Experimental Study of Dynamic Permeability in Porous Media

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We report experimental measurements of ac permeability made on a variety of fused glass beads and crushed glass in a frequency range of 0.1 Hz to 1 kHz. We observe a rollover in $\kappa(\omega)$ from a viscous flow regime to an inertial one, and determine the values of the transport coefficients in both the high-frequency limit and the low-frequency limit. A good agreement is found with the scaling theory $\kappa/\kappa_0 = f(\omega/\omega_c)$ with only two independent parameters.

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The Darcy permeability of porous media is of prime importance in many applications of chemical and petroleum engineering, and several historical attempts have been made to relate it to geometrical parameters of the porous structure. Recently, it was recognized theoretically that ac permeability is also a powerful tool in the characterization of porous media, since it demonstrates the analogies between flow and other transport phenomena and points out the relations between the coefficients controlling them.[1-3] Oscillatory flows have been studied in straight capillaries and between two parallel plates,[4,5] and are characterized by the dynamic permeability $\kappa(\omega)$ defined by $\mathbf{U}(\omega) = -\kappa(\omega)\nabla P(\omega)/\eta$, where η is the fluid viscosity, $\mathbf{U}(\omega)$ is the average fluid velocity, and $\nabla P(\omega) = \nabla P e^{-i\omega t}$ is the ac pressure gradient applied between the two extremities of the system. For small-amplitude oscillations and negligible fluid compressibility, the only forces acting on the fluid are inertia and viscous stress, which result in a local pressure gradient given by the Stokes equation:

$$\nabla P(r) = \eta \Delta \mathbf{u}(r) - \rho d\mathbf{u}(r,\omega)/dt = \eta \Delta \mathbf{u}(r) - i\omega\rho \mathbf{u}(r).$$

The key feature of $\kappa(\omega)$ is that it undergoes a crossover from a viscous dominated regime to an inertial dominated one, for the value ω_c of the frequency at which the viscous penetration depth $\delta = (2\eta/\rho\omega)^{1/2}$ becomes of order of the space in between the solid walls. We first discuss the case of a straight capillary[5] of radius r. At low frequencies ($\omega \ll \omega_c$), the pressure drop is controlled by viscous dissipation and $\kappa(\omega)$ is dominated by the dc permeability of value $\kappa_0 = r^2/8$. However, the inertial acceleration term, $i\omega\rho u$, acts as an out-of-phase force on the fluid and induces an imaginary contribution to $\kappa(\omega)$, growing as $i\omega r^4\rho/48\eta$. At high frequencies ($\omega \gg \omega_c$), the acceleration term takes over in the bulk volume where the flow becomes potential, and $\kappa(\omega)$ becomes dominated by the imaginary part $i\eta/\rho\omega$. The viscous effects are concentrated in a viscous penetration depth of thickness δ at the wall, and result in a real part of $\kappa(\omega)$ decreasing as $(\delta/r)(\eta/\rho\omega)\omega^{-3/2}$. In the case of a porous medium, it was shown to leading order that the asymptotic behavior of $\kappa(\omega)$ is given by[1-3]

$$\kappa(\omega) = \kappa_0 + iC_1(\omega\rho/\eta), \quad \omega \to 0, \tag{1}$$

$$\kappa(\omega) = i\eta\phi/\alpha\rho\omega(1 - i\delta/\Lambda), \quad \omega \to \infty, \tag{2}$$

where κ_0 is the dc value of the permeability, C_1 is a coefficient with the dimension of $(\text{area})^2$, ϕ is the porosity of the medium, α its tortuosity, and Λ is an effective pore size (weighted volume/surface area) which dominates the transport properties. Equation (2) reflects the similarity between inertial flow and other potential transport phenomena such as electrical conductivity in the fluid phase or steady-state diffusion.

Since a porous medium has pores of different sizes, and hence of different rollover frequencies, one may expect $\kappa(\omega)$ to depend on the disorder of the porous medium. However, an important property of $\kappa(\omega)$ proposed in Ref. 3 is that it is scaled by only two independent parameters, κ_0 and $\omega_c = \eta\phi/\rho\alpha\kappa_0$, and a universal function independent of the disorder and pore microstructure. The set of parameters controlling viscous and potential transport are thus related by two ratios: $F_1 = \alpha\kappa_0^2/C_1\phi$ and $F_2 = (\phi\Lambda^2/\alpha\kappa_0)^{1/2}$, whose numerical values are nearly constant. A relation similar to the first of these ratios has also been obtained from percolation arguments.[6] Such scaling implies that the relation between viscous and potential transport may be independent of the microstructure and disorder of a porous medium, and that the dc value of the permeability may be obtained from high-frequency measurements, or from other properties (electrical and acoustical) of the porous medium.

