Homoclinic and Heteroclinic Chaos in a Single-Mode Laser

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Heterodyne measurements of the field of Lorenz-type far-infrared–laser intensity pulsations reveal two different kinds of chaotic attractors. At high pressure, a symmetric attractor is found (two saddle foci), representing (heteroclinic) Lorenz-type chaos. At lower pressure, in accordance with theoretical modeling, an asymmetric attractor (one saddle focus) is found, generating (homoclinic) Roessler- or Shilnikov-type chaos. Detuned lasers exhibit phase switches smaller than π as predicted.

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Far-infrared (FIR) lasers have been proposed[1] as examples of a realization of the Lorenz model[2] which describes not only convective fluid dynamics but also the behavior of a single-mode laser field interacting with a homogeneously broadened two-level medium.[3,4] Experiments on such lasers in certain pressure and pump-power ranges have shown remarkable similarities with the predictions of the Lorenz model, including characteristic time-dependent behavior (spiral intensity pulsations, odd periodic windows, inverse period doublings upon detuning of the laser) and appropriate instability thresholds.[5]

Nevertheless, considerable controversy has emerged as to whether such optically pumped lasers can be properly modeled as incoherently pumped two-level lasers.[6] The principal objection is that the laser excitation leads to coherences in the three-level system that cannot be adiabatically eliminated to reduce the equations to an equivalent two-level formulation. The three-level models differ from the Lorenz model in having lower instability thresholds, supercritical bifurcations to periodic pulsations, and (more typical) asymmetric periodic and chaotic attractors.

However, the actual laser systems are more complex than simple coherently coupled three-level systems. Complications arise from (a) detuned pumping of the Doppler-broadened pump transition to achieve unidirectional operation,[7] (b) operation in the backward emission direction, and (c) involvement of high angular momentum states with the magnetic sublevels mixed by orthogonally polarized fields.

In a recent model of single-mode FIR laser dynamics,[8] (a) and (b) have been incorporated with the result that experimental thresholds, the detuning-pump bifurcation diagram,[4] and intensity pulsation forms were satisfactorily recovered. The remaining discrepancy is that the "Lorenz-type" intensity pulsations predicted for high-pressure operation in the model arise from asymmetric "spiral" chaotic attractors of the Shilnikov or Roessler type whose trajectories revolve about only one of the two saddle foci. In contrast, the Lorenz model predicts symmetric spiral attractors which involve a switching between and spiraling about the two saddle foci. These different attractors produce the same intensity pulsations since the field sign changes of the Lorenz model at the jumps from one attractor leaf to the other are lost when squaring ($I \sim E^2$). Heterodyne measurements are therefore necessary to distinguish between these two attractor types by directly measuring the laser field.

For the measurements the $aR(4,4)$ rotational transition in $^{15}NH_3$ at 153 μm was used, pumped by the $^{13}CO_2$ 10-μm $R(18)$ line. The FIR laser was a ring resonator operated in the backward emission direction, very similar to the one described in Ref. 5. The heterodyne reference frequency was provided by the 27th harmonic of a millimeter-wave klystron (72 GHZ) whose output illuminated the Schottky barrier diode used to detect the FIR laser radiation. The 27th harmonic was generated by the nonlinear response of the diode. The heterodyne reference frequency was offset by 80 MHz from the FIR laser frequency to separate the intensity pulsation spectrum (pulsing frequency about 1 MHz with harmonics up to about 10 MHz) from the heterodyne spectrum. The intensity pulsations were recorded separately but were filtered out from the heterodyne signal which was then split and separately mixed with two signals (differing in phase by 90°) of an rf reference oscillator also set to 80 MHz. The outputs of the mixers provided in-phase and in-quadrature signals corresponding to the electric field of the laser. Thus, in principle, from the two heterodyne signals the laser phase and amplitude could be calculated as a function of time. It was found, however, that the amplitude of the harmonic generated in the diode depended on the (relatively strong) laser power incident on the diode. Thus the amplitude of the heterodyne signals was not exactly proportional to the laser field. The field amplitude therefore could not be determined as reliably as the phase.

The field, polarization, and population decay rates ($k, \gamma_\perp$, and γ_u, respectively), which are keys to the dynamics, were determined. The ratio $\gamma_\parallel/\gamma_\perp$ has been measured to be 0.25.[9] The heterodyne measurement procedure permitted us to determine the mode pulling of stable laser operation and thereby to determine the cavity linewidth. At the higher of the two pressures used for the instability studies the ratio $k/\gamma_\perp \approx 4.5$ was thereby inferred, satisfying the "bad cavity" ($k > \gamma_\perp + \gamma_\parallel$) prerequisite condition for instabilities in the Lorenz model. In terms of the standard parameters of the Lorenz model $\sigma = k/\gamma_\perp$ and $b = \gamma_\parallel/\gamma_\perp$.

