe}$, a condition that we assume henceforth, the maximum growth rate occurs in the so-called supersonic regime, in which the wave number k_{max} satisfied $k_{max} \gg k_0$. For this reason we take $k_0 = 0$ and the inequality $k_0 \ll k_{max}$ is *a posteriori* found to be satisfied in domains (2) and (3) of Fig. 1. In this limit $k_0 = 0$, one can use the oscillation reference frame technique from which one may easily derive the dispersion relation.[4] In the limit $k\lambda_{De} \ll 1$, $kv_{ti} \ll \gamma$, it reads

$$\gamma(k)^2 = \omega_{pi}^2 \left\{ J_1^2[-1/(1+\chi_e^{(1)}) + -1/(1+\chi_e^{(-1)})] + (2/kx_L)J_0J_1 - \tfrac{3}{2}J_1^2 - 1 - 1/\chi_e^{(0)} \right\}, \tag{1}$$

where $\gamma(k)$ denotes the growth rate, $\lambda_{De} = v_{te}/\omega_{pe}$ is the Debye length, and v_{ti} the ion thermal velocity; $\chi_e^{(\pm 1)}$ and $\chi_e^{(0)}$ are the electron susceptibilities $\chi_e(i\gamma \pm \omega_{pe}, k)$ and $\chi_e(i\gamma, k)$, respectively; the argument of the Bessel functions J_n is $z = kv_L/\omega_{pe} \equiv kx_L$. Though we have solved Eq. (1) numerically with all terms, it is interesting to compare its exact solution with a convenient approximation. It is indeed easily seen that only the two first terms are dominant in Eq. (1) since they correspond to the plasma-wave resonance $|\chi_e^{(\pm 1)} - 1| \ll 1$. In the limit $k\lambda_{De} \ll 1$, Eq. (1) can thus be reduced to

$$\gamma(k)^2 = \omega_{pi}^2 \frac{6k^2\lambda_{De}^2}{9k^4\lambda_{De}^4 + 4\gamma^2/\omega_{pe}^2} J_1^2(kx_L). \tag{2}$$

Approximating $J_1(z)$ by $J_1^a(z) \equiv (z/2)(1 - z^2/8)$, the maximum growth rate γ_{max} is found to correspond approximately to a wave number $k \approx k_a \equiv k_1(1 + k_1x_L/2)^{-1}$, with

$$k_1\lambda_{De} \equiv \frac{2^{1/6}}{3^{2/3}} \left[\frac{\omega_{pi}}{\omega_{pe}} \left(\frac{v_L}{V_{te}} \right)^2 \right]^{1/3}.$$

In the case of moderately large amplitude, namely for $(k_a\lambda_{De})^2 \geq (\gamma_{max}/\omega_{pe})$, one recovers the usual expression of the maximum growth rate of the instability,

$$\gamma_{max}^{wf} = 6^{-1/2}\omega_{pi}(v_L/v_{te}), \tag{3}$$

where the superscript wf stands for "weak field." The validity for this expression is easily found *a posteriori* to be $(v_L/v_{te}) \leq (\omega_{pe}/\omega_{pi})^{1/3}$. Thus the standard expression (3) has a range of applicability which extends surprisingly beyond the naive estimate $v_L/v_{te} < 1$.

In the opposite case of large amplitude plasma waves, i.e., for $(v_L/v_{te}) \geq (\omega_{pe}/\omega_{pi})^{1/3}$, one quickly obtains the following expression for the maximum growth rate:

$$\gamma_{max}^{sf} = (\tfrac{3}{2})^{1/4}\omega_{pi}(\omega_{pe}v_{te}/\omega_{pi}v_L)^{1/2}, \tag{4}$$

where the superscript sf stands for "strong field."