We report here experimental measurements of dynamic permeability, performed on capillary tubes and model porous media made of fused glass beads and crushed glass of various sizes. We observe the rollover of $\kappa(\omega)$ in going from the viscous to the inertial regime. The values of the formation factor $F = \alpha/\phi$ obtained from the high-frequency behavior of $\kappa(\omega)$ is in good agreement with electrical conductivity measurements made on the same samples. Values of Λ are comparable to those obtained from acoustic measurements in similar porous materials.[1] We also observe a reasonably good two-parameter scaling of the dynamic permeability in the crossover region and over 4 decades of frequency.

Our experiments are done on samples of glass beads, or crushed glass, slightly sintered together and to the wall of a glass tube container in order to avoid free motion of the solid material. We purposely use two different type of particles so as to vary the microstructure of the sintered porous medium. The glass beads that we study have three different mean diameters: 1.7, 0.95, and 0.5 mm, with a variation of roughly 20% around that mean. For the crushed glass we use two different particle-size ranges: 350–709 μm with a mean of 530 μm, and 709–1050 μm with a mean of 880 μm. The size of the samples themselves is from 5 to 7 cm in length and 4.3 mm in diameter, and the porosity is about 50% $\pm$ 3%. We also measure the dynamic permeability of capillaries of diameter 0.5 and 1 mm.

An oscillatory flow is induced in the system by means of an audio speaker driving a soft latex membrane, as described in Ref. 7. The pressures P_2 and P_1 on each side of the sample are measured with Omega model PX170 differential pressure transducers made out of a piezoresistive silicon membrane. The high sensitivity of these transducers allows the use of small-amplitude displacements. In order to minimize dead volumes, only one side of the membrane is connected to the liquid; P_2 and P_1 are thus relative to atmospheric pressure and the pressure drop across the sample is $P_2 - P_1$. For measurement of the flow rate, a capillary of diameter 0.7 mm (referred to as the "velocity capillary") is added in series with the sample, between the transducer measuring P_1 and a reservoir at atmospheric pressure. This is the exact analog of current measurement by measuring the voltage drop across a calibrated series resistor. The flow rate $Q_{\mathrm{cap}}(\omega)$ inside this capillary is thus

$$Q_{\mathrm{cap}}(\omega) = -S_{\mathrm{cap}}\kappa_{\mathrm{cap}}(\omega)P_1(\omega)/\eta L_{\mathrm{cap}},$$

where S, $\kappa(\omega)$, and L are, respectively, the cross-sectional area, dynamic permeability, and length of the capillary. However, the flow rate in the velocity capillary is not rigourously equal to the flow rate Q in the porous sample, since the piezoresistive membrane of the transducer undergoes a small displacement with applied pressure. We correct this effect by assuming a linear relation between the volume change dV associated with the membrane deformation and the applied pressure: $dV = C dP$. The extra impedance introduced by the transducer is thus equivalent to an electrical capacitance connected to ground, and the flow rate Q in the porous sample is related to the flow rate Q_{cap} in the velocity capillary by the relation

$$Q(\omega) - Q_{\mathrm{cap}}(\omega) = dV/dt = -iC\omega P_1(\omega).$$

The validity of this linear approximation and the value of C are established by calibration of the experimental setup with a capillary of the same diameter as the velocity capillary.

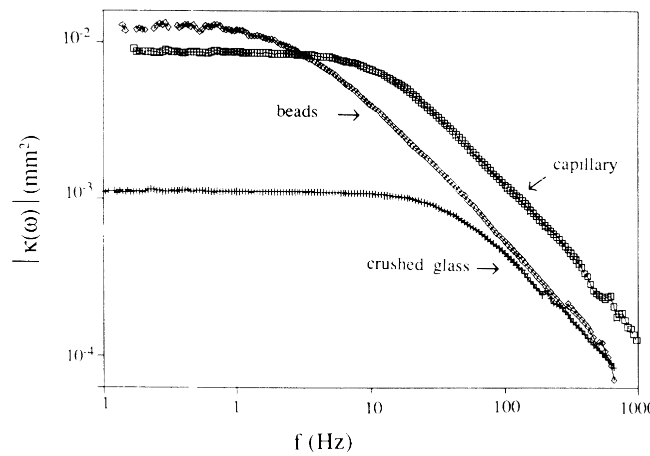

FIG. 1. The modulus of the ac permeability in mm^2 plotted as a function of frequency. □, capillary of diameter 0.508 mm; ◇, beads of diameter 1.7 mm; +, crushed glass with a particle-size range 0.350–0.710 mm.