Sample results for resonant tuning of the laser resonator to the center of the emission line shape are shown in Fig. 1. Under these conditions the five-equation laser model with detuning, which leads to a "complex Lorenz model,"[4] reduces to the usual Lorenz model with three real variables. Figure 1(c) shows spiral Lorenz-type intensity pulsations. The phase of the electric field shown in Fig. 1(d) is reconstructed from the in-phase and in-quadrature signals [Figs. 1(a) and 1(b)]. At the end of each expanding spiral in the intensity trace we see that the field jumps in phase by π indicating a change in sign of the field amplitude. This behavior is more nearly that of the Lorenz model than that of the most exact FIR three-level model.[8]

Frequency offsets between the heterodyne signal and the rf oscillator cause linear drifts in the phase plot which can be subtracted out. Residual variations in the frequency, resulting from the absence of active or passive stabilization of the klystron and the pump and FIR laser resonators, cause frequency changes of the order of 10 kHz/ms.

In the central 20% of the intensity pulsation plot of Fig. 1(c) there is an apparent spiral followed by two slightly shorter and one large pulse. The in-phase and in-quadrature signals do not give a clear visual explanation, but the reconstructed phase shows that what seemed to be the last pulse of the spiral was a single loop around one side of the attractor (corresponding to the spike in the phase plot), while the preceding part of the spiral and the succeeding three pulses were on the same side. This indicates the difficulty of reliably determining by eye the locations of every phase shift in an intensity plot.

For a symmetric attractor the phase jumps occur at zero amplitude and thus should alternate as $+\pi$ and $-\pi$. However, the phase is a multivalued function of the heterodyne amplitudes and thus phase jumps by $+\pi$ or $-\pi$ are indistinguishable. In the experiment, noise gives the algorithm a clear choice at each phase switching point. In addition we note that nearly symmetric attractors (indistinguishable in the form of their intensity pulsations) result from slight detunings in the Lorenz model and these have phase jumps of slightly less than π with a definite sign.[10] The nonsystematic nature of the jumps observed (neither alternate nor of the same sign) may indicate the influence of both noise and small detunings.

In contrast to Fig. 1, Fig. 2 shows results for lower pressure, $k/\gamma_\perp \sim 3.0$, where it was shown earlier[5] that the instability threshold is lower than for the Lorenz model but still consistent with the FIR model.[8] In this case, for suitably high excitation, one again finds "spiral, Lorenz-type" intensity pulsations as shown in Fig. 2(c). However, the reconstructed phase [Fig. 2(d)] shows no switching by π at the end of each spiral, though some correlated phase variations can be discerned. In this case the attractor must be "one sided" (asymmetric in the field amplitude), as predicted in the FIR model.[8]

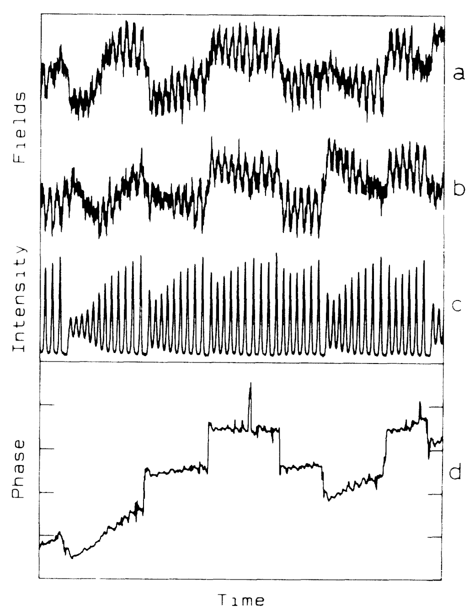

FIG. 1. High pressure ($p = 9$ Pa) chaotic pulsing of 153-μm $^{15}NH_3$ laser emission for resonant tuning. Trace marked "intensity": laser intensity pulses. Pulsing period is 1 μs. Traces marked "fields": in-phase and in-quadrature heterodyne signals measuring the laser field. Trace marked "phase": phase changes of the laser field as a function of time, reconstructed from field traces. One division on the vertical axis corresponds to a phase change of π rad.

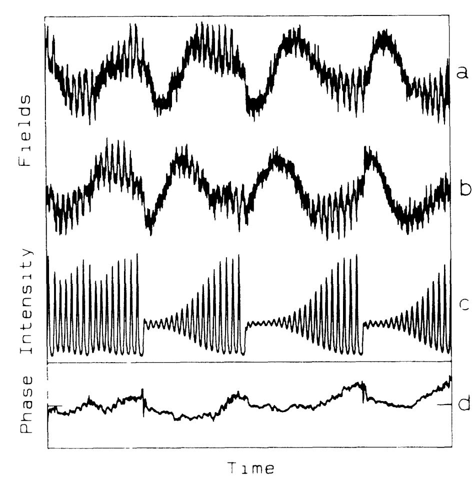

FIG. 2. Chaotic pulsing of 153-um $^{15}NH_3$-laser emission, for resonant tuning and $p = 5$ Pa. Traces marked as in Fig. 1, with same time and phase scales.

One may conclude that the pressure broadening at high pressure causes the laser to behave in a more nearly Lorenz-type fashion. Perhaps the collisional dephasing of the optically mixed m sublevels is sufficient to turn a complex three-level system into an effective two-level one.