In Fig. 2, the solid line represents the exact solution of Eq. (1) as a function of the wave number k of the unstable fluctuation for $v_L/v_{te} = 25$. For comparison, also shown are the solution of the approximate Eq. (2) (dotted line), and the usual weak-field approximation (3)

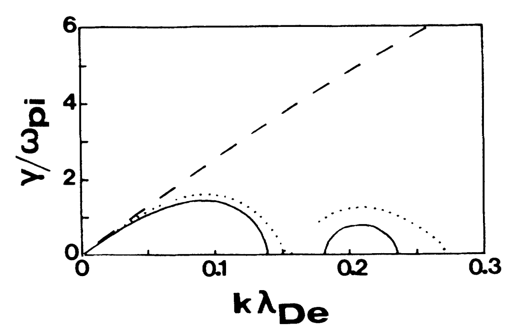

FIG. 2. The growth rate $\gamma(k)$ in units of ω_{pi} as a function of $k\lambda_{De}$ for $v_L/v_{te} = 25$ and $Z/A = 1$. The solid line is the solution of the exact equation (1), the dotted line is the solution of the approximate equation (2), and the dashed line is the usual weak-field expression of the growth rate.

where $J_1(z)$ is replaced by $z/2$ in Eq. (2) (dashed line). These curves exhibit the smoothing effect of the Bessel functions on the instability: Physically, when the particle excursion in the main wave x_L becomes comparable with, or larger than the perturbation wavelength k^{-1}, the usual ponderomotive potential which gives rise to the instability is in some sense averaged over the spatial excursion of the electrons; therefore the latter cannot fully experience the ponderomotive potential and the growth rate is consequently reduced.

Figure 3 shows the maximum growth rate $\gamma_{\max}$ as a function of v_L/v_{te} (solid lines). For comparison the results of the two limiting cases are plotted, namely Eqs. (3) and (4), as dotted lines; lastly the dashed line represents the result obtained by setting $k=k_a$ and $J_1=J_1^q$ in Eq. (2). It can be seen that this simple approximation provides a remarkably good estimate of the maximum growth rate. For completeness we also plot the second maximum of $\gamma(k)$; we observe that the latter becomes larger than the first maximum for $v_L/v_{te}>38(A/Z)^{1/6}$. Actually a careful comparison between the exact solution of Eq. (1) and our approximation (4) shows that the latter—corresponding to the first maximum of $\gamma(k)$—remains excellent for large v_L/v_{te} up to $v_L/v_{te}\approx 50(A/Z)^{1/6}$. The wave number $k_{\max}$ corresponding to the first maximum of $\gamma(k)$ is found to reach its largest value $k_{\max}\approx 0.2\lambda_{De}^{-1}$ for $v_L/v_{te}\approx 4(A/Z)^{1/6}$, justifying *a posteriori* the first validity condition $k_{\max}\lambda_{De}\ll 1$ for Eq. (1) (from this result it follows also that the Landau damping could actually be neglected). On the other hand, the ratio $\gamma_{\max}/k_{\max}v_{ti}$ can be checked and found to be very large compared with $(ZT_e/T_i)^{1/2}$ in the supersonic regime $v_L/v_{te}\gg\omega_{pi}/\omega_{pe}$; the second validity condition $\gamma_{\max}\gg k_{\max}v_{ti}$ for Eq. (1) is thus satisfied for plasmas with $T_i\lesssim ZT_e$. We therefore conclude that the result obtained by setting $k=k_a$ and $J_1=J_1^q$ in Eq. (2) is a very good approximation of the maximum growth rate $\gamma_{\max}$ of the supersonic modulational instability, for $v_L/v_{te}\lesssim 50(A/Z)^{1/6}$ and $T_i\lesssim ZT_e$. Lastly, the growth rate $\gamma_{\max}$ is maximum for $v_L/v_{te}\approx 10(A/Z)^{1/6}$ with $\gamma_{\max}\approx 1.8(A/Z)^{1/6}\omega_{pi}$.