Our experiments are made mostly with water, of kinematic viscosity $\nu = 0.96 \times 10^{-2}$ cm^2/s, but measurements in capillaries have also been made with decane ($\nu = 1.21 \times 10^{-2}$ cm^2/s). The porous samples need to be carefully saturated, since bubbles, even of small volume (a fraction of a mm^3), introduce noticeable compression effects and meniscus resonances.[7] For this reason, air is first removed from the samples by several hours of flushing with carbon dioxide, which then dissolves completely into water. The experiment is run for different displacement amplitudes, corresponding to Reynolds numbers $\mathrm{Re} = Ud/\nu$ usually less than 1 (d is the diameter of the beads or the mean particle size for crushed glass). The pressure responses as a function of frequency are recorded by a Hewlett-Packard dynamic signal analyzer, model 3625A, whose source output is used to drive the system. The frequency range that we study spans from 0.1 Hz to 1 kHz. At higher frequencies, acoustic resonances appear in the system as a result of the breakdown of the incompressibility condition.

The results obtained for the 0.5-mm capillary, for the 1.7-mm-diam beads, and for the smaller crushed glass are displayed in Fig. 1. One sees the rollover of the modulus of $\kappa(\omega)$ from a constant value at low frequencies to a ω^{-1} dependence at high frequencies. The rollover frequency ω_c increases as the dc permeability of the sample decreases. In the last frequency decade, the data are corrected for transducer dilation; the remaining small peaks give a relatively large error in the evaluation of the high-frequency coefficients which are obtained with a precision of $\pm$5%. The frequency dependences of the real and imaginary parts of $\kappa(\omega)$ are shown in Fig. 2: They are consistent with a linear dependence of $\mathrm{Im}[\kappa(\omega)]$ at low frequencies and a $\omega^{-3/2}$ dependence of

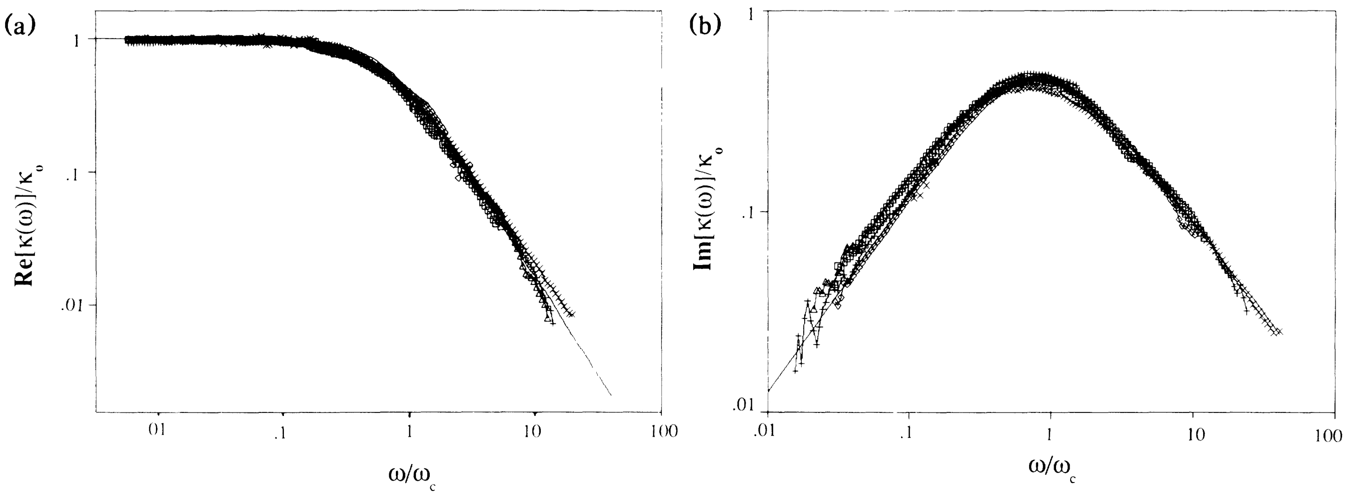

FIG. 2. (a) The real part and (b) the imaginary part of the scaled ac permeability $\kappa(\omega)/\kappa_0$ as a function of the scaled frequency ω/ω_c, for beads of diameter 1.7, 0.95, and 0.5 mm and for crushed glass with particle-size ranges 0.35–0.71 mm and 0.71–1.05 mm. The line without symbols is the theoretical scaling function. The values of κ_0 and ω_c of the samples are listed in Table I.

$\mathrm{Re}[\kappa(\omega)]$ at high frequencies, as expected from theory.[1]

Table I shows the coefficients κ_0, $F=\alpha/\phi$, Λ, and C_1 derived from the variations of $\kappa(\omega)$ with frequency. Since the media that we study are very lightly sintered, the ratio κ_0/r^2 (r being the radius of the beads or half the mean particle size in crushed glass) and the formation factor F do not vary over a wide range. The values found for F are close to those of random bead packs obtained from electrical or diffusion measurements.[8,9] In order to have a more precise comparison, we have also measured the electrical conductivity of the samples saturated with a brine solution. The values obtained tend to be slightly lower than the ones derived from ac permeability (5%), which we attribute to the fact that the pressure measurements cannot be made as close to the inlet and outlet of the medium as can be done with the electrical measurements. The Λ values vary over a wider range, which is expected since Λ measures a typical pore size. The ratio Λ/r varies between 0.4 and 0.52. These values can be compared with acoustic measurements in materials of lower porosities and much smaller grain sizes,[1,10] which yield values of Λ/r ranging between 0.15 and 0.35. They are also in very good agreement with the numerical simulations on periodic media of Ref. 3.