Figure 3 shows apparent period-three intensity pulsations for the higher pressure corresponding to a symmetric attractor. The spiraling frequency about the saddle foci has locked to the sixth harmonic of the switching frequency. We have observed many such integer lockings as periodic windows in the phase space. In the nomenclature of Ref. 2b this is a "symmetric x^3y^3 attractor."

Figure 4(b) shows period-two pulsing measured with a detuned laser at high pressure while Fig. 4(a) shows period-two pulsing measured on resonance at low pressure. Phase switches different from π and different for successive pulses are observed. The results of Fig. 4(b) are in agreement with the predictions of the Lorenz model with detuning.[10]

The detuning of the laser resonator or the pump can give sufficiently large distortions that attractor symmetry does not provide an exact or approximate nonmenclature. Of course, in any of these cases one can choose the optical reference frequency to make the phase relatively constant during the intensity pulses or to make the phase periodic (requiring that it vary during each pulse). We have chosen the former for the presentations as that corresponds to using the center of the heterodyne spectrum as our reference frequency.

In summary, at high pressure, where three-level coherences are most probably destroyed by collisions, we find regular and chaotic field pulses corresponding to a double-sided attractor like the Lorenz attractor. The spiral chaos here represents a case of "heteroclinic chaos" because it involves two saddle foci. It is in this parameter region where all other experimental results agree with the predictions of the Lorenz model.[5]

At lower pressure, where presumably coherence effects are less strongly suppressed, a one-sided attractor is found in accordance with the predictions of the three-level treatments[6b,6c,8] though the results most accurately agree with the appropriate FIR model.[8] This represents "homoclinic chaos" involving only a single saddle focus. We might expect a "gluing bifurcation"[11] (which cements two asymmetric attractors into a symmetric one) such as we observe with increasing pressure to be found in a suitably more elaborate FIR laser model, as it is known generically (but not for our parameter values) in both the Lorenz[2b] and resonant three-level laser models.[6b,6c]

We believe that our low-pressure result is the first experimental observation of homoclinic chaos in a laser

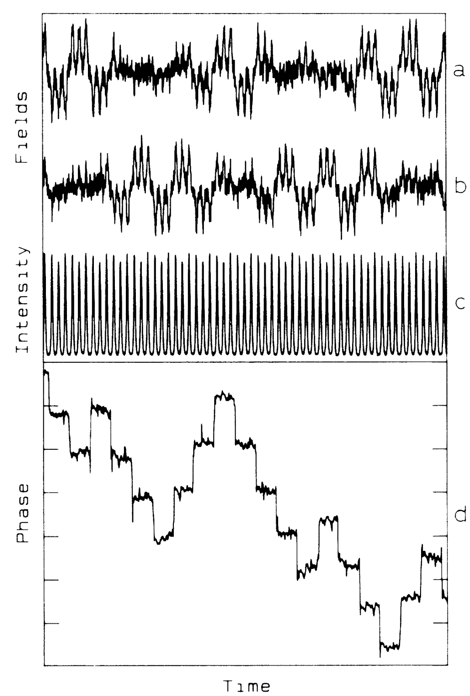

FIG. 3. A periodic window in the high-pressure chaotic range. Same conditions as Fig. 1 but slightly changed pump intensity. Traces and units as in Fig. 1. Note phase changes of π as in Fig. 1 indicating a symmetric attractor as expected for the Lorenz model.

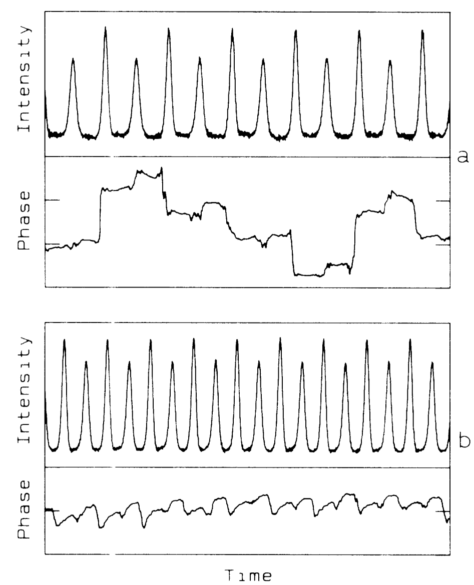

FIG. 4. Period-two pulsing. Upper part: low pressure ($p = 4$ Pa), resonant tuning. Large intensity pulses are associated with π changes of phase; small pulses are associated with changes of phase $< \pi$. Lower part: high pressure as in Fig. 1, detuned resonator. Phase changes associated with the pulses are different for the large and small pulses and always $< \pi$ indicating a one-sided attractor. This is in accordance, as in Fig. 1, with the Lorenz model extended for resonator detuning (Ref. 4).

system. Here only one saddle focus determines the dynamics. Observations of so-called Shilnikov chaos[12] in a laser with feedback and of similar intensity pulsations in models and experiments with a laser with saturable absorber[13-15] involve a homoclinic *cycle* between a saddle focus and the saddle point of the zero-intensity solutions.

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