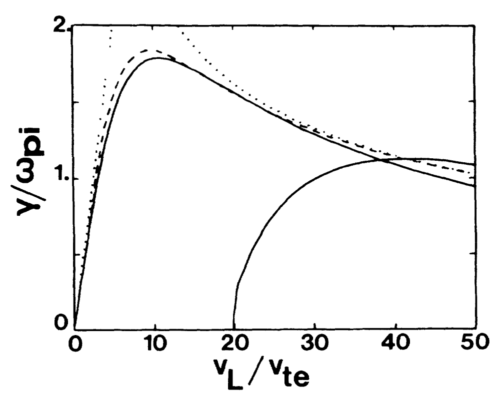

FIG. 3. The maximum growth rate $\gamma_{\max}$ in units of ω_{pi} as a function of v_L/v_{te} for $Z/A=1$. The two curves with solid lines represent the exact solution of Eq. (1) [the line corresponding to the first maximum of $\gamma(k)$ begins at $v_L=0$ and the one to the second maximum at $v_L/v_{te}=20$]. The dotted lines represent the results of the two limiting expressions, namely Eq. (3) for small v_L/v_{te} and Eq. (4) for large v_L/v_{te}. The dashed line is the result obtained by setting $k=k_a$ and $J_1=J_1^q$ in Eq. (2).

We now study the effect of this instability on the growth of the Langmuir wave in the PBWA case. We assume that the amplitude of the beat wave grows linearly with time until its coherency is destroyed by the instability. We assume that this happens for $\max_k\int^t\gamma(k,t')dt'\equiv e_c\approx 5$; this latter critical e folding has been found to reproduce correctly the results obtained from particle simulations[8] performed with $10^{14}<\rho I\lambda^2<2\times 10^{16}$ W cm^{-2} μm^2 and $15<T_e<150$ eV. In domains (1) and (2) where the usual weak-field expressions apply, the calculation can be performed analytically; in Ref. 5, the maximum density amplitude that the beat plasma wave can reach coherently has been found to be

$$(\delta n_L/n_0)_{\text{mod decay}}=3.2\times 10^{-2}(\rho I_{14}\lambda^2)^{3/5}(A/Z)^{1/5}$$

in domain (1) (corresponding to a limitation by the modified decay instability), and

$$(\delta n_L/n_0)_{\text{modul}}^{\text{wf}}=3.2\times 10^{-2}(A/Z)^{1/4}T_e^{1/4}(\rho I_{14}\lambda^2)^{1/2}$$

in domain (2) (corresponding to a limitation by the modulational instability in the weak-field regime). Here I_{14} is the laser flux in units of 10^{14} W/cm^2 and λ is in μm. The boundary between the two domains (1) and (2) (the dashed line in Fig. 1) corresponds to a temperature T_e given by $T_{\text{decay/modul}}=(Z/A)^{1/5}(\rho I_{14}\lambda^2)^{2/5}$. Let us recall that the maximum beat plasmon amplitude in the relativistic domain (4) is given by[3]

$$(\delta n_L/n_0)_{\text{rel}}=7.3\times 10^{-2}(\rho I_{14}\lambda^2)^{1/3}.$$

The boundary between the domains where the beat plasmon growth is limited by the ions instabilities and the one where it is due to the relativistic detuning is represented in Fig. 1 by a solid line: Between domains (1) and (4) it corresponds to $T_e>T_{\text{decay/modul}}$ and

$$\rho I_{14}\lambda^2<(\rho I_{14}\lambda^2)_{\text{moderate}}\equiv 22(Z/A)^{3/4};$$

between domains (2) and (4) it corresponds to $T_e<T_{\text{decay/modul}}$,

$$\rho I_{14}\lambda^2<(\rho I_{14}\lambda^2)_{\text{modul/rel}}^{\text{wf}}=1.4\times 10^2(Z/A)^{3/2}/T_e^{3/2},$$

and

$$T_e>T_{\text{wf/sf}}=1.8\times 10^{-3}(A/Z)^{1/3}(\rho I_{14}\lambda^2)^2.$$

The latter temperature is the lower limit below which the smoothing effect of the Bessel functions upon the modulational growth has to be retained. We have not been able to derive a simple analytical expression

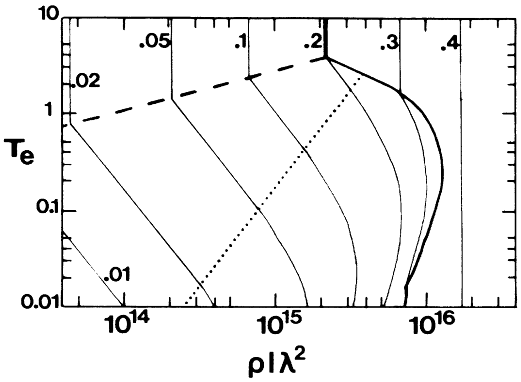

FIG. 4. Same figure as Fig. 1 in which are drawn the lines of isolevels for the beat plasma-wave amplitude $(\delta n_L/n_0)$. The numbers labeling each curve represent the value $(\delta n_L/n_0)$ reached by the beat plasma wave.

