Universality Class of the $d=2$ Θ Point of Linear Polymers

Duplantier and Saleur[1] obtained recently exact bulk and surface exponents at the tricritical Θ' point of a self-avoiding walk (SAW) on a hexagonal lattice with random forbidden hexagons. This is not the standard model for polymers at the Θ point, and, as already pointed out,[2,3] contrary to previous conjectures,[1,4] the Θ- and Θ'-point behaviors could belong to different universality classes. Here we show that this is most probably the case and that the Θ point could correspond to a nonunitary superconformal field theory[5] with central charge $C=\frac{1}{2}$. The exponents at the Θ' point, on the other hand, are obtained from a Kac table with $C=0$,[1] the value appropriate for critical SAW's.[5]

An extensive numerical investigation of the Θ point for a SAW on a square lattice with nearest-neighbor interactions was undertaken recently.[6] A very accurate Monte Carlo strategy and different methods of enumeration analysis, including an original renormalization group, allowed us to obtain $\nu=0.57\pm0.015$, $\phi=0.52\pm0.07$, and $\gamma=1.075\pm0.04$. These values, apart from slight discrepancies for ϕ and γ, do not appear inconsistent with $\nu=\frac{4}{7}$, $\phi=\frac{3}{7}$, and $\gamma=\frac{8}{7}$ of Ref. 1. The situation, however, changes drastically if one looks at the entropic γ_1 and γ_{11} exponents for walks starting at a linear boundary and never trespassing it. γ_{11} refers in particular to the subset of walks which also end at the boundary. For these Θ-point exponents, which were never investigated before, we obtain[6] $\gamma_1=0.57\pm0.09$ and $\gamma_{11}=-0.53\pm0.10$, which seem hardly compatible with the Θ'-point values $\gamma_1=\frac{8}{7}$ and $\gamma_{11}=\frac{4}{7}$.[1] This is strong evidence against the possible identity of Θ- and Θ'-point behaviors.

The above findings also give further support to a recent conjecture, according to which tricritical $O(n)$ models should correspond to superconformal invariant field theories.[7] This is sure for $n=1$ (Ref. 5) and is also consistent with numerical calculations for $n=2$.[7] The existing information leads naturally to the assumption that C becomes $\frac{1}{2}$ in the $n\to0$ limit, with bulk exponents $\nu=\frac{4}{7}$, $\phi=\frac{3}{7}$, and $\gamma=\frac{15}{14}=1.07\ldots$. As to surface exponents, the most natural assumption within the superconformal table[5,6] for $n=0$ seems to be $\gamma_1=\frac{15}{28}=0.53\ldots$ and $\gamma_{11}=-\frac{4}{7}=-0.57\ldots$. Indeed this choice is the only one satisfying $x_h'=4x_h/x_t$, a relation which is always found to be satisfied by the dimensions of energy (x_t) and bulk (x_h) and surface (x_h') magnetization operators in $d=2$ conformal invariant theories.[8] This relation is also satisfied by the exponents of the Θ' point.[1] The agreement of the above numerical values with these proposed exponents is remarkable.

Let us consider the $O(n)$ model (n-component spins $\mathbf{S}_i$, $\mathbf{S}_i^2=n$ at each site i) with annealed vacancies. This model should reproduce linear polymer statistics, including the Θ point, for $n\to0$. The partition function reads

$$Z=\mathrm{Tr}_{\{t\}}\int\prod_i d\Omega_i \exp\left[K\sum_{\langle ij\rangle}t_it_j\mathbf{S}_i\cdot\mathbf{S}_j+L\sum_{\langle ij\rangle}t_it_j-\Delta\sum_i t_i\right], \tag{1}$$

where the trace is over the lattice-gas variables ($t_i=0,1$) and angles Ω_i of the spins. It is easy to show that in the $n\to0$ limit, Z reduces to the partition function of an Ising lattice gas, while $\langle\mathbf{S}_i\cdot\mathbf{S}_j\rangle/n$ becomes an interacting SAW correlation function. The C of the tricritical $O(n\to0)$ model, as given, e.g., by the finite-size scaling correction to the free energy of an infinite strip of width L, must then be identified with the C of one of the possible fixed points of the Ising model. Inspection of $\langle\mathbf{S}_i\cdot\mathbf{S}_j\rangle/n$ at the paramagnetic and infinite- and zero-temperature fixed points singles out the critical Ising fixed point as the natural candidate to represent the Θ point and leads one to assume for it $C=\frac{1}{2}$.[7] Indeed the correlations at the other fixed points with $C=0$ represent either compact or purely self-repelling chains, and the critical Ising fixed point is left by exclusion as the only possibility.

In conclusion, we have presented strong evidence that Θ and Θ' points belong to different universality classes. Numerical results and theoretical arguments also support the conjecture that the Θ point could correspond to a superconformal field theory with $C=\frac{1}{2}$.

F. Seno,[(1)] A. L. Stella,[(2)] and C. Vanderzande[(3)]
[(1)]Instituut voor Theoretische Fysica
Katholieke Universiteit
B-3030 Heverlee, Belgium
[(2)]Dipartimento di Fisica dell'Università and
Centro Interuniversitario di Struttura della Materia
I-35131 Padova, Italy
[(3)]Limburgs Universitair Centrum
Universitaire Campus
B-3610 Diepenbeek, Belgium

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[8]The dimension of an operator is given by $x=d-y$, where y is the scaling exponent of the conjugated field.