TABLE I. Experimental and theoretical values of the coefficients κ_0, C_1, F, and Λ, of the ratios $F_1=\kappa_0^2F/C_1$ and $F_2=(\Lambda^2/F\kappa_0)^{1/2}$, and of the rollover frequency $f_c=\eta/2\pi\rho F\kappa_0$, obtained from the variations of $\kappa(\omega)$ with ω in beads and crushed glass [Eqs. (1) and (2) in text]. The ranges of the grain sizes in crushed glass are 0.35–0.71 mm and 0.71–1.05 mm, and the "radius" r designates half of the mean particle size. F_{elec} is the formation factor obtained from electrical conductivity measurements. The two last lines as well as the last column, marked by *, are theoretical values obtained in Ref. 3 from numerical simulations of periodic porous media with microgeometries of the grain shape similar to beads (model II) or to crushed glass (model III). The values of the porosity are 0.475 (model II) and 0.5 (model III). The theoretical rollover frequency $f_c^*=\eta/2\pi\rho F\kappa_0$ is calculated for the value of the dynamic viscosity of water and values of the radii listed in column 2.

Medium	Radius r (μm)	$(\kappa_0/r)^{1/2}$	$F=\alpha/\Phi$	F_{elec}	Λ/r	F_1	C_1/κ_0^2	F_2	f_c (Hz)	f_c^* (Hz)
Beads	850	0.107	2.98	2.85	0.527	0.74	4.02	2.83	6.2	7
Beads	475	0.09	3.29	3.22	0.459	0.72	4.57	2.81	25	22.4
Beads	250	0.085	3.13	2.96	0.443	0.71	4.4	2.95	108	81
Crushed glass	440	0.074	3.24	3.04	0.42	0.62	5.22	3.17	44	41
Crushed glass	265	0.079	3.33	3.19	0.44	0.68	4.89	3.04	103	113
Model II*		0.099	3.12		0.464	0.68	4.56	2.64		
Model III*		0.084	2.81		0.64	0.556	5.057	4.53		

The two-parameter scaling of $\kappa(\omega)$ appears in Fig. 2 which shows κ/κ_0 as a function of ω/ω_c, $\omega_c = \eta/\rho F \kappa_0$. These parameters are natural scaling variables for $\kappa(\omega)$ in the low- and high-frequency limits but we find that they also scale the real and imaginary parts in the whole crossover region. The theoretically predicted scaling function, derived from the ac permeability of a straight capillary, agrees well with the experimental data over the entire frequency range. The values of the ratios $F_1 = \kappa_0^2 F/C_1$ and $F_2 = (\Lambda^2/F\kappa_0)^{1/2}$ which relate the low-frequency to the high-frequency coefficients, are, respectively, $F_1 = 0.62$ to 0.75 and $F_2 = 2.8$ to 3.2. The corresponding values for a capillary are $F_1 = 0.75$ and $F_2 = \sqrt{8}$. We find a wider variation in the ratio F_1 than in the ratio F_2. At the present we believe that this reflects the difficulty in measuring the imaginary part of $\kappa(\omega)$ at low frequency, which appears only as a correction to the dc flow and requires high-amplitude motion to be detected. The lower values of F_1 are found in crushed glass.

Our measurements of ac permeability on fused glass beads and crushed glass directly demonstrate the similarity of ac responses of a capillary and of a porous medium. The randomness, as well as the geometry of the pore microstructure, does not seem to affect the shape of $\kappa(\omega)$, which is well described by the scaling theory over an appreciable range of variation of the grain size. This result contradicts the intuition that disorder should spread out the transition region as well as the two branches of the imaginary part. However, permeability is actually determined by moments of the pore-size distribution, such as the average pore size and the average of the inverse pore size, which tend to cancel out the effect of disorder.[3] As a result, permeability appears to be controlled primarily by one typical pore size. However, our experiments have been done for a relatively narrow range of variation of the tortuosity, and future experiments will include the investigation of more disordered porous media, as well as direct comparison of the inertial response with the Biot slow-wave measurements. It would also be important to study porous media of drastically different microgeometry (such as spherical cavities connected by small holes) since a breakdown of the simple scaling theory is expected when the spatial variation of the cross section of pores is very rapid.

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