$(\rho I_{14}\lambda^2)^{\rm sf}_{\rm modul/rel}(T_e)$ corresponding to the solid line in Fig. 1 separating the domains (3) and (4), i.e., such that for $(\rho I_{14}\lambda^2) < (\rho I_{14}\lambda^2)^{\rm sf}_{\rm modul/rel}$ the plasma growth is limited by the modulational instability in its strong-field regime. This boundary has been determined numerically by solving Eq. (1) and by taking $e_c = 5$. By doing so, we also obtained the maximum amplitude $(\delta n_L/n_0)$ reached by the beat plasma wave. In Fig. 4 are drawn the lines of constant values for the beat plasma-wave maximum amplitude. As can be seen in Figs. 1 and 4, the important result is that due to the reduction of the modulational instability in its strong-field regime, there exists a maximum flux $(\rho I_{14}\lambda^2)_{\rm max}$ above which the modulational instability can no longer inhibit the plasmon growth. This maximum flux is found to be $(\rho I\lambda^2)_{\rm max} \approx 1.3(Z/A)^{1/2}10^{16}$ W/cm^2, and it occurs for $T_e = T^* \approx 0.2(Z/A)^{2/3}$ keV.

In conclusion, we have derived convenient approximate expressions for the modulational instability growth rate which are in excellent agreement with the exact result for $v_L/v_{te} \lesssim 50(A/Z)^{1/6}$. Secondly, we have computed the maximum amplitude $(\delta n_L/n_0)$ reached by the Langmuir wave in the context of PBWA as a function of the laser fluxes and of the plasma temperature. Finally, we have determined the flux $(\rho I\lambda^2)_{\rm max}$ above which the modulation has no time to grow to inhibit the beat plasma-wave generation. Our results are all one dimensional and therefore cannot account for the effects caused by the finite transverse size of the laser beams. Such two-dimensional effects have been investigated elsewhere.[6,9] Lastly, it should be noted that the above results apply only in the type of beat-wave accelerator that uses a neutral plasma as the wave supporting medium, and do not apply in another type of beat-wave accelerator that uses a relativistic counterstreaming electron beam as the wave supporting medium.[10]

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[1]T. Tajima and J. M. Dawson, Phys. Lett. **43**, 267 (1979).

[2]*New Developments in Particle Acceleration Techniques,* edited by S. Turner (CERN, Genève, 1987), Vol. 2.

[3]M. N. Rosenbluth and C. S. Liu, Phys. Rev. Lett. **29**, 701 (1972).

[4]V. P. Silin, Zh. Eksp. Teor. Fiz. **48**, 1679 (1965) , and **51**, 1842 (1966) [Sov. Phys. JETP **21**, 1127 (1965), and **24**, 1242 (1967)]; J. R. Sanmartin, Phys. Fluids **13**, 1523 (1970); R. Pellat, in *Laser-Plasma Interaction,* Proceedings of the Les Houches Summer School, Session XXXIV, edited by R. Balian and J. C. Adam (North-Holland, Amsterdam, 1982), p. 411.

[5]D. Pesme, S. J. Karttunen, R. R. E. Salomaa, G. Laval, and N. Silvestre, Ref. 2, p. 418.

[6]P. Mora, Ref. 2, p. 412.

[7]V. E. Zakkarov, Zh. Eksp. Teor. Fiz. **62**, 1745 (1972) [Sov. Phys. JETP **35**, 908 (1972)]; A. Hasegawa, Phys. Rev. A **1**, 1746 (1970).

[8]J. C. Adam and A. Héron, private communication.

[9]W. B. Mori, C. Joshi, J. M. Dawson, D. W. Forslund, and J. M. Kindel, Phys. Rev. Lett. **60**, 1298 (1988).

[10]Y. T. Yan, C. J. McKinstrie, T. Katsouleas, and J. M. Dawson, Phys. Rev. A **36**, 5455 (